# Research Article <br> On Heuristic Approach for Solution of Scheduling Problem Involving Transportation Time and Break-down Times for Three Machines 

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#### Abstract

In most manufacturing and distribution systems, semi-finished jobs are transferred from one processing facility to another by transporters such as automated guided vehicles and conveyors and finished jobs are delivered to customers or warehouses by vehicles such as trucks. Most machine scheduling models assume either that there are a finite number of transporters for delivering jobs or that jobs are delivered instantaneously from one location to another without transportation time involved. In this study we study a new simple heuristic algorithm for a ' 3 machine, n-job' flow shop scheduling problem in which transportation time and break down times of machines are considered. A heuristic approach method to find optimal and near optimal sequence minimizing the total elapsed time.


Keywords: Break down time, flow-shop scheduling problem, optimal sequence, transportation time

## INTRODUCTION

Now-a-days, the decision makers for the manufacturing plant must find a way to successfully manage resources in order to produce products in the most efficient way in the complex manufacturing setting, with multiple lines of products, each requiring many different steps and machines for completion. Also, they need to design a production schedule that promotes on-time delivery as well as minimizes the flow time of a product. Out of these concerns grew an area of studies known as the scheduling problems. In the scheduling problem, one of the central tasks in highlevel synthesis is the problem of determining the order in which the operations in the behavioral description will execute. It involves solving for the optimal schedule under various objectives, different machine environments and characteristics of the jobs. The number of possible schedules of the flow-shop scheduling problem involving n -jobs and m -machines is ( $\mathrm{n}!$ ). The optimal solution to the problem is to find the sequence of jobs on each machine in order to complete all the jobs on all the machines in the minimum total time provided each job is processed on machines 1,2 , $3, \ldots ., m$ in that order.

Flow shop scheduling problems have been dealt by many authors when machines operating job are smooth, that is, when machines do not get predetermined or random break-downs. This assumption is not realistic. Practical importance of scheduling problem depends upon two factors i.e., job transportation time and break down machine time. These two concepts were separately studied by different researchers as Johnson (1954), Jackson (1956), Maggu and Das (1980) and

Maggu and Khodadadi (1988). They consider a two machine flow shop make span problem where for solution of a scheduling problem involving weights of jobs and break-down times for machines (Chandramouli, 2005). Consider a new simple heuristic algorithm for a 3-machine, n-job flow-shop scheduling problem in which jobs are attached with weights to indicate their relative importance and the transportation time and break-down intervals of machine.

Other related problems have been studied by Mitten (1959), Maggu et al. (1981), Langston (1987) and Yu (1996). Presents a simple rule to solve a twomachine flow-shop make span problem in which each job has a starting time lag and a completion time lag. A starting (completion) time lag forces the starting (completion) time of a job on the second machine to be at least some time later than that on the first machine. Clearly, if each job has either a starting time lag or a completion time lag, but not both, the resulting problem is then equivalent to the problem considered by Maggu and Das (1980) show that a simple rule can solve a more general problem that has both the features of the problem (Maggu and Das, 1980) and those of problem of Mitten (1959). Langston (1987) analyses some heuristics for a k-station flow-shop make span problem where each station has a number of machines that can be used to process jobs and there is only one transporter with a capacity of one transport jobs with transportation times depend on the physical locations of the origin and destination machines.

## NOTATION

The flow-shop model can be stated as follows:

Table 1: Three machines A, B, C with processing times and two

|  | Machines with processing time and transportation time |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | $\mathrm{A}_{\mathrm{i}}$ | $\mathrm{t}_{\mathrm{i}}$ | $\mathrm{B}_{\mathrm{i}}$ | $\mathrm{g}_{\mathrm{i}}$ | $\mathrm{C}_{\mathrm{i}}$ |
| 1 | $\mathrm{A}_{1}$ | $\mathrm{t}_{1}$ | $\mathrm{B}_{1}$ | $\mathrm{g}_{1}$ | $\mathrm{C}_{1}$ |
| 2 | $\mathrm{A}_{2}$ | $\mathrm{t}_{2}$ | $\mathrm{B}_{2}$ | $\mathrm{g}_{2}$ | $\mathrm{C}_{2}$ |
| - | . |  | . | . | . |
| - | . | . | . | . | . |
| - | - | . | . |  | . |
| $\underline{\square}$ | $\mathrm{A}_{\mathrm{n}}$ | $\mathrm{t}_{\mathrm{n}}$ | $\mathrm{B}_{\mathrm{n}}$ | $\mathrm{g}_{\mathrm{n}}$ | $\mathrm{C}_{\mathrm{n}}$ |

- Let n -job be processed through three machines A , $B$ and $C$ in order $A B C$
Letter " $i$ " denote the job in $S$ where $S$ is an arbitrary sequence. A job is available for processing at time zero
- Let each job be completed through the same production stage, i.e., ABC . In order words, passing is not allowed in the flow-shop
- Let $A_{i}, B_{i}, C_{i}$ denote the processing time of job " $i$ " on machine $\mathrm{A}, \mathrm{B}$ and C respectively $t_{i}$ and $g_{i}$ denote the transportation time of job " $i$ " from A to $B$ and from $B$ to $C$, respectively
- Find optimal schedule for the given problem when break-down effect-time interval ( $\mathrm{a}, \mathrm{b}$ ) is ignored by using Johnson's (1954) technique
- Read the effect of break-down time interval (a, b) on all jobs of the optimal schedule in step (d). For this, find total elapsed for the optimal schedule, say, $\mathrm{S}=\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right)$ in step (d)

Then our aim is to find out optimal or near optimal sequence of jobs so as to minimize the total elapsed time.

The given problem in the tabular time form may be stated as in Table 1.

## ALGORITHM

Suppose that either one or both of the following structural conditions involving the processing time and transportation time of jobs hold:

$$
\begin{aligned}
& \min _{i}\left(A_{i}+t_{i}\right) \geq \max _{i}\left(t_{i}+B_{i}\right) \\
& \min _{i}\left(g_{i}+c_{i}\right) \geq \max _{i}\left(t_{i}+B_{i}\right)
\end{aligned}
$$

Then the following steps of algorithm are:
Step 1: Modify the problem into two machines flowshop problem by introducing two fictitious machine G and H with processing time $G_{i}$ and $H_{i}$ respectively such that:
$G_{i}=A_{i}+t_{i}+B_{i}+g_{i}$ and $H_{i}=t_{i}+B_{i}+g_{i}+C_{i}$
The modified problem in the tabular form is (Table 2).
Step 2: Determine the optimal sequence by using Johnson's algorithm for the new reduced problem obtained in Step 1 and see the effect

Table 2:Two fictitious machines G and H with processing time and respectively such that $=++$ and $=++$

|  | Machines with processing time and transportation time |  |
| :---: | :---: | :---: |
| I | $\mathrm{G}_{\mathrm{i}}$ | $\mathrm{H}_{\mathrm{i}}$ |
| 1 | $\mathrm{G}_{1}$ | $\mathrm{H}_{1}$ |
| 2 | $\mathrm{G}_{2}$ | $\mathrm{H}_{2}$ |
| . | . | . |
| . | . | . |
| . | . | . |
| n | $\mathrm{G}_{\mathrm{n}}$ | $\mathrm{H}_{\mathrm{n}}$ |

Table 3: Determine the optimal sequence and the effect of break down interval $(\mathrm{a}, \mathrm{b})$ on the different jobs

| Job | Machines with processing time and transportation time |  |  |
| :---: | :---: | :---: | :---: |
| I | $\mathrm{A}_{\mathrm{i}}$ | $\mathrm{B}_{\mathrm{i}}$ | $\mathrm{C}_{i}$ |
|  | In-out | In-out | In-out |
| $\alpha_{1}$ | $\mathrm{U}_{1 \mathrm{~A}}-\mathrm{T}_{1 \mathrm{~A}}$ | $\mathrm{U}_{1 \mathrm{~B}}-\mathrm{T}_{1 \mathrm{~B}}$ | $\mathrm{U}_{1 \mathrm{C}}-\mathrm{T}_{1 \mathrm{C}}$ |
| $\alpha_{2}$ | $\mathrm{U}_{2 \mathrm{~A}}-\mathrm{T}_{2 \mathrm{~A}}$ | $\mathrm{U}_{2 \mathrm{~B}}-\mathrm{T}_{2 \mathrm{~B}}$ | $\mathrm{U}_{2 \mathrm{C}}-\mathrm{T}_{2 \mathrm{C}}$ |
| . | . | - | . |
| - | . | . | . |
| ; | U T | - |  |
| $\alpha_{n}$ | $\mathrm{U}_{\mathrm{nA}}-\mathrm{T}_{\mathrm{nA}}$ | $\mathrm{UnB}-\mathrm{T}_{\mathrm{nB}}$ | $\mathrm{UnC}-\mathrm{T}_{\mathrm{nc}}$ |

Table 4: The different way of Table 3

| Job | Machines with processing time and transportation time |  |  |
| :---: | :---: | :---: | :---: |
| I | $\mathrm{A}_{\mathrm{i}}$ | $\mathrm{B}_{\mathrm{i}}$ | $\mathrm{C}_{\mathrm{i}}$ |
| $\alpha_{1}$ | $\left(\mathrm{U}_{1 \mathrm{~A}}, \mathrm{~T}_{1 \mathrm{~A}}\right)$ | $\left(\mathrm{U}_{1 \mathrm{~B}}, \mathrm{~T}_{1 \mathrm{~B}}\right)$ | $\left(\mathrm{U}_{1 \mathrm{C}}, \mathrm{T}_{1 \mathrm{C}}\right)$ |
| $\alpha_{2}$ | $\left(U_{2 A}, T_{2 A}\right)$ | $\left(U_{2 B}, T_{2 B}\right)$ | $\left(U_{2 C}, T_{2 C}\right)$ |
| . | . | . | . |
| - | . | . | . |
| $\alpha$ |  |  | ( U , T ) |
| $\alpha_{n}$ | $\left(\mathrm{U}_{\mathrm{nA}}, \mathrm{I}_{\mathrm{nA}}\right)$ | $\left(\mathrm{UnB}, \mathrm{T}_{\mathrm{nB}}\right)$ | $\left(\mathrm{n}_{\mathrm{nC}}, \mathrm{T}_{\mathrm{nc}}\right)$ |

of break-down interval $(a, b)$ on the different jobs (Table 3).
Table 3 can be written as follow as in Table 4: where, $\left(U_{i x}, T_{i x}\right),\left(i=\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n} ; \mathrm{X}=\mathrm{A}\right.$, $\mathrm{B}, \mathrm{C})$ denote intervals of processing times of jobs $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ on machine X with the first co-ordinate $U_{i x}$ denoting as start-time and the second co-ordinate $T_{i x}$ denoting completiontime of the concerned job on machine X .

## Step 3:

Case (1):
$\operatorname{If}\left(U_{i x}, T_{i x}\right) \cap(\mathrm{a}, \mathrm{b})=\phi$
Then the optimal sequence $S$ is optimal and the given break-down time interval $(a, b)$ is not effective on the optimal schedule S .

## Case (2):

$$
\operatorname{If}\left(U_{i x}, T_{i x}\right) \cap(\mathrm{a}, \mathrm{~b}) \neq \phi
$$

Then formulate a new processing time $A_{l}, \dot{B}_{l}, \dot{C}_{l}$ where,

$$
\begin{aligned}
& \dot{A}_{l}=\dot{A}+(\mathrm{b}-\mathrm{a}) \\
& \dot{B}_{l}=\dot{B}+(\mathrm{b}-\mathrm{a}) \\
& \dot{C}_{l}=\dot{C}+(\mathrm{b}-\mathrm{a})
\end{aligned}
$$

| Job | Machines with processing time and transportation time |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | $\mathrm{A}_{\mathrm{i}}$ | $\mathrm{t}_{\mathrm{i}}$ | $\mathrm{B}_{\mathrm{i}}$ | $\mathrm{gi}_{\text {i }}$ | $\mathrm{C}_{\mathrm{i}}$ |
| 1 | 18 | 1 | 10 | 2 | 10 |
| 2 | 13 | 3 | 9 | 5 | 14 |
| 3 | 12 | 2 | 6 | 4 | 10 |
| 4 | 10 | 5 | 5 | 1 | 11 |
| 5 | 8 | 4 | 3 | 3 | 9 |
| 6 | 6 | 6 | 2 | 6 | 5 |


| Job | Machines with processing time and transportation time |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | $\mathrm{A}_{\mathrm{i}}$ | $\mathrm{t}_{\mathrm{i}}$ | $\mathrm{B}_{\mathrm{i}}$ | $\mathrm{g}_{\mathrm{i}}$ | $\mathrm{C}_{\mathrm{i}}$ |
| 5 | 0-8 | 4 | 12-16 | 3 | 19-28 |
| 4 | 8-18 | 5 | 23-28 | 1 | 29-40 |
| 2 | 18-31 | 3 | 34-43 | 5 | 48-62 |
| 1 | 31-49 | 1 | 50-60 | 2 | 62-72 |
| 3 | 49-61 | 2 | 63-69 | 4 | 73-83 |
| 6 | 61-67 | 6 | 73-75 | 6 | 83-88 |

Table 7: A new problem after effect of break-down interval

|  | Machines with processing time and transportation time |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | $\mathrm{A}_{\mathrm{i}}$ | $\mathrm{t}_{\mathrm{i}}$ | $\mathrm{B}_{\text {i }}$ | $\mathrm{g}_{\text {i }}$ | $\mathrm{C}_{\text {i }}$ |
| 1 | 25 | 1 | 10 | 2 | 10 |
| 2 | 13 | 3 | 9 | 5 | 14 |
| 3 | 12 | 2 | 6 | 4 | 10 |
| 4 | 10 | 5 | 12 | 1 | 11 |
| 5 | 8 | 4 | 3 | 3 | 16 |
| 6 | 6 | 6 | 2 | 6 | 5 |

Table 8: Optimal sequence
Machines with processing time and transportation time

| Job |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | $\mathrm{A}_{\mathrm{i}}$ | $\mathrm{t}_{\text {i }}$ | B́ $_{\text {i }}$ | $\mathrm{g}_{\mathrm{i}}$ | Ć $_{\text {i }}$ |
| 5 | 0-8 | 4 | 12-15 | 3 | 18-34 |
| 4 | 8-18 | 5 | 23-35 | 1 | 36-47 |
| 2 | 18-31 | 3 | 35-44 | 5 | 49-59 |
| 1 | 31-56 | 1 | 57-67 | 2 | 69-80 |
| 3 | 56-68 | 2 | 70-76 | 4 | 80-84 |
| 6 | 68-74 | 6 | 80-82 | 6 | 88-94 |

Step 4: Now repeat the procedure to get the optimal sequence.

This sequence is either optimal or near optimal for the original problem. By this sequence we can determine the total elapsed time.

## NUMERICAL EXAMPLE

Consider a six job and three machine problems with processing time, transportation time of jobs is given as shown in Table 5.

Find optimal or near optimal sequence when the break-down interval is $(\mathrm{a}, \mathrm{b})=(20,32)$. Also calculate the total elapsed time.

Solution: Now Min $\left(A_{i}+t_{i}\right)=12$

$$
\operatorname{Max}\left(t_{i}+B_{i}\right)=12
$$

Hence, (I) structural condition is satisfied. Then, using the steps 1 to 5 and applying Johnson technique for optimal or near optimal sequence then we get the sequence $S=(5,4,2,1,3,6)$.

Now to check the effect of break down interval $(22,29)$ on sequence $S=(5,4,2,1,3,6)$ is read as shown in Table 6.

Hence, with the effect of break-down interval the original problem becomes a new problem as per step (e) in Table 7.

The final Table 8 now, repeating the procedure we get the sequence $(5,4,2,1,3,6)$, which is optimal or near optimal and is?

## CONCLUSION

This method is proposed to obtain an optimal flowshop scheduling problems of 6-jobs on 3-machines. This method is very easy to understand and apply it. It will help the managers in a simple manner. Determining a best schedule for given sets of jobs can help decision making process to provide an optimal or near optimal scheduling sequence to control job flows and a solution for job sequencing. Hence 94 h is an optimal or near optimal sequence for the original problem. With any other schedule we can never find less than 94 h .

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