Research Journal of Applied Sciences, Engineering and Technology 13(1): 64-74, 2016
DOI:10.19026/rjaset.13.2891
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: December 30, 2015 Accepted: April 22, 2016 Published: July 05, 2016

Research Article

The Multi-objective Optimization by the Restricted Area Method to Determine the Technological Mode of Cold Drying Process of Carrot Product

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Abstract: Finding the technological mode of cold drying process of carrot product was the major aim of this study. The experiments were carried out according to experimental plannings. Results obtained were to build the multi-objective optimization problem to describe relationships between objective functions with technological factors (temperature of moisture condensation, temperature of cold drying chamber, velocity air (or drying agents) and time of cold drying) of cold drying process of carrot product. By the Restricted Area Method (RAM), solving the multi-objective optimization problem was found out the technological mode of the cold drying process of carrot product as follows: temperature of moisture condensation was $Z_1^{opt} = 15.62°C$, temperature of cold drying chamber was $Z_2^{opt} = 35.79°C$, velocity air (or drying agents) was $Z_3^{opt} = 11.74m/s$ and the time of cold drying process was $Z_4^{opt} = 16.05h$. Corresponding to these optimal factors, the objective functions reached the minimum value in terms of the final product, including the energy consumption of $y_{1P}^R = 1.62kWh/kg$, the residual water content of $y_{2P}^R = 4.52\%$, the anti-rehydration capacity of $y_{3P}^R = 6.43\%$ (Correspondingly IR = 93.57%) and the loss of total β-caroten inside carrot of $y_{4P}^R = 4.45\%$.

Keywords: Carrot cold drying, dried carrot, multi-objective optimization problem for cold drying process of carrot, optimization the cold drying process, optimization the cold drying process of carrot

INTRODUCTION

The carrots are a kind of vegetable, they have been grown very popularly for thousands of years. Originally, carrots have been cultivated in central Asian, Middle Eastern countries and along with parts of Europe. These original carrots are only the bright orangre and look different from common carrots. They have many colour such as featuring red, purple and yellow that we find them in supermarkets. Carrots were cultivated widely in Europe during the 15th and 16th centuries. Firstly they were brought over to North America to grow during this same general time period (Benjamin *et al*., 1997). Currently, carrots are grown popularly in Southeast Asia (such as Malaysia, Indonesia, Laos, Cambodia, Thailand, Myanmar and Vietnam), tropical countries, China and Brazil. In general, in Vietnam, carrots are very pupolarly planted from north to south.

The carrots have many important nutritional substances for human's health, including: protein, lipid, sugar, carbohydrate, dietary fiber and mineral salts. In addition, they contain many bioactive compounds that have extremely good effect on human's health such as β-carotene, vitamins and enzymes (Rubatsky *et al*., 1999; Ross, 2005; Bradeen and Simon, 2007; Simon *et al*., 2008), carrot product in Viet Nam can see in Fig. 1. The ratio of β-carotene components of dry weight in carrot is very high. According to analytical results of Lab room at HCMC University of Technology and Education, the basic chemical composition of carrot product in Viet Nam is presented in Table 1 to 3.

Fig. 1: Carrots were harvested

From Table 1 to 3, they are obvious that carrot product in Viet Nam contains many important bioactive

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Table 1: The basic chemical composition of carrot product in Viet Nam

Substance	Unit	Its weight contents 100g of initial material
Water	g	79.06
Carbohydrate	g	9.6
Sugar	g	4.7
Dietary fiber	g	2.8
Protein	g	2.6
Fat	g	0.24
Minerals (Ash)	g	1.0
β-caroten	μg	8285

Table 2: The vitamins composition of carrot product in Viet Nam

Substance	Unit	Its weight contents 100g of initial material
Vitamin A	μg	835.0
Vitamin B1	mg	0.066
Vitamin B2	mg	0.058
Vitamin B3	mg	0.983
Vitamin B5	mg	0.275
Vitamin B6	mg	0.138
Vitamin B_c	μg	19.00
Vitamin C	mg	5.900
Vitamin E	mg	0.660
Vitamin K	μg	13.20

Table 3: The minerals composition of carrot product in Viet Nam

Substance	Unit	Its weight contents 100g of initial material
Calcium	mg	33
Iron	mg	0.3
Magnesium	mg	12
Manganese	mg	0.143
Phosphorus	mg	35
Potassium	mg	320
Sodium	mg	69
Zinc	mg	0.24
Fluoride	μg	3.2

compounds and they have high ratio inside carrot product but the most improtant compound inside carrot product is still β-caroten. Because carrot's characteristic has bright orange colour from β-carotene. On the other hand, β-carotene is not only easily metabolized but also antioxidized. For this reason, β-carotene is very good for human's health. However, carrot product is a very advantageous environment in order that microorganism grows up and develops. If carrots (or carrot product) are not preserved, they will be easily decomposed or hydrolyzed and oxidized, they will be no longer value of use (Ross, 2005; Simon *et al*., 2008; Sharma *et al*., 2012).

In the fact that, there are two methods to apply for preserving carrot product, those are the cooling preservation method and the drying method. Fristly the cooling preservation method, carrot product must be preserved in suitable environment. Temperature of preservation environment is maintained from 0^0C to 10°C during use time and export time. As a result, it makes to increase the expenditure of preservation carrot product. Secondly, the drying method are used the most popular. The carrot product after the drying placed in nylon bags and seaming, it is preserved in usual environment of 25°C. Thus, it will be not lost the expenditure for preservation process (Dzung and Ba, 2007; Haugvalstad *et al*., 2005). Currently, there are many different drying methods to preserve carrot product, quality carrot produc after drying depend on very much temperature of drying chamber. Therefore, the aim of this research work is study to apply the cold drying method to preserve carrot product because this method can reduce temperature of cold drying chamber as well as reduce the loss of quality carrot product, (Holman, 1986).

According to research results of Luikov (1975), Holman (1986), Gebhart (1993), Heldman and Lund (1992), Dzung *et al*. (2012) and Dzung (2014), they were obvious that these researches established and solved the mathematical models about heat and mass transfer in the cold drying process of many different types of drying materials. Results obtained were used to describe the kinetics and set up the technological mode of the cold drying process, but the assessment of the qualified products via the cold drying mode reaching the objectives such as minimum energy consumption or residual water content or the anti-rehydration capacity or the loss of total β-carotene in carrot of cold-dried product (final product) still remained unsolved.

According to Dzung and Dzung (2011), Dzung *et al*. (2011a, 2011b), Dzung (2011, 2012b) and Dzung and Ba (2007), the cold drying process is very complicated, it depends on very technological factors such as: temperature of moisture condensation (Z_1, °C), temperature of cold drying chamber (Z_2, °C), velocity drying agents (Z_3, m/s) and time of cold drying process (Z_4, h). The problem posed here is how to determine the technological mode for the cold drying process of carrot product in order that carrot after cold drying have the best quality, which mean we determine optimal technological factors in order that the outputs reach the minimal level (Fig. 2), including: the energy consumption per weight (y_1, kWh/kg), the residual water content (y_2, %), the anti-rehydration capacity (y_3, %) and the loss of total β-carotene in carrot (y_4, %) of the final product), (Dzung, 2011).

From Fig. 2, it can be obvious that problem determine the technological mode of cold drying process, which mean we need to solve the multi-objective optimization problem. This is problem that appears regularly in reality and in different fields. In this study, the multi-objective optimization problem for the cold drying process of carrot product was solved by the RAM. The rsults obtained were used to establish the technological mode of cold drying process of carrot product which was the closest to the utopian point but the furthest from the restricted area C, (Dzung, 2012a, 2012b, 2014; Dzung *et al*., 2012, 2015; Luc *et al*., 2013).

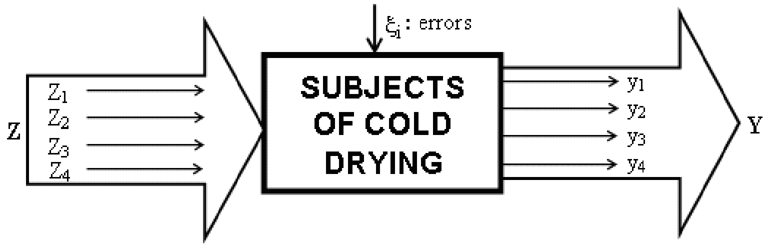

Fig. 2: Diagram of subjects of cold drying process

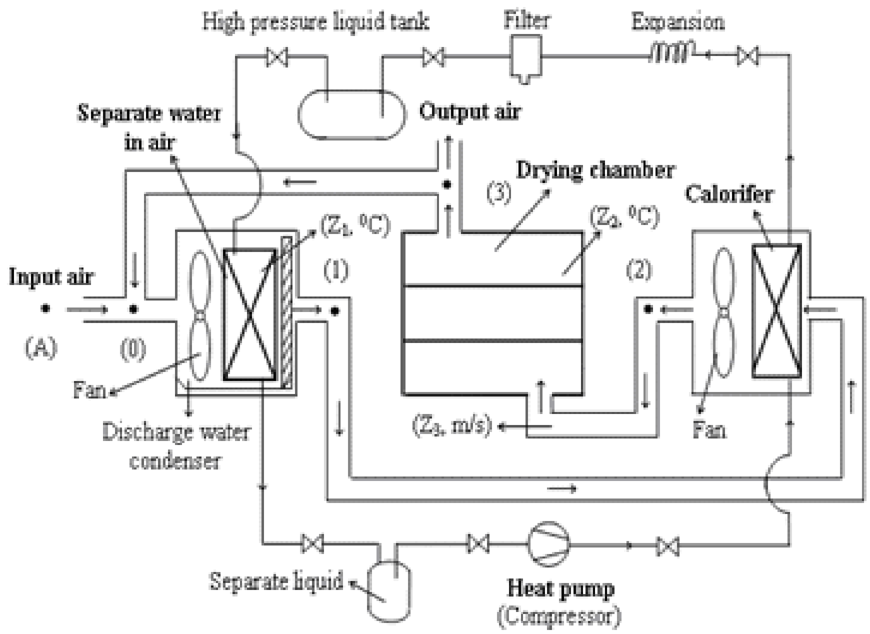

Fig. 3a: The cold drying system DSL–02

Fig. 3b: The cold drying system DSL–02

MATERIALS AND METHODS

Materials:

- The materials used for the cold drying experiments were nature carrot, mainly grown in Viet Nam (Dzung and Ba, 2007).
- Before the cold drying process, carrots were separated skin and washed, put on shells to remove water, after that cutting thin slice of carrot, It's the water content was 79.06% (Dzung and Ba, 2007).

Apparatus:

- The cold drying system DSL-02 controlled by computer was used to dry carrot product (Fig. 3a, 3b and 4).
- Determining the weight of samples by Satoriusbasic Type BA310S and mass sensor with the range of 0 to 300g and the error of 0.1g.
- Determining the volume of samples by Cylinders with the range of 0 to 500ml and the error of 0.1g.
- Dual digital thermometer (T.P.34-23) and temperature sensor were used to determine the temperature of moisture condensation, the temperature of cold drying chamber during the cold drying process with the range of 0 to 100^0C and the error of 0.5 °C.
- Determining time of the cold drying process of carrot product by timer.
- Determining velocity drying agents by veloccity sensor (DMK-045) with the error of 0.01m/s.
- The equipments of High Performance Liquid Chromatography (HPLC) were used to determine the content of β-carotene inside carrot product.

Methods:

The energy consumption: (y_1, kWh/kg final product of dried carrot) for 1 kg final product was determined by Watt meter, (Figura and Teixeira, 2007; Dzung, 2011; Dzung *et al*., 2011a):

$$y_1 = \frac{U.I.\tau.\cos\varphi}{G} \quad (1)$$

where,

G (kg): Weight of the final product (weight of carrot product after cold drying)
U (V) : Number of Voltmeter
I (A) : Number of Amperemeter
τ (s) : Second; cosφ-power factor

The residual water content of the final product (y_2, %) was determined by the mass sensor controlled by computer, (Figura and Teixeira, 2007; Dzung and Dzung, 2011; Dzung *et al*., 2011a, 2011b, 2012; Dzung and Du, 2012; Dzung, 2011, 2012a, 2012b):

Fig. 4: Final carrot product of cold drying process

$$y_2 = 100 - \frac{G_i}{G_e}\left(100 - W_i\right) \quad (2)$$

The anti-rehydration capacity of the final product (y_3, %) was indirectly determined by IR (%), which is the rehydration capacity of the final product: y_3 = 100-IR, (Figura and Teixeira, 2007; Dzung and Dzung, 2011; Dzung *et al*., 2011a, 2011b, 2012; Dzung and Du, 2012; Dzung, 2011, 2012a, 2012b):

$$IR = \frac{G_1 - G_e}{G_i - G_e}.100\% \quad (3)$$

$$y_3 = 100 - IR = \frac{G_i - G_1}{G_i - G_e}100\% \quad (4)$$

where,

G_i (kg) : Weight of carrot product before cold drying
G_e (kg) : Weight of the final product (weight of carrot product after cold drying)
G_1 (kg) : Weight of the final product which was soaked into the water at 25°C until the constant mass (the saturation of the water content)
W_i (%) : Initial water content of carrot product before cold drying (the material).

The ideal rehyration capacity of the product means that the in-water content is equal to the out-water content of the product, i.e. $G_1 = G_i$ and $IR_{max} = 1 = 100\%$, $y_{3min} = 0$. In fact, $y_3 > 0$, IR<100%.

The loss of total β-carotene in carrot of the final product (y_4, %) was determined by HPLC method in TCVN 4715–90 (Figura and Teixeira, 2007; Dzung and Dzung, 2011; Dzung *et al*., 2011a, 2011b, 2012; Dzung and Du, 2012; Dzung, 2011, 2012a, 2012b):

$$y_4 = \frac{m_1 - m_2}{m_1}100\% = \frac{\Delta m}{m_1}100\% \quad (5)$$

where, y_4-The loss of total β-carotene in carrot after cold drying; m_1 and m_2 (mg%) the total β-carotene in carrot before and after cold drying respectively. The fact that the product achieves the best quality means $y_{4min} = 0$. In fact, $y_4 > 0$.

- Orthogonal experimental planning method with degree 2 (Dzung and Dzung, 2011; Dzung *et al*., 2011a, 2011b, 2012, 2015; Dzung and Du, 2012; Luc *et al*., 2013; Dzung, 2011, 2012a, 2012b, 2014).
- Using quadratic orthogonal experimental planning method (Dzung and Dzung, 2011; Dzung *et al*., 2011b, 2012; Dzung and Du, 2012; Dzung, 2011, 2012a, 2012b) to build the mathematical model about relationships between y_j (j = 1÷4) and technological factors effect on the cold drying process (Z_1, Z_2, Z_3, Z_4). These mathematical models of y_j (j = 1÷ 4) were written as follow (Dzung, 2014; Dzung *et al*., 2015; Luc *et al*., 2013):

$$y_j = b_0 + \sum_{u=1}^{k} b_u x_u + \sum_{u\neq i;u,i=1}^{k} b_{ui} x_u x_i + \sum_{u=1}^{k} b_{uu}\left(x_u^2 - \lambda\right) \quad (6)$$

These variables x_1, x_2, x_3 and x_4 were coded by variables of Z_1, Z_2, Z_3 and Z_4 presented as follow:

$$x_i = (Z_i - Z_i^0)/\Delta Z_i; \quad Z_i = x_i.\Delta Z_i + Z_i^0 \quad (7)$$

where,

$$Z_i^0 = (Z_i^{max} + Z_i^{min})/2$$

$$\Delta Z_i = (Z_i^{max} - Z_i^{min})/2 \quad (8)$$

$$Z_i^{min} \le Z_i \le Z_i^{max}; \ i = 1 \text{ to } 4$$

The experimental number is determined, (Dzung and Dzung, 2011; Dzung, 2011, 2014):

$$N = n_k + n_* + n_0 = 2^k + 2k + n_0 = 25 \quad (9)$$

With:

$$k = 4;\ n_k = 2^k = 2^4 = 16;\ n_* = 2k = 2x4 = 8;\ n_0 = 1$$

The value of the star point:

$$\alpha = \sqrt{\sqrt{N2^{(k-2)}} - 2^{(k-1)}} = \sqrt{\sqrt{25.2^{(4-2)}} - 2^{(4-1)}} = 1.414 \quad (10)$$

The condition of the orthogonal matrix:

$$\lambda = \frac{1}{N}\left(2^k + 2\alpha^2\right) = \frac{1}{25}\left(2^4 + 2\left(\sqrt{2}\right)^2\right) = 0.8 \quad (11)$$

- Building and solving 4-objective optimization problem by the RAM (Dzung *et al*., 2011b; Dzung, 2011, 2012b)

RESULTS AND DISCUSSION

Develop the mathematical models of the cold drying process of carrot product: In the fact that, all objective functions of the cold drying process of carrot product as the energy consumption per weight (y_1, kWh/kg), the residual water content (y_2, %), the anti-rehydration capacity (y_3, %) and the loss of total β-carotene in carrot (y_4, %) of the cold-dried product always depended on the technological factors, including: temperature of moisture condensation (Z_1, °C), temperature of cold drying chamber (Z_2, °C), velocity drying agents (Z_3, m/s) and time of cold drying process (Z_4, h). Therefore, these all objective functions were established by the experimental planning method with the quadratic orthogonal experimental matrix (k = 4, n_0 = 1). In addition, the experimental factors were established by conditions of the technological cold drying of carrot product (Dzung and Dzung, 2011; Dzung *et al*., 2011b, 2015; Dzung, 2011, 2012b), they were summarized in Table 4.

The experiments were carried out with all of the factor levels in Table 4 and all of the experimental planning in Table 5 to determine the value of the objective functions according to technological factors in the cold drying process of carrot product, $y_j = f_j(x_1, x_2, x_3, x_4)$ with j = 1 to 4, (Dzung and Dzung, 2011; Dzung *et al*., 2011a, 2011b; Dzung, 2011, 2012a, 2012b, 2014; Dzung *et al*., 2012, 2015; Dzung and Du, 2012; Luc *et al*., 2013). The results were summarized in Table 5.

Carrying out processing the experimental data in Table 5, calculating the coefficients, testing the significance of the coefficients by the Student criterion and testing the regression equations for the fitness of the experimental results by Fisher criterion (Dzung *et al*., 2011b, 2015; Dzung, 2011, 2012b, 2014) were building the regression equations y_j, j = 1 to 4, from Eq. (12) to Eq. (15) as follows:

The energy consumption of 1 kg final carrot product after cold drying process:

$$y_1 = f_1(x_1, x_2, x_3, x_4) = 1.682+0.133x_1+0.145x_2-0.074x_3+0.466x_4-0.19x_1x_3+0.214x_1x_4-0.187x_2x_3+0.227x_2x_4-0.028x_3x_4+0.036x_1^2-0.043x_4^2 \quad (12)$$

Table 4: The technological factors levels design

	Levels					
Parameters	-α (-1.414)	Low -1	Central 0	High+1	+α (1.414)	Deviation ΔZ_i
Z_1 (°C)	7.93	10	15	20	22.07	5
Z_2 (°C)	28.93	31	36	41	43.07	5
Z_3 (m/s)	4.76	6	9	12	13.24	3
Z_4 (h)	10.344	12	16	20	21.656	4

Table 5: The orthogonal experimental matrix level 2 ($k = 4$, $n_0 = 1$)

N		Value of real variables				Value of coded variables					Value of objective functions			
		Z_1	Z_2	Z_3	Z_4	x_0	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4
2^k	1	10	31	6	12	1	-1	-1	-1	-1	1.21	8.10	11.48	3.54
	2	20	31	6	12	1	1	-1	-1	-1	1.31	7.39	10.47	4.20
	3	10	41	6	12	1	-1	1	-1	-1	1.46	6.67	9.46	5.60
	4	20	41	6	12	1	1	1	-1	-1	1.32	6.24	8.84	5.24
	5	10	31	12	12	1	-1	-1	1	-1	1.31	6.60	9.36	5.54
	6	20	31	12	12	1	1	-1	1	-1	1.53	6.11	8.67	5.14
	7	10	41	12	12	1	-1	1	1	-1	1.40	6.52	9.24	5.48
	8	20	41	6	20	1	1	1	-1	1	1.71	7.02	9.95	5.90
	9	10	31	6	20	1	-1	-1	-1	1	1.83	5.18	7.34	4.35
	10	20	31	6	20	1	1	-1	-1	1	1.92	5.74	8.15	4.82
	11	10	41	6	20	1	-1	1	-1	1	1.83	5.96	8.44	5.01
	12	20	41	6	20	1	1	1	-1	1	2.09	4.53	6.42	4.80
	13	10	31	12	20	1	-1	-1	1	1	1.77	4.61	6.54	4.88
	14	20	31	12	20	1	1	-1	1	1	1.87	5.33	7.55	4.48
	15	10	41	12	20	1	-1	1	1	1	2.09	5.52	7.83	4.64
	16	20	41	12	20	1	1	1	1	1	2.14	4.21	5.97	6.23
2k	17	22.07	36	9	16	1	1.414	0	0	0	2.43	5.33	7.55	4.48
	18	7.93	36	9	16	1	-1.414	0	0	0	1.25	5.04	7.14	4.23
	19	15	43.07	9	16	1	0	1.414	0	0	2.35	6.04	8.56	5.07
	20	15	28.93	9	16	1	0	-1.414	0	0	1.21	5.94	8.42	4.99
	21	15	36	13.24	16	1	0	0	1.414	0	2.18	5.40	7.66	4.54
	22	15	36	4.76	16	1	0	0	-1.414	0	1.41	6.11	8.67	5.14
	23	15	36	9	21.656	1	0	0	0	1.414	2.25	4.23	6.42	6.81
	24	15	36	9	10.344	1	0	0	0	-1.414	1.11	5.73	8.12	4.81
n_0	25	15	36	9	16	1	0	0	0	0	1.34	5.49	7.64	4.53

The residual water content of final carrot product after cold drying process:

$$y_2 = f_2(x_1, x_2, x_3, x_4) = 5.163 – 0.946x_3 – 0.172x_1x_2 – 0.788x_1x_3 + 0.857x_1x_4 – 0.651x_2x_3 + 0.946x_2x_4 + 0.456x_2^2 + 0.342x_3^2 \quad (13)$$

The anti-rehydration capacity of final carrot product after cold drying process:

$$y_3 = f_3(x_1, x_2, x_3, x_4) = 7.373 – 0.156x_1 – 0.16x_2 – 1.341x_3 – 0.244x_1x_2 – 1.118x_1x_3 + 1.216x_1x_4 – 0.922x_2x_3 + 1.342x_2x_4 + 0.62x_2^2 + 0.458x_3^2 \quad (14)$$

The loss of total β-carotene in carrot of final product after cold drying process:

$$y_3 = f_3(x_1, x_2, x_3, x_4) = 4.854 + 0.106x_1 + 0.303x_2 – 0.396x_3 + 0.66x_4 – 0.697x_1x_3 + 0.808x_1x_4 – 0.833x_2x_3 + 0.634x_2x_4 – 0.139x_3x_4 – 0.286x_1^2 + 0.441x_4^2 \quad (15)$$

One-objective optimization problems for the cold drying process of carrot product: From Fig. 2 (*Diagram of subjects of cold drying process*) was obvious that all objective functions (y_j, j = 1 to 4) for the cold drying process of carrot product depended on the technological factors (x_i, i = 1 to 4). If every objective function was individually surveyed, these one-objective functions along with the technological factors would constitute the one-objective optimization problems. Because all the one-objective functions were to find the minimal value, the one-objective optimization problems were restated as follow (Dzung *et al*., 2011a, Dzung, 2011, 2012b; Dzung *et al*., 2015): Finding in common the test $x^{jopt} = (x_1^{jopt}, x_2^{jopt}, x_3^{jopt}, x_4^{jopt}) \in \Omega_x = \{-1.414 \le x_1, x_2, x_3, x_4 \le 1.414\}$ in order that:

$$\begin{cases} y_j = f_{jmin}\left(x_1^{jopt}, x_2^{jopt}, x_3^{jopt}, x_4^{jopt}\right) \\ \quad = \min f_j\left(x_1, x_2, x_3, x_4\right) \\ \forall x \in \Omega_x = \left\{-1.414 \le x_1, x_2, x_3, x_4 \le 1.414\right\}; \\ j = 1 \div 4 \end{cases} \quad (16)$$

According to the results of Dzung *et al*. (2011b) and Dzung (2011), if all the one-objective optimization problems (16) have the same roots: $(x_1^{jopt}, x_2^{jopt}, x_3^{jopt}, x_4^{jopt}) = (x_1^{kopt}, x_2^{kopt}, x_3^{kopt}, x_4^{kopt})$ with $k \ne j$, these roots called are utopian roots and also roots of multi-objective optimization problem (17). The optimal plan of utopian roots called is utopian plan. If the utopian roots and the utopian plan do not exist, multi-objective optimization problem (17) will be solved to find the optimal *Pareto* roots and the optimal *Pareto* plan. Therefore, solving one-objective optimization problems (16) were found to achieve: $y_{jmin} = \min f_j(x_1, x_2, x_3, x_4)$, $j = 1 \div 4$, with the identified domain $\Omega_x = \{-1.414 \le x_1, x_2, x_3, x_4 \le 1.414\}$.

By using the meshing method programmed in Matlab R2008a software, the results of the optimal parameters of every objective function from (12) to (15) limited in the experimental domain were summarized in Table 6, (Dzung *et al*., 2011b; Dzung, 2011, 2012b; Dzung *et al*., 2015; Luc *et al*., 2013):

Table 6: Minimum roots of each one-objective optimization problems

j	Value of roots of one-objective optimization problems $x_1^{j\,opt}$	$x_2^{j\,opt}$	$x_3^{j\,opt}$	$x_4^{j\,opt}$	Value of functions y_{jmin}
1	-1.414	-1.414	-1.414	-1.414	0.79
2	0.000	0.000	1.414	0.000	4.51
3	0.143	-0.021	1.114	0.223	6.31
4	0.124	-0.152	1.414	0.123	4.38

In Table 6, it was obvious that the utopian root of Eq. (16) and the utopian plan of Eq. (16) did not exist, because of $x^{jopt} = (x_1^{jopt}, x_2^{jopt}, x_3^{jopt}, x_4^{jopt}) \neq x^{kopt} = (x_1^{kopt}, x_2^{kopt}, x_3^{kopt}, x_4^{kopt})$ with j, k = 1÷4, j ≠ k (Which mean, Eq. (16) had not a general root). However, the utopian point was also indentified: $f^{UT} = (f_{1min}, f_{2min}, f_{3min}, f_{4min}) = (0.79, 4.51, 6.31, 4.38)$.

From Table 6, it was also obvious that the utopian root and utopian plan did not exist. Therefore, by the RAM, multi-objective optimization problems (17) must be solved to find the optimal *Pareto* root and the optimal *Pareto* plan in order that optimal *Pareto* effect $y_P^R = (y_{1P}^R, y_{2P}^R, y_{3P}^R, y_{4P}^R)$ closest to the utopian point f^{UT} (Dzung *et al*., 2011b; Dzung, 2011, 2012b; Dzung *et al*., 2015).

Multi-objective optimization problems for cold drying process of carrot product: It was easilly obvious that all objective functions (y_j, j = 1 to 4) always depened on the technological factors (x_1, x_2, x_3 and x_4) of the cold drying process of carrot product, with the identified domain $\Omega_x = \{-1.414 \le x_1, x_2, x_3, x_4 \le 1.414\}$. Consequently, the multi-objective optimization problem to determine the technological mode of the cold drying process of carrot product appeared in this case and it was restated as follow:

Finding in common the root $x = (x_1^{opt}, x_2^{opt}, x_3^{opt}, x_4^{opt}) \in \Omega_x = \{-1.414 \le x_1, x_2, x_3, x_4 \le 1.414\}$ in order that (Dzung *et al*., 2011b, 2015; Dzung, 2011, 2014; Luc *et al*., 2013):

$$\begin{cases} y_j = f_{jmin}\left(x_1^{opt}, x_2^{opt}, x_3^{opt}, x_4^{opt}\right) \\ \qquad = \min f_j\left(x_1, x_2, x_3, x_4\right) \\ \forall x \in \Omega_x = \{-1.414 \le x_1, x_2, x_3, x_4 \le 1.414\}; \\ y_j < C_j;\ j = 1 \div 4 \end{cases} \tag{17}$$

where,

$$y_1<C_1 = 2.5;\ y_2<C_2 = 5.2;\ y_3<C_3 = 10;\ y_4<C_4 = 6.5 \tag{18}$$

The cold drying mode of carrot product established was based on factors including: economic, technicality and quality of the product obtained. Experimental results were obvious that: if the energy consumption for 1 kg final carrot product was over C_1 = 2.5kWh, it would increase the final carrot product price and difficult commercialization. If the residual water content of the final carrot product was over C_2 = 5.2%, the microorganisms would be capable to grow and develope and damage products. Besides, If the anti-rehydration capacity of the final carrot product was over C_3 = 10%, carrot would be denatured, not be able to recover the original its quality. As a result, quality of product reduced. In addition, if the loss of total β-carotene in carrot of the final product was over C_4 = 6.5%, natural color and flavor of carrot would be destroyed and nutritional value of product reduced. According to Dzung *et al*. (2011a), if the multi-objective optimization problem was solved by the utopian point method, value of of the objective functions (y_1, y_2, y_3 and y_4) would not satisfy conditions (17), so the multi-objective optimization problem have to be solved by the RAM (Dzung *et al*., 2011b, 2015; Dzung, 2011).

The purpose of the experiment was to reach the targets of the cold drying process of carrot product which were expressed by 4 regression Eq. (12), (13), (14) and (15), but the tests satisfying all function values (y_{1min}, y_{2min}, y_{3min}, y_{4min}) could not be found. Hence, the idea of the four-objective optimization problem was to find the optimal *Pareto* test for the optimal *Pareto* effect $y(x^R) = y_P^R = (y_{1P}^R, y_{2P}^R, y_{3P}^R, y_{4P}^R)$ closest to the utopian point $y^{UT} = (y_{1min}, y_{2min}, y_{3min}, y_{4min}) = (0.79, 4.51, 6.31, 4.38)$.

The RAM established the R-objective combination function $R(y_1, y_2, y_3, y_4) = R(x_1, x_2, x_3, x_4) = R(x)$ as the followings:

$$\begin{cases} R(x) = \sqrt[4]{r_1(x).r_2(x).r_3(x).r_4(x)} = \left[\prod_{j=1}^{4} r_j(x)\right]^{1/4} \\ \Omega_x = \{-1.414 \le x_1, x_2, x_3, x_4 \le 1.414\} \end{cases} \tag{19}$$

Where: with conditions (19), thus $r_1(x)$, $r_2(x)$, $r_3(x)$ and $r_4(x)$ can be established as follows:

$$r_j(x) = (C_j - y_j)/(C_j - y_{jmin}) \text{ when } y_j < C_j \tag{20}$$

$$r_j(x) = 0 \text{ when } y_j \ge C_j \tag{21}$$

Or:

$r_1(x) = (2.5-y_1)/(2.5-y_{1min})$ when $y_1<2.5$;
$r_1(x) = 0$ when $y_1\ge 2.5$
$r_2(x) = (5.2-y_1)/(5.2-y_{1min})$ when $y_1<5.2$;
$r_2(x) = 0$ when $y_2\ge 5.2$
$r_3(x) = (10-y_3)/(10-y_{3min})$ when $y_3<10$
$r_3(x) = 0$ when $y_3\ge 10$
$r_4(x) = (6.5-y_4)/(6.5-y_{4min})$ when $y_4<6.5$

Table 7: Minimum roots of multi-objective optimization problems

Value of optimal *Pareto* roots of multi-objective optimization problem				Minimun value of R-objective combination function	Value of the optimal *Pareto* effects of multi-objective optimization problem			
x_1^R	x_2^R	x_3^R	x_4^R	$R_{max}(x)$	y_{1P}^R	y_{2P}^R	y_{3P}^R	y_{3P}^R
0.124	-0.042	0.914	0.013	0.830	1.62	4.52	6.43	4.45

$r_4(x) = 0$ when $y_4 \geq 5.5$

With: y_{1min}, y_{2min}, y_{3min}, y_{4min} were showed in Table 6. It was easily obvious that if $y_j = f_j(x) \to f_{j\,min}$ and $y_j = f_j(x) < C_{2j}$), $r_j(x) \to r_{jmax} = 1$ (Luc *et al*., 2013).

By choosing R(x) as the objective function, the m-objective optimization problem is restated as: *Find $x^R = (x_1^R, x_2^R, x_3^R, x_4^R) \in \Omega_x$ in order that R(x) reaches the maximum value:*

$$\begin{cases} R_{max} = R(x^R) = \max\{R(x)\} = \max\left\{ \sqrt[4]{\left[\prod_{j=1}^{4} r_j(x) \right]} \right\} \\ \forall x = (x_1, x_2, x_3, x_4) \in \Omega_x \end{cases} \tag{22}$$

From (20), it can be seen: $0 \leq R(x^R) \leq 1$. If $R(x^R) = 1$, $x^R = Z^{UT}$–the utopian test. If $R(x^R) = 0$, one of the values of $f_j(x)$ violates (21), which means that $f_j(Z)$ belongs to the restricted area C (18). The four-objective optimization problem needed to indentify $x^R = (x_1^R, x_2^R, x_3^R, x_4^R) \in \Omega_x$ in order that $R(x_1^R, x_2^R, x_3^R, x_4^R) = \text{Max}\{R(x_1, x_2, x_3, x_4)\} = \text{Max}\{R(x)\}$. The maximum value of (19) was determined by the meshing method programmed in Matlab 7.0, results were pretened in Table 7.

$$R_{max} = \text{Max}\{R(x_1, x_2, x_3, x_4)\} = 0.830$$

With,

$$x_1^R = 0.124;\ x_2^R = -0.042$$
$$x_3^R = 0.914;\ x_4^R = 0.013$$

Then, transforming into real variables:

$$Z_1^{opt} = 15.62°C$$
$$Z_2^{opt} = 35.79°C$$
$$Z_3^{opt} = 11.74 \text{ m/s}$$
$$Z_4^{opt} = 16.05h$$

Substituting x_1^R, x_2^R x_3^R, x_4^R into these Eq. (12), (13), (14) and (15), the optimal *Pareto* effect was obtained as: $y_{1P}^R = 1.62$; $y_{2P}^R = 4.52$; $y_{3P}^R = 6.43$; $y_{4P}^R = 4.45$;

The rehydration capacity of the cold-dried product was determined as:

$$IR = 100 - y_{3P}^R = 100 - 6.43 = 93.57\%$$

By the RAM, solving the multi-objective optimization problem with R-optimal combination criterion which satisfied the maximum R-optimal combination criterion ($R_{min} = 0.83$) was determined the optimal *Pareto* test (or the technological mode of cold drying process of carrot product) as: temperature of moisture condensation was $Z_1^{opt} = 15.62^0C$, temperature of cold drying chamber was $Z_2^{opt} = 35.79^0C$, the velocity drying agents was $Z_3^{opt} = 11.74$m/s and the time of cold drying process was $Z_4^{opt} = 16.05$h. Corresponding with the optimal *Pareto* test was also determined the optimal *Pareto* effect as: the energy consumption per weight of 1 kg final product was $y_{1P}^R = 1.62$kWh/kg; the residual water content of the final product was $y_{2P}^R = 4.52\%$; the anti-rehydration capacity of the final product was $y_{3P}^R = 6.43\%$ (Correspondingly IR = 93.57%) and the loss of total β-carotene in carrot of the final product was $y_{4P}^R = 4.45\%$.

Compared with the experimental results from the Table 5, these results above were suitable and satisfying with the objectives of the problem.

Experiment to test the results of multi-objective optimization problem: The cold drying process of carrot product was carried out at the optimal *Pareto* test: temperature of moisture condensation of $Z_1^{opt} = 15.62°C$, temperature of cold drying chamber of $Z_2^{opt} = 35.79°C$, the velocity drying agents of $Z_3^{opt} = 11.74$m/s and the time of cold drying process of $Z_4^{opt} = 16.05$ hours. The experimental results were determined: The energy consumption per weight of $y_1 = 1.65$kWh/kg; the residual water content of $y_2 = 4.59\%$; the anti-rehydration capacity of $y_3 = 6.92\%$ (or the rehydration capacity of $IR = 100 - y_3 = 93.08\%$) and the loss of total β-carotene in carrot of $y_4 = 4.67\%$ of the cold-dried product.

Consequently, it was very noticeable that the results from the optimization problems of cold drying process of carrot product had the approximation to the experimental results.

Carrot product was dried by the cold drying method at the optimal *Pareto* test: $Z_1^{opt} = 15.62°C$; $Z_2^{opt} = 35.79°C$; $Z_3^{opt} = 11.74$m/s; $Z_4^{opt} = 16.05$h. The final product obtained could be seen in Fig. 4. It was certain that the optimal *Pareto* test and the optimal *Pareto* effect of the multi-objective optimization problem of cold drying process of carrot product be possibly applied to determine the technological mode of cold drying process of carrot product for using in the industry.

The relationship between y_1, y_2, y_3 and y_4 with 2 variables x_2, x_4, $x_1^R = 0.124$ and $x_3^R = 0.914$; was

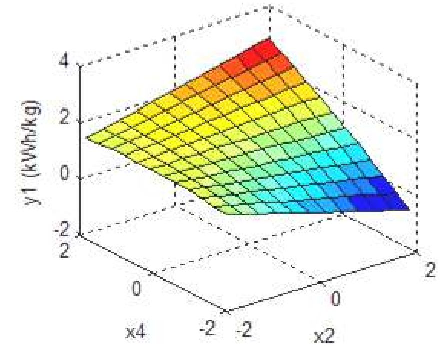

Fig. 5: Relationship between y_1 and x_2, x_4 (in 3D)

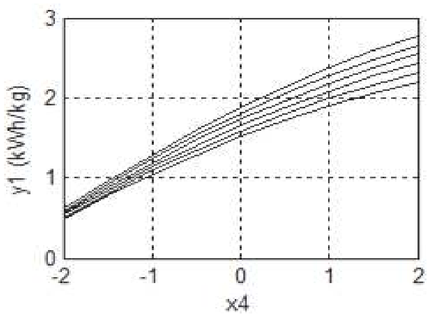

Fig. 6: Relationship between y_1 and x_4 (in 2D)

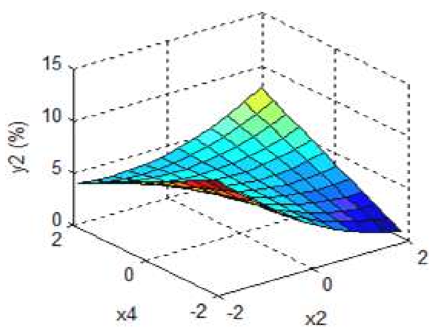

Fig. 7: Relationship between y_2 and x_2, x_4 (in 3D)

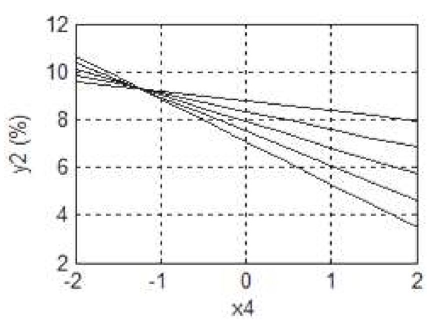

Fig. 8: Relationship between y_2 and x_4 (in 2D)

performed geometrically in 3D. The relationship between y_1, y_2, y_3 and y_4 with 2 variables x_4, $x_1^R = 0.124$; $x_2^R = -0.042$ and $x_3^R = 0.914$; was performed geometrically in 2D (Fig. 5 to 12).

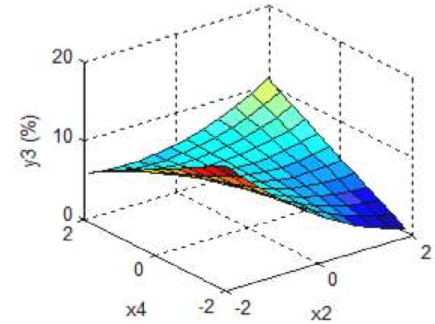

Fig. 9: Relationship between y_3 and x_2, x_4 (in 3D)

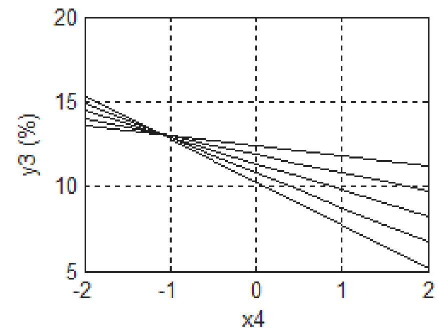

Fig. 10: Relationship between y_3 and x_4 (in 2D)

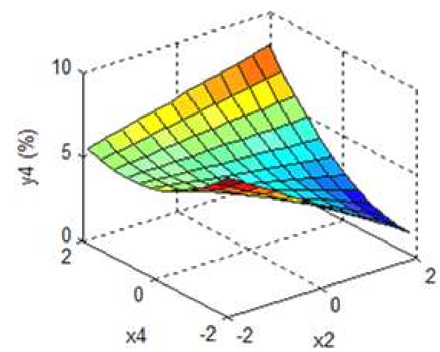

Fig. 11: Relationship between y_4 and x_2, x_4 (in 3D)

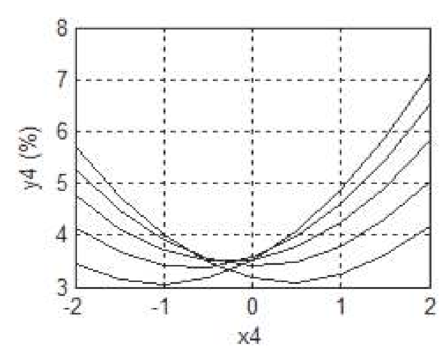

Fig. 12: Relationship between y_4 and x_4 (in 2D)

All Figures on above were obvious that all objective functions were completely suitable with experimental results. Therefore, it proved that relationships between objective functions with effect

Table 8: The technological mode of the cold drying process of carrot product

No	Technological parameters	Symbol and unit	Value
1	Temperature of moisture condensation	Z_1, (°C)	15.62
2	Temperature of cold drying chamber	Z_2, (°C)	35.79
3	The velocity drying agents	Z_3, (m/s)	11.74
4	The time of cold drying process	Z_4, (h)	16.05
The standards of final carrot product after drying			
5	The energy consumption per weight of 1kg final product	y_{1P}^R, (kWh/kg)	1.62
6	The residual water content of final product	y_{2P}^R, (%)	4.52
7	The anti-rehydration capacity of final product	y_{3P}^R, (%)	6.43
8	The loss of total β-carotene in carrot of the cold-dried product	y_{4P}^R, (%)	4.45

factors very well described for cold drying process of carrot product.

Determining technological mode of cold drying process of carrot product: From results on above, it allowed to set up the technological mode during the cold drying process of carrot product in Table 8 as follow:

The optimal technological mode of cold drying process of carrot product in Table 8 was obvious, when carrot was carried out cold drying at the optimal technological mode, the quality of carrot product after drying had very good quality (Fig. 4). The technological mode of cold drying process of carrot product was found out in Table 8, it can be completely applied for preservation of carrot product in order to be prolonged use time and export time.

CONCLUSION

The mathematical models (12), (13), (14) and (12) which were established from the experiments, they have quite well described the relationship between temperature of moisture condensation, temperature of cold drying chamber, the velocity drying agents and the time of cold drying process with the energy consumption of 1 kg final product; the residual water content of final product; the anti-rehydration capacity of final product (or the rehydration capacity of final product) and the loss of total β-carotene in carrot of the cold-dried product. The system of Eq. (17) and (18) was the multi-objective optimization problems of the cold drying process of carrot product. These mathematical models were suitably used for calculating and setting up the technological mode of the cold drying process of carrot product. Solving the multi-objective optimization problems (17) and (18) determined the technological mode of the cold drying process of carrot product. The results were presented in Table 8.

ACKNOWLEDGMENT

The authors thank Head of Lab Food Engineering and Technology, Department of Food Technology, Faculty of Chemical and Food Technology, HCMC University of Technical Education, Viet Nam, for help with experiments carrying out.

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