Research Journal of Applied Sciences, Engineering and Technology 7(5): 944-949, 2014
DOI:10.19026/rjaset.7.339
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: May 01, 2013 Accepted: June 11, 2013 Published: February 05, 2014

Research Article

A Quantitative Evaluation Method for Transportation Network Efficiency

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Abstract: The efficiency of a transportation network is the comprehensive performance index of the network. In order to evaluate the operation situation of the transportation network objectively, the scientific quantitative evaluation method for the network efficiency is very important. In this study, a new quantitative evaluation method for transportation network efficiency is developed, which could evaluate the transportation network performance comprehensively and reasonably. The method is defined in the context of network equilibrium, which could reflect the influences of travel behavior, travel cost, travel demands and link flows, all in equilibrium state, on network efficiency. The computation results compared with a previously proposed one by numerical example, which denote that the new method can quantitatively reflect the influence on the transportation network efficiency induced by traffic flows, user behavior and network structure, which accords with the practical situation.

Keywords: Efficiency, quantitative evaluation method, traffic equilibrium, transportation network

INTRODUCTION

The transportation network is one of the most critical infrastructures in urban structures and plays an essential role in meeting normal city operation. Its operational situation is the main basis for the traffic management and traffic planning. Accompanied by the developing of the social economic, the vehicles on the roads in the cities have increased sharply. But it is well-known that the land and other resources are limited in cities, which resulted in more conflicts between the supply and demand in transportation system. And the congestion problem in cities had been the worldwide problem.

It is proved that it couldn't solve the more seriously congestion in the cities which rely only on the increased investments on the infrastructures. So in order to solve the problem, many researchers have deeply studied the management of the transportation network and many models were developed in theory and used in practice. But these existed traditional methods are all lack of the feedback of the operation performance of the transportation network in their operation process. So it is difficult to make sure the results of these methods are always rational. This occurs because there is no scientific method for evaluating the transportation network is available.

Network efficiency is an accounting statement that reflects comprehensively the operational situation of transportation network. In order to make scientific analysis of situation of transportation network, we should evaluate the transportation network efficiency objectively at first. Therefore, it is more and more important to develop an evaluation method that can measure the transportation network efficiency quantitatively, directly and objectively.

Roughgarden (2002) used the total travel time in the network to represent the transportation network efficiency and the loss of efficiency under different conditions was also studied in the paper. Xiaolei *et al*. (2010) also studied the similar problem based on the method. Vito and Massimo (2001) proposed a quantitative measure for the efficiency of the weighted network, which computed by the shortest distances between any two nodes. The method were used in their researches of complex network (Vito and Massimo, 2003, 2002, 2004; Paolo *et al*., 2003). But the congestion effects on the roads are not considered in the method, so it couldn't be used directly to measure the efficiency of the transportation network. This method also is used to evaluate the efficiency of the airline network (Chaug-Ing and Hsien-Hung, 2008). Although the congestion effects don't need be considered in the airline network, the other factors which could have influence on the network efficiency, such as demand and route choice, are also ignored. David (2003) described the network efficiency as five indexes: mobility, utility, productivity, accessibility and travelers

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(subjectivity and equity). Obviously, the method is a qualitative method, which couldn't be used in the models to measure the network efficiency objectively. Anna and Qiang (2007, 2008) proposed an efficiency measure for the congested network based on the path travel time.

In this study, a new quantitative evaluation method for transportation network efficiency is developed, which captures traffic flows, travel costs, user behavior and network configuration etc. The method could be used to measure the performance of a transportation network. We also compare the results derived from the new method to the existing methods.

LITERATURE REVIEW

Given a transportation network $G = (N, A)$, where N is set of nodes and A is set of links. The traffic demand for OD pair (r, s) is q_{rs}, where r is a origin node and s is a destination node. The set of paths joining an OD pair (r, s) is K_{rs} and $\cup K_{rs} = K$. The traffic flows on path $p \in K_{rs}$ and link $a \in A$ are denoted by f^p_{rs} and x_a, respectively.

We define a binary variable δ_a^{p}, if link a is contained in path p, $\delta_a^{\mathrm{p}} = 1$, otherwise $\delta_a^{\mathrm{p}} = 0$.
The travel time on link a is:

$$t_a = t_a(x_a),\ a \in A$$

We group the link flow into the vector x and the OD traffic demands into the vector q.

The traffic equilibrium model can be defined as follows:

$$Min\ \ Z(\mathbf{x}) = \sum_{a \in A} \int_0^{x_a} t_a(y)\, d_y \tag{1}$$

Subject to:

$$\sum_{p \in K_{rs}} f_{rs}^p = q_{rs} \tag{2}$$

$$x_a = \sum_{(r,s)} \sum_{p \in K_{rs}} f_{rs}^p \delta_a^p \quad , \quad \forall a \in A \tag{3}$$

$$f_{rs}^p \geq 0, \forall r, s, p \tag{4}$$

In the model, the objective function (1) is to minimize the total travel cost. The constraint (2) is the conservation on flows. The link flows are related to the path flows through the conservation of flow Eq. (3), constraint (4) are the standard nonnegative constraints on flows.

There are many solution approaches for the solutions of the above model (1)-(4), such as the Frank-Wolfe algorithm (Yossi, 1984; Masao, 1984), the Gradient Projection algorithm (Anthony *et al*., 2002). And the classical Frank-Wolfe algorithm is adopted to solve the model in this study, which could be described as following:

Step 0: Initialization: Set $t_a = t_a(0), a \in A$. Perform All-or-nothing assignment based on the $\{t_a\}$. This yields a link flow pattern x^1, set $n = 1$

Step 1: Update: $t_a^n = t_a\left(x_a^n\right),\ a \in A$

Step 2: Direction finding: Perform all-or-nothing assignment based on the $\{t^n_a\}$. This yields a set of (auxiliary) link flows y^n

Step 3: Line search: Find α_n that solves:

$$\min_{0 \leq \alpha \leq 1} \sum_{a \in A} \int_0^{x_a^n + \alpha\left(y_a^n - x_a^n\right)} t_a(w)\, d_w$$

Step 4: Move: Set $x_a^{n+1} = x_a^n + \alpha_n\left(y_a^n - x_a^n\right),\ a \in A$

Step 5: Convergence check: If the given convergence criterion is met, stop and output the current flow $\{x_a^{n+1}\}$ as the equilibrium link flow; otherwise, set n = n + 1 and go to step 1

QUANTITATIVE NETWORK EFFICIENCY EVALUATION METHOD

The reasonable transportation network efficiency should be able to reflect the following characteristics of the network:

- The efficiency should show fluctuation when network components like structures and demands distribution change. Because in the given network, its resource is limited, so the number of travelers it could service is limited too.
- For a given transportation network, even its operation efficiency fluctuates when the traffic demands change, it may have many extreme values, but there is one and only one global maximum between those possible extreme points. Furthermore, the number of extreme points should relate directly to the number of paths between the OD pair.
- The network efficiency could be used to explain some paradox phenomena in transportation networks, such as Braess paradox.

Here we propose a new network efficiency measure which is defined in equilibrium state and capable of taking into account traffic demand, travel time and traveler behavior as well as conforms to the characteristics above.

For a given transportation network G and OD demand vector q, we define the network efficiency ε as:

$$\varepsilon = \frac{1}{n} \sum_{a \in A} \bar{x}_a \big/ \bar{t}_a \tag{5}$$

where,

n = The number of links in the network

$\bar{x}_a$ and $\bar{t}_a$ = Flow and travel time on link a in the context of network equilibrium, respectively

Furthermore, we could extend the travel cost $\bar{t}_a$ into general travel cost. In other words, it could include factors such as environment cost. In addition, a few links in the physical network may need special consideration. Hence, we may point weighting factor $\lambda_a \geq 0$ for certain links a to increase or decrease their influence on efficiency. Nevertheless, we will not discuss these aspects since they do not exert an impact on the following analysis.

The above method has a fine economic meaning since it measures performance verses cost or price, with the performance being measured by potential number of users and cost or price by each unit of travel cost on each link. In other words, the efficiency/performance is measured by link flows and travel disutility. For example, suppose that there is only one link a in a network and $\bar{x}_a$ = 50 (vehicles), $\bar{t}_a$ = 0.5 (h). According to exp. (6), network efficiency ε = 50 (vehicle/h), which means this network can deal with 100 vehicles/h. If $\bar{t}_a$ = 1 h, then ε = 50 (vehicle/h), which means the efficiency of this network is half of the former one. But it is worth noting that due to the differences of congestion network, the unit of measurement should be in accordance with the type of flow in the network.

Vito and Massimo (2001) proposed a network efficiency measure method (we refer to it as LM method in this study), which can be used as a comparison to the new evaluation method we proposed. LM method is a weighted efficiency measure in analyzing small-world behavior that has been applied to a variety of networks (Vito and Massimo, 2003, 2004). It is defined as:

$$E = \frac{1}{m(m-1)} \sum_{i \neq j \in G} 1/d_{ij} \tag{6}$$

where,

m = The number of nodes in network G

d_{ij} = The shortest path length between nodes i and j

When there is no path in the network between i and j, $d_{ij} = +\infty$.

Obviously, exp. (6) only uses the shortest path lengths between the nodes and other important factors which may affect the network efficiency are ignored. As the travel flow becomes greater, the shortest path lengths will increase either. As a result, the efficiency E by exp. (6) will become increasingly smaller. In other words, E is a strictly monotone regression function on demand. Hence we could find it unreasonable that efficiency of a given network reaches maximum when its travel flow is 0.

NETWORK EXAMPLES

Consider the transportation network in Fig. 1, in which there are four nodes 1, 2, 3, 4 and five links a, b, c, d, e.

Assume that the link cost functions are all BPR functions:

$$t_a(x_a) = 10\left(1 + 0.15\left(x_a/4\right)^4\right)$$

$$t_b(x_b) = 15\left(1 + 0.15\left(x_b/6\right)^4\right)$$

$$t_c(x_c) = 12\left(1 + 0.15\left(x_c/3\right)^4\right)$$

$$t_d(x_d) = 15\left(1 + 0.15\left(x_d/10\right)^4\right)$$

$$t_a(x_e) = 20\left(1 + 0.15\left(x_e/8\right)^4\right)$$

There are five OD pairs as (1, 2), (1, 3), (1, 4), (3, 2), (3, 4) with demands given respectively by q_{12} = 11, q_{13} = 2, q_{14} = 6, q_{32} = 3 and q_{34} = 1.

The link flows in equilibrium situation can be easily computed using Frank-Wolfe Method as:

$$\bar{\mathbf{x}} = \{\bar{x}_a, \bar{x}_b, \bar{x}_c, \bar{x}_d, \bar{x}_e\}$$
$$= \{8.23, 4.93, 2.07, 5.77, 5.84\}$$

Then efficiency according to exp. (5) is ε = 0.249. While E = 0.020 and the total travel cost is 114.3. However, the comparison of absolute value of these two results cannot explain those relating phenomena. So we will analyze the rationality of the new method by analyzing the impact on efficiency of changing components in the network.

Influence of the traveler behavior: First we will analyze the impact of the different traveler behaviors on network efficiency, which means the influence of different OD traffic flows on efficiency when total traffic demand is fixed.

For example, if all demands in the network are transferred to OD pair (1, 2), which means q_{12} = 23, q_{13} = 0, q_{14} = 0, q_{32} = 0, q_{34} = 0. In the corresponding equilibrium network, link b and c will not be used,

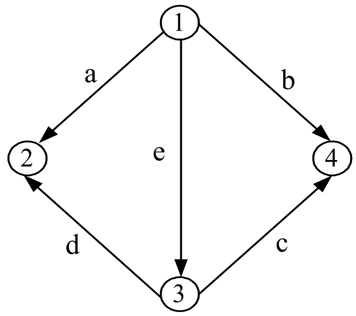

Fig. 1: Transportation network

Table 1: Network efficiency vs. traveler behavior q_{12}

Efficiency	Before transfer to q_{12}	After transfer to q_{12}	Gap (%)
ε	0.249	0.191	23.3
E	0.020	0.018	10.0
Total travel time	114.300	158.200	38.4

Table 2: Network efficiency vs. traveler behavior q_{14}

Efficiency	Before transfer to q_{14}	After transfer to q_{14}	Gap (%)
ε	0.249	0.102	59.100
E	0.020	0.017	14.500
Total travel time	114.300	277.100	142.430

which means in fact efficiency should drop considerably. And we could get that $E' = 0.018$, $\varepsilon' = 0.191$, which decrease by 12 and 23.3%, respectively while the travel time is 158.2. The results show in Table 1.

Similarly, if all demands in the network are transferred to OD pairs (1, 4), which means $q_{14} = 23$, $q_{12} = 0$, $q_{13} = 0$, $q_{32} = 0$, $q_{34} = 0$. Then we could get $E'' = 0.017$, $\varepsilon'' = 0.102$, decreasing by 14.5 and 59.1%, respectively comparing to the original efficiency. And the travel time is 277.1. According to the change of travel cost, the method we demonstrated given by exp. (5) could better reflect the actual traffic state comparing to the LM method. The results could find in Table 2.

In order to better analyze the problem, we observe the variation trend of efficiency while demand q_{12} is transferring to q_{14} gradually.

Denote Δ_1 as the transferred volume from q_{12} to q_{14} ($0 \le \Delta_1 \le q_{12}$). In this way, we could get the network efficiency trend illustrated in Fig. 2. Similarly, denote Δ_2 as the transferred traffic volume from q_{14} to q_{12} ($0 \le \Delta_2 \le q_{12}$), we could get another network efficiency trend illustrated in Fig. 3.

As is clearly demonstrated in the Fig. 2 and 3, when the demand on a particular OD pair changes while that of the network is fixed, the efficiency ε will fluctuate according to the change of the travel behavior in the network while the efficiency E referring to LM measure approximately stays the same. As a result, the evaluation method we proposed can better demonstrate

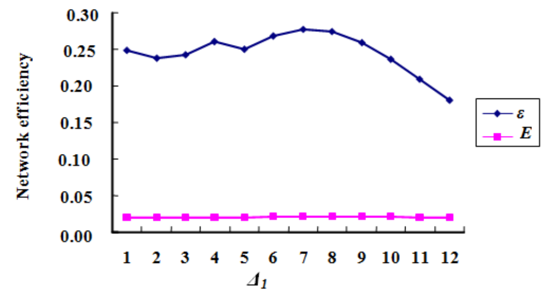

Fig. 2: Δ_1 vs. network efficiency

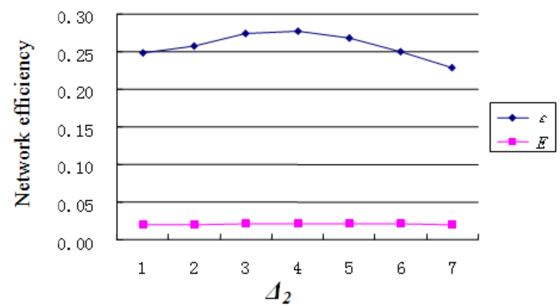

Fig. 3: Δ_2 vs. network efficiency

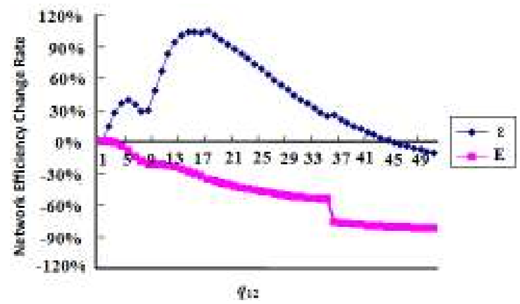

Fig. 4: q_{12} vs. network efficiency

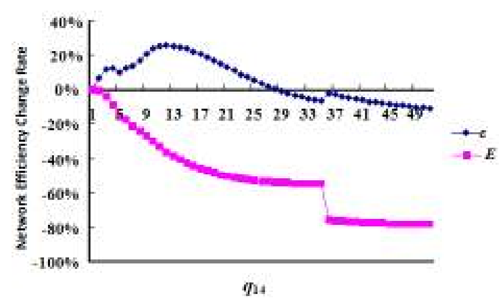

Fig. 5: q_{14} vs. network efficiency

the impact exerted by travel demand structure on the network efficiency.

Influence of traffic demand: The Fig. 4 and 5 describe the relative change rates of network efficiency ε and E (comparing to their original value) while q_{12}, q_{14} increase from 1 to 50, respectively.

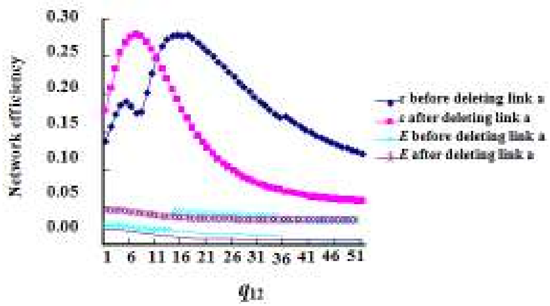

Fig. 6: Efficiency before and after removing link a

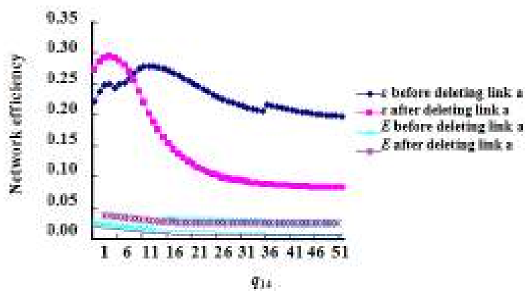

Fig. 7: Efficiency before and after removing link b

As is clearly demonstrated in the figures, the efficiency E is monotonically decreasing while demand is increasing, which means E is a monotonic regression function. But the efficiency ε fluctuate with the change of demand (all have the process increase at first and then decrease) and all have maximums. This apparently, better coordinates with the concept of transportation network efficiency in the real world. As a result, it is reasonable that we conclude that ε could better reflect transportation network efficiency in the real world.

Figure 4 to 5 also illustrate that for a given transportation network configuration, there always exists an optimal traffic flow which could maximize the network efficiency. So in the actual daily traffic management, it is possible to effectively increase network efficiency by using some methods such as improving network structure (add or close certain links), changing the behaviors of network user (implement traffic guide or other control policies).

Influence of network topology: Figure 6 and 7 demonstrate the trend of network efficiency ε and E before and after the removal of link a and link b. The efficiency E is still the monotonically decreasing and it changes so slightly while removing links that it could not reflect the impact of the network topology on network efficiency. However the efficiency ε reflects the impact more rationally. And from the variation of ε, the famous Braess Paradox phenomenon can be explained.

If the transportation network efficiency after removing certain links is higher than that before removing those links, we could easily know that it means the Braess Paradox occurs.

For example, in the Fig. 7, when q_{14} = 2 and other OD demands are fixed, the efficiency ε before and after removing link b are 0.237 and 0.285, the travel cost is 95.58 and 88.76, respectively. In other words, the removal of link b will reduce travel cost rather than increasing it, that's the Braess Paradox phenomenon in transportation network.

It is also demonstrated in Fig. 6 that when q_{14} is in the range (1, 8), the efficiency ε will increase after removing link b. That is to say, the Braess Paradox occurs. When q_{14} is in the range (8, $+\infty$), the efficiency ε will decrease after removing link b. Meanwhile, the Braess Paradox will not occur. And the same situation coordinates with the removal of link a.

This phenomenon that efficiency increases after removing certain link only exists when the traffic demand in certain range accords with the conclusion of Dietrich *et al*. (2005), which is, the Braess paradox occurs in a certain range of demand and with the change of demand it can be eliminated.

In addition, in the Fig. 4 to 7, it is noticeable that the efficiency ε has two efficiency maximum point before removing the link a or b, this attributes to that there are two paths between the OD pair (1, 3). But after link b is removed, there is only one path between OD pair (1, 3). As a result, ε has only one maximum point exists in this case, which is in accordance with the fact. At the same time, it is clearly seen that E has one maximum all along, which is not reasonable.

CONCLUSION

A new evaluation method for transportation network efficiency which computed by link flows and link travel times at equilibrium is developed. It captures the influence of traffic flows, travel costs and user behavior on the network. Actually there are many definitions of traffic equilibrium model for transportation network. We only discuss the classic static equilibrium model with fixed demand. However, after the analysis we could find that it does not matter which definition of equilibrium we adopt, for all the information we use for calculation is acquired at equilibrium.

The evaluation method is well-defined, even in the case of disconnected networks. And the numerical example results indicate that it is more scientifically dependable than the existed method. Moreover, the experimental results reflect the impaction of travel demand and network topology on the efficiency of a transportation network. As a result, in the practical traffic management, the methods such as improving network configuration, guiding behaviors of travelers and controlling traffic demand, all may be adopted to increase network efficiency. Of course we will develop further application of this measure in transportation network efficiency evaluation.

ACKNOWLEDGMENT

This study was supported by the National Natural Science Foundation of China, under the Grant Nos. 71101155.

REFERENCES

Anna, N. and Q. Qiang, 2007. A network efficiency measure for congested networks. Europhys. Lett., 79: 1-5.

Anna, N. and Q. Qiang, 2008. A network efficiency measure with application to critical infrastructure networks. J. Glob. Optim., 40(1): 261-275.

Anthony, C., L. Derhorng and R. Jayakrishnan, 2002. Computational study of state-of-the-art path-based traffic assignment algorithms. Math. Comput. Simulat., 59(6): 509-518.

Chaug-Ing, H. and S. Hsien-Hung, 2008. Small-world network theory in the study of network connectivity and efficiency of complementary international airline alliances. J. Air Transp. Manag., 14(3): 123-129.

David, L., 2003. Perspectives on efficiency in transportation. Int. J. Transp. Manage., 1(3): 45-155.

Dietrich, B., N. Anna and W. Tina, 2005. On a paradox of traffic planning. Transport. Sci., 39(4): 446-450.

Masao, F., 1984. A modified Frank-Wolfe algorithm for solving the traffic assignment problem. Transport. Res. B-Meth., 18(2): 169-177.

Paolo, C., L. Vito, M. Massimo and R. Andrea, 2003. Efficiency of scale-free networks: Error and attack tolerance. Phys. A, 320(3): 622-642.

Roughgarden, T., 2002. How bad is selfish routings. J. ACM, 49: 236-259.

Vito, L. and M. Massimo, 2001. Efficient behavior of small-world networks. Phys. Rev. Lett., 87(19): 198701.

Vito, L. and M. Massimo, 2002. Is the Boston subway a small-world networks. Phys. Rev. Lett., 87: 198701.

Vito, L. and M. Massimo, 2003. Economic small-world behavior in weighted networks [J]. Eur. Phys. J. B, 32(2): 249-263.

Vito, L. and M. Massimo, 2004. How the science of complex networks can help developing strategies against terrorism. Chaos Solition. Fract., 20(1): 69-75.

Xiaolei, G., Y. Hai and L. Tianliang, 2010. Bounding the inefficiency of logit-based stochastic user equilibrium. Eur. J. Oper. Res., 201(2): 463-469.

Yossi, S., 1984. Urban Transportation Network: Equilibrium Analysis with Mathematical Programming Methods. Prentice-Hall, Englewood Cliffs, NJ.