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Research Article A Novel Fuzzy Mathematical Method to the Sensory Evaluation of Wine

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Abstract: The aim of this study is to propose a new sensory evaluation method of wine. Sensory evaluation of wine contains four evaluating factors, which are color, aroma, taste and style. Sensory evaluation values given by experts are usually expressed with linguistic terms, which are more suitable depicted by triangular numbers than crisp numbers. The new evaluation method is developed with the concept of TOPSIS combining with fuzzy AHP method. The fuzzy AHP method is used to determine the weights of attributes and TOPSIS method is used to rank all the wine samples. An applied example is given to verify the new method and the result shows that the proposed method is effective and feasible.

Keywords: AHP method, fuzzy, linguistic term, sensory evaluation, TOPSIS, triangular fuzzy number

INTRODUCTION

Tasting is the main method to judge the quality of wine. However, there is a lot of uncertainty during the judgment. Sensory evaluation of wine depends on different fermenting conditions with color, aroma, taste and style used as the evaluating factors (Huang et al., 2014). In recent years, the method of sensory evaluation has been received great attention for determining the quality of products in various fields, such as food, cosmetic, medical, chemical and textile (Zolfaghari et al., 2014). Seneory evaluation method is defined as a scientific evaluation method, which is firstly gathering the information provided by a group of experts through the senses of sight, smell, touch, taste, touch and hearing and the using some evaluation rules to give the final result (Stone and Sidel, 2004; Martínez, 2007). The sensory evaluation technique has also many applications for the assessment of food quality (Debjani et al., 2013; Sinija and Mishra, 2011; Martínez et al., 2008; Wei et al., 2014). For the assessment of wine quality, Ren and Yang (2014) developed a fuzzy multiattribute group decision model for wine evaluation based on the sensory data expressed with interval numbers. Wang and Ren (2014) put forward a evaluation method combining TOPSIS with group eigenvalue method for wine evaluation model. For a sensory evaluation problem, there are many evaluation criteria (attributes), such as color, aroma, taste, etc. Thus a sensory evaluation problem is actually a Multi-Attribute Decision Making (MADM) problem. In recent years, various MADM methods are developed form crisp environment to fuzzy environment. Fuzzy MADM methods are also be applied to solve various decision problems, such as material selection, medical diagnosis, supplier selection and water quality

evaluation (Jahan and Edwards, 2013; Chen *et al.*, 2013; Igoulalene *et al.*, 2015; Ji *et al.*, 2015).

The main task of the article is to propose a new sensory evaluation method, which is developed from the concept of TOPSIS method with the theory of triangular fuzzy numbers. The quality evaluation of wine samples is used as an application of the new fuzzy sensory evaluation method.

MATERIALS AND METHODS

In this section, we firstly recall some concepts and operation laws of triangular fuzzy numbers and then we will construct a multi-attribute decision making model for wine sensory evaluation problem on the basis of sensory data.

Definition 1: For three given real numbers *a*, *b* and *c* and they satisfy $a \le b \le c$. A triple $\tilde{A} = (a, b, c)$ is called triangular fuzzy number, if its membership function defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x \le a \\ \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \\ 0, x \ge c \end{cases}$$
(1)

The figure of a triangular fuzzy number $\tilde{A} = (a, b, c)$ is illustrated in Fig. 1.

Definition 2: Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the operation laws of these two fuzzy numbers are defined as follows:

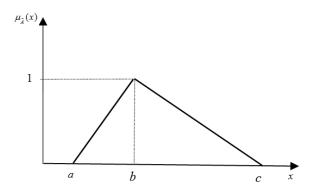


Fig. 1: The membership function of triangular fuzzy number $\tilde{A} = (a, b, c)$

Table 1: Linguistic terms with corresponding triangular fuzzy number
Linguistic terms of rating of sensory

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attribute	Triangular fuzzy number
Poor (P)	(0,0,25)
Fair (F)	(0,25,50)
Good (G)	(25,50,75)
Very good (VG)	(50,75,100)
Excellent (E)	(75,100,100)

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$k\tilde{A} = (ka_1, ka_2, ka_3), k \in \mathbb{R}$$

$$\tilde{A} / \tilde{B} = (a_1 / b_3, a_2 / b_2, a_3 / b_1)$$

Definition 3: Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then the distance between them is defined as follows (Chen, 2000):

$$d(\tilde{A},\tilde{B}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}$$

Owing to the fuzziness of the sensory evaluation problem, the ratings of sensory attribute are often represented with linguistic terms provided by experts. In this study, the linguistic terms and corresponding triangular fuzzy numbers are illustrated in Table 1.

Now, we will construct a MADM model for wine evaluation based on sensory data as follows:

Consider a sensory evaluation problem with respect to wine. Let $X = \{x_1, x_2, ..., x_m\}$ be possible alternatives (wine sample) set and $O = \{o_1, o_2, ..., o_n\}$ be the sensory evaluation attribute set with which alternative wine samples are evaluated. $D = \{D_1, D_2, ..., D_s\}$ are the expert set. Suppose the rating of alternative $x_i(i = 1, 2, ..., m)$ with respect to attribute $o_j(j = 1, 2, ..., n)$ given by expert $D_k(k = 1, 2, ..., s)$ is a linguistic term noted by \tilde{S}_{ij}^k , which belongs to the linguistic terms set {Poor, Fair, Good, Very Good, Excellent}. Then by using Table 1, \tilde{S}_{ij}^k can be expressed with triangular fuzzy number $\tilde{S}_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k)$. Let \tilde{S}_{ij} be the total sensory score of sample x_i with respect to attribute o_j and it is defined as:

$$\tilde{s}_{ij} = (s_{ij}^{l}, s_{ij}^{m}, s_{ij}^{u}) - \frac{1}{s} \sum_{k=1}^{s} \tilde{s}_{ij}^{k}$$
(2)

Then the wine evaluation model can be regarded as a MADM model with the following decision matrix format:

$$\tilde{S} = (\tilde{s}_{ij})_{m \times n} = \begin{array}{cccc} & o_1 & o_2 & \cdots & o_n \\ & x_1 \begin{pmatrix} \tilde{s}_{11} & \tilde{s}_{12} & \cdots & \tilde{s}_{1n} \\ & \tilde{s}_{21} & \tilde{s}_{22} & \cdots & \tilde{s}_{2n} \\ & \vdots & \vdots & \vdots & \vdots \\ & x_m \begin{pmatrix} \tilde{s}_{m1} & \tilde{s}_{m2} & \cdots & \tilde{s}_{mn} \end{pmatrix} \end{array}$$

Let $\widetilde{w} = (\widetilde{w}_1, \widetilde{w}_2, ..., \widetilde{w}_n)$ be the attribute weight vector and each element \widetilde{w}_j is depicted with a linguistic term and it represents the important degree of attribute. Similarily \widetilde{w}_j can also be rewritten with triangular fuzzy number $\widetilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$. In the following discussion, we will first give the description of Fuzzy AHP method and then develop a new sensory evaluation method for wine quality evaluation.

Fuzzy AHP method: AHP method is well-known decision making method and it has been widely applied in various decision problems. With the increasing complexity of objective thing and the limitation of time and knowledge of decision, in the evaluation process, the decision makers often assign fuzzy concept to the rating of evaluation attribute. Thus Chang (1996) developed the fuzzy AHP (FAHP) method to decision making problem in which attribute values expressed with triangular fuzzy numbers. Tang and Beynon (2005) used FAHP method to the application of a capital investment study. Hsieh et al. (2004) presented a fuzzy multi-criteria analysis approach for selecting of planning and design alternatives in public office buildings. The FAHP method is used to determine the weightings for evaluation criteria. Naghadehi et al. (2009) used FAHP method to the selection of optimum underground mining problem. The detail description of FAHP is given as follows:

Let $X = \{x_1, x_2, ..., x_n\}$ be an object set and $U = \{g_1, g_2, ..., g_m\}$ be a goal set, according to the method of Chang (1996) extent analysis, each object is taken and extent analysis is performed for each goal g_i , respectively. Therefore, *m* extent analysis values for each object can be obtained and shown as follows:

$$M_{gi}^{1}, M_{gi}^{2}, ..., M_{gi}^{n}, i = 1, 2, ..., m$$

where all the M_{gi}^{j} (j = 1, 2, ..., m) are triangular fuzzy numbers. Then the weighting method based on FAHP is given as follows:

• Compute the value of fuzzy synthetic extent with respect to the *i*th object according to the following equation:

$$\tilde{S}_{i} = \left(\sum_{j=1}^{n} M_{gi}^{j}\right) \otimes \left(\sum_{i=1}^{m} \sum_{j=1}^{n} M_{gi}^{j}\right)^{-1}, i = 1, 2, ..., m$$
(3)

To obtain $\sum_{j=1}^{n} M_{gi}^{j}$ perform the fuzzy addition

operation of m extent analysis values for a particular matrix such that:

$$\sum_{j=1}^{n} M_{gi}^{j} = \sum_{j=1}^{n} \tilde{a}_{ij} = \left(\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij}\right)$$
(4)

Then using the operation law of fuzzy numbers, we can obtained the values of \tilde{S}_i .

• Compute the degree of possibility of $\tilde{S}_2 \geq \tilde{S}_1$ as follows:

$$V(\tilde{S}_{2} \ge \tilde{S}_{1}) = \begin{cases} 1, & m_{2} \ge m_{1} \\ \\ \frac{u_{2} - l_{1}}{(u_{2} - m_{2}) - (l_{1} - m_{1})}, otherwise \\ 0, & l_{1} \ge u_{2} \end{cases}$$

where, $\tilde{S}_{i} = (l_{i}, m_{i}, u_{i})$ and i = 1, 2.

• Compute the weights $w_i(i = 1, 2, ..., n)$ with the following formula:

$$w_{i} = \frac{M_{i}}{\sum_{i=1}^{n} M_{i}} (i = 1, 2, ..., n)$$
(5)

where, $M_i = V(\tilde{S}_i \ge \tilde{S}_j | j = 1, 2, ..., n; j \ne i$. That is:

$$M_{i} = \min_{j=1,2,\dots,n; j \neq i} V(\tilde{S}_{i} \ge \tilde{S}_{j})$$
(6)

and M_i represents the degree of possibility of \tilde{S}_i be greater than all the other fuzzy number $\tilde{S}_j (j = 1, 2, ..., n, j \neq i)$.

Note that, in the process of using the FAHP method, we need first transform the linguistic scales into triangular fuzzy numbers as reported in Table 2.

New fuzzy sensory evaluation method: This section will develop a new fuzzy sensory evaluation method of

Table 2: FAHP scales and corresponding to Triangular Fuzzy Numbers (TFN)

1 (41)		
AHP scales	Linguistic scales	TFN scales
1, 1, 1	Equal importance(EI)	(1, 1, 1)
2, 3, 4	Weak importance (WMI)	(1/2, 1, 3/2)
4, 5, 6	Strong importance (SMI)	(3/2, 2, 5/2)
6, 7, 8	Demonstrated importance (VSMI)	(2, 5/2, 3)
8, 9, 10	Absolute importance (AMI)	(5/2, 3, 7/2)

wine based on the concept of close value method. The detail steps of the new method are given as follows:

- Step 1: Collect the attribute values into a fuzzy decision matrix $\tilde{S} = (\tilde{s}_{ii})_{m \times n}$, where
 - $\tilde{s}_{ij} = \frac{1}{s} \sum_{k=1}^{s} \tilde{s}_{ij}^{k}$ be the total sensory score of

sample x_i with respect to attribute o_j .

- **Step 2:** Calculate the sensory attributes' weights using FAHP method.
- **Step 3:** Calculate weighted decision matrix $\tilde{Z} = (\tilde{z}_{ii})_{m \times n}$, where $\tilde{z}_{ii} = w_i \tilde{s}_{ii}$
- **Step 4:** Calculate the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS):

The PIS is defined as $x^+ = (\tilde{z}_1^+, \tilde{z}_2^+, ..., \tilde{z}_n^+)$ and the NIS is defined as $x^- = (\tilde{z}_1^-, \tilde{z}_2^-, ..., \tilde{z}_n^-)$, where for any j(j = 1, 2, ..., n),

$$\tilde{z}_{j}^{+} = (z_{j}^{+l}, z_{j}^{+m}, z_{j}^{+u}) = (\max_{i} z_{ij}^{l}, \max_{i} z_{ij}^{m}, \max_{i} z_{ij}^{u})$$
$$\tilde{z}_{j}^{+} = (z_{j}^{-l}, z_{j}^{-m}, z_{j}^{-u}) = (\min_{i} z_{ij}^{l}, \min_{i} z_{ij}^{m}, \min_{i} z_{ij}^{u})$$

Step 5: Calculate the distance between alternative x_i with the PIS and the distance between alternative x_i with NIS as follows:

$$d(x_i, x^+) = \sum_{j=1}^n d(\tilde{z}_{ij}, \tilde{z}_j^+), d(x_i, x^-) = \sum_{j=1}^n d(\tilde{z}_{ij}, \tilde{z}_j^-)$$

where the distance $d(\cdot, \cdot)$ is defined as in definition 3.

Step 6: Calculate the relative closeness degree.

$$C_{i} = \frac{d(x_{i}, x^{-})}{d(x_{i}, x^{+}) + d(x_{i}, x^{-})}, \quad i = 1, 2, ..., m$$

Step 7: Rank all the wine samples $x_i(i = 1, 2, ..., m)$ according to $C_i(i = 1, 2, ..., m)$. The larger of the value of C_i the better of the sample.

RESULTS AND DISCUSSION

To illustrate the effectiveness and feasibility of the proposed sensory evaluation method, an example of

Table 3: Relative importance degree of the four attributes

Sensory attributes	Color	Aroma	Taste	Style
Color	EI	WMI	WMI	EI
Aroma	SMI	EI	WMI	SMI
Taste	VSMI	SMI	EI	VSMI
Style	EI	WMI	WMI	EI

Table 4: Sum of number of judges for particular quality attributes of wine samples

		Sum of number of judges				
Sensory attribute	Sample	 Р	F	G	VG	E
o_1	x_1	0	1	4	8	5
	x_2	0	2	6	8	2
	x_3	1	5	8	2	2
	x_4	0	2	8	6	2
o_2	x_1	0	1	10	4	3
	x_2	0	2	4	9	3
	x_3	0	2	8	6	2
	x_4	0	2	10	5	1
03	x_1	0	2	5	5	6
	x_2	1	0	4	10	3
	x_3	0	1	8	7	2
	x_4	0	2	9	4	3
04	x_1	0	7	5	5	1
	x_2	0	1	3	12	2
	x_3	0	2	6	6	4
	x_4	0	1	8	8	1

wine quality sensory evaluation is used to analysis. The example is given as follow:

Sensory preference of wine was dependent on its color, aroma, taste and style (Huang et al., 2014). A company wants to evaluate the wine quality of four produced wine samples x_1 , x_2 , x_3 , x_4 . They hire 18 judges (experts) to evaluate these samples according to four sensory attribute: color (o_1) , aroma (o_2) , taste (o_3) and style (o_4) . Their relative importance given by decision maker is reported in Table 3. Sensory evaluation values assigned to each of the quality attributes are linguistic terms: Poor (P), Fair (F), Good (G), Very good (VG) and Excellent (E), the evaluation values are shown in Table 4.

To rank the quality of these wine samples, the proposed sensory evaluation method is used to solve as follows:

Step 1: Calculate the total sensory score \tilde{S}_{ij} of sample x_i with respect to attribute o_i according to Eq.

	01	02	03	04
x_1	(45.8333,68.0556,83.3333)	(31.9444,54.1667,72.2222)	(40.2778,62.5000,76.3889)	(20.8333,43.0556,63.8889)
x_2	(38.8889,61.1111,80.5556)	(40.2778,62.5000,80.5556)	(43.0556,63.8889,81.9444)	(44.4444,66.6667,86.1111)
x_3	(16.6667,37.5000,59.7222)	(31.9444,54.1667,75.0000)	(34.7222,56.9444,76.3889)	(38.8889,61.1111,77.7778)
x_4	(30.5556,52.7778,72.2222)	(26.3889,48.6111,69.4444)	(30.5556,52.7778,72.2222)	(36.1111,58.3333,79.1667)

(2). The sensory evaluation decision matrix $\tilde{S} = (\tilde{s}_{ii})_{4 \times 4}$ is reported in Table 5.

Step 2: According to Table 2 and 3 and FAHP method, the weights of attributes are obtained as follows:

 $w_1 = 0.1060, w_2 = 0.3212, w_3 = 0.4669, w_4 = 0.1060$

Step 3: The weighted decision matrix
$$\tilde{Z} = (\tilde{z}_{ij})_{4\times 4} = (\tilde{w}_j \tilde{s}_{ij})_{4\times 4}$$
 is reported in Table 6.

Step 4: The PIS and NIS are obtained as:

$$\begin{split} x^- &= (\tilde{z}_1^-, \tilde{z}_2^-, \tilde{z}_3^-, \tilde{z}_4^-) \\ &= ((1.7660, 3.9734, 6.3280), (8.4761, 15.6139, 22.3056), \\ &\rightarrow (14.2659, 24.6411, 33.7194), (2.2075, 4.5621, 6.7695)) \\ x^+ &= (\tilde{z}_1^+, \tilde{z}_2^+, \tilde{z}_3^+, \tilde{z}_4^+) \\ &= ((4.8564, 7.2110, 8.8298), (12.9372, 20.0750, 25.8745), \\ &\rightarrow (20.1019, 29.8287, 38.2585), (4.7092, 7.0639, 9.1242)) \end{split}$$

Step 5: The distances of alternative x_i with the PIS and NIS are respectively obtained as follows:

> $d(x_1, x^+) = 6.8460, d(x_2, x^+) = 0.6244$ $d(x_3, x^+) = 9.3624, d(x_4, x^+) = 11.7225$ $d(x_1, x^-) = 8.3784, d(x_2, x^-) = 14.2108,$ $d(x_3, x^-) = 5.5080, d(x_4, x^-) = 3.0953$

Step 6: The relative closeness degrees are obtained as follows:

$$C_1 = 0.5503, C_2 = 0.9579,$$

 $C_3 = 0.3704, C_4 = 0.2089$

Then we can rank the wine samples as $x_2 \succ x_1 \succ x_3 \succ x_4$ and the quality of sample x_2 is the best wine sample.

Table 6: Weighted sensory evaluation decision matrix

	01	02	03	04
x_1	(4.8564,7.2110,8.8298)	(10.2606,17.3984,23.1978)	(18.8050,29.1802,35.6647)	(2.2075, 4.5621, 6.7695)
<i>к</i> ₂	(4.1206,6.4752,8.5355)	(12.9372,20.0750,25.8745)	(20.1019,29.8287,38.2585)	(4.7092,7.0639,9.1242)
3	(1.7660,3.9734,6.3280)	(10.2606,17.3984,24.0900)	(16.2112,26.5864,35.6647)	(4.1206, 6.4752, 8.2412)
4	(3.2376,5.5922,7.6525)	(8.4761,15.6139,22.3056)	(14.2659,24.6411,33.7194)	(3.8263,6.1809,8.3883)

CONCLUSION

The problem of wine sensory evaluation contains many quality attributes which can not or be difficult to be depicted by crisp numbers. Linguistic terms are suitable to deal with this situation and in real decision process, they are often be transformed into triangular fuzzy numbers. Thus this study proposed a new sensory evaluation method based on TOPSIS combining with fuzzy AHP method. Fuzzy AHP method is used to determine the weights of sensory attributes. An applied example of wine quality evaluation shows that the proposed method is effective and feasible. The proposed sensory evaluation method can also be to applied to other decision problems, such as material selection, risk project investment and supplier selection.

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