

Research Article

Effect of Er⁺³ Concentration on the Small Signal Gain Coefficient and the Gain in the Erbium Doped Fiber Amplifier

¹O. Mahran, ¹Mohammed S. Helmi, ¹Gamal D. Roston, ²Naglaa El. Sayed and ²Engy M. Gerges

¹Faculty of Science,

²Faculty of Education, University of Alexandria, Egypt

Abstract: The small signal gain coefficient and the gain of Erbium-Doped Fiber Amplifier (EDFA) in the wavelength range (1400-1700 nm) for different erbium concentrations and different amplifier lengths are calculated and studied. A core graded-index and erbium-doped concentration, are optimized for an EDFA in simplified two-level model. There is evidence to show that, the gain increases with the erbium concentration and the amplifier length. Where the relation between the gain and the amplifier length at different wavelengths is linear with the maximum gain at $\lambda = 1530$ nm. Also the temperature dependence of the small signal gain coefficient and the gain at the peak wavelength of EDFA was studied which shows, slightly increase in the values of both with temperature. The value of the signal wavelength was chosen in the gain window of EDFA at 1530 nm.

Keywords: Core-index, EDFA, erbium concentration, gain, small signal gain coefficient

INTRODUCTION

Rare earth doped fiber amplifiers and laser are important tools in understanding and designing new optical devices. Erbium-Doped Fiber Amplifiers (EDFAs) are attractive devices for single-mode fibers in optical communication systems in the 1530 nm wavelength band which is known as a third window for fiber optic communication. EDFAs have many advantages such as high gain and low noise in optical communication networks. EDFAs are characterized by gain which depends on erbium concentrations and this feature is very interesting in the modern optical transmission systems which use Wavelength Division Multiplexing (WDM) (Kemtchou *et al.*, 1997).

Erbium-Doped Fiber Amplifiers (EDFAs), as key components in Wavelength Division Multiplexing (WDM) systems in optical telecommunication, have received great attention over the past 10 years. The rapid growth and the future commercial importance of multi-wave length optical networking create strong incentives for the development of EDFAs with higher gain and broader bandwidth. Many interesting research results were reported in recent years. For example, Chernyak and Qian (2002) established modeling high-concentration L-band EDF A at high optical powers based on inversion function. Johannes (2004) reported spatial distribution effects and laser efficiency in Er/Yb-doped fibers. Wei *et al.* (2004) utilized a genetic algorithm to optimize multistage erbium-doped fiber amplifier systems with complex structures. A remarkable modeling was introduced by Giles and Desurville

(1991), which established the propagation and rate equations for a two-level homogeneous laser medium. This approximated model is suitable for analyzing open-loop optical fiber amplifiers and also the steady-state operation of the optical fiber networks.

For radial effects of the fibers on EDFA performances of the, there have been a few researches reported in recent years. One of the researches was presented by Martin (2001), who studied erbium transversal distribution influence on the effectiveness of a doped fiber by introducing a simple mathematical function and significant gain differences were observed in active fibers.

It is apparent that the transmission performances are manipulated and optimized by controlling the optical and geometrical parameters in the fiber structures. As a result, any undesired variation in the fiber structure parameters, can perturb the transference performances. The refractive index variation as a function of temperature (dn/dT) is the important feature in the optical fibers. This factor determines the temperature characteristics of an optical fiber transmission system (Osama, 2010a). Aerial optical systems are expected to face changes of temperature in several areas of the planet, which compel the essential demand to contemplate the thermal effect in the design of high-speed optical communication systems (Rostami and Makouei, 2008).

In this research the dependence of the erbium concentration on the core graded refractive index is done, also the small signal gain coefficient and the gain spectrum in the wavelength range (1400-1700) nm for

different erbium concentrations and different amplifier lengths are studied. So we can expect an increase for the gain with the erbium concentration and the amplifier length. Where the relation between the gain and the amplifier length at different wavelengths is linear with the maximum gain at $\lambda = 1530$ nm. Also the temperature dependence of the small signal gain coefficient and the gain at the peak wavelength of EDFA was studied which shows, slightly increase in the values of both with temperature. This is calculated by theoretical model which made by the authors.

MATERIALS AND METHODS

This theoretical research of the dependence of the erbium concentration on the core graded refractive index is done.

Model: There are several functional forms that can be chosen to describe the radial distribution of the erbium ions. We use an exponential function as follows (Osama, 2010a):

$$E_r(r) = E_{r0} \exp\left(\frac{-r}{\beta}\right)^\delta \quad (\beta, \delta > 0) \quad (1)$$

where, E_{r0} is the center concentration, β and δ are the parameters required to be optimized in the genetic algorithm. Equation (1) can show various radial profiles with different values of β and δ . At $r = 0$, E_r reaches a maximum, which is coincident with actual erbium-doped concentration.

For the fibers with radial distributions of the core refractive index (i.e., graded-index fibers), some parameters (e.g., cut-off frequency) describing light propagation in a step-index are no longer available since the graded-index alters with the core radius. Some detail analyses on this aspect were already developed and one of these is a usual variational method (Cheng and Xiao, 2005), in which a graded-index is reduced into an equivalent step-index. Here, a useful formula for the core refractive index is given by Osama (2010a):

$$n_{core}^2 = n_{clad}^2 \left[1 + 2\Delta n H\left(\frac{r}{a}\right) \right] \quad (r \leq a) \quad (2)$$

where, a is the fiber core radius and Δn is the relative refractive-index difference. The function H has the following form Osama (2010b):

$$H\left(\frac{r}{a}\right) = 1 - \left(\frac{r}{a}\right)^\alpha \quad 0 < \alpha < \infty \quad (3)$$

Then the core radius can give as:

$$r = \left[1 - \frac{1}{2\Delta n} \left(\frac{n_{core}^2}{n_{clad}^2} - 1 \right) \right]^{1/\alpha} a \quad (4)$$

Table 1: The optimized values of β , δ , α , a and Δn of the EDFAs

Central concentration	β (μm)	δ	α	a (μm)	Δn
6.43×10^{19}	2.6	2	0.108	4.1	0.0063

Table 2: Interpolated coefficient of the Eq. (8)

K (10^{-9})	J (10^{-6})	λ_g (μm)	α_t (10^{-6})
-1.6548	31.7794	0.109	0.45

The relation between the erbium concentration and the refractive index of the core can give by substituting the value of r in Eq. (1), we can give:

$$E_r(n_{core}) = E_{r0} \exp\left[-\frac{a}{\beta} \left[1 - \frac{1}{2\Delta n} \left(\frac{n_{core}^2}{n_{clad}^2} - 1 \right) \right]^{1/\alpha}\right]^\delta \quad (5)$$

The values of a , Δn , α , β and δ are chosen to optimizing an Erbium-Doped Fiber Amplifier (EDFA), (Osama, 2010b) and are given in Table 1.

The refractive index variation of the core due to temperature and wavelength is considered in this section. The method used in this study has been introduced by Ghosh (Rostami and Makouei, 2008). This model is based on the subscription of both electrons and optical phonons. The optical constants computed from this model are then used to calculate the refractive indexes at any operating temperature or wavelength for the optical fiber transmission system. Thermo-optic coefficient dn_{core}/dT contains the contribution of electrons and optical phonons. Consequently, it can be described in the optical transmission range in terms of linear expansion coefficient α_t and the temperature variation of energy gap (dE_g/dT) by the following relation as Osama (2010b):

$$2n \left(\frac{dn_{core}}{dT} \right) = \chi_e \left[-3\alpha_t - \frac{2}{E_g} \cdot \frac{dE_g}{dT} \cdot \frac{E_g^2}{(E_g^2 - E^2)} \right] \quad (6)$$

where, χ_e , E and E_g are the electronic susceptibility, photon energy and the suitable energy gap lying in the vacuum ultraviolet region, respectively. Equation (6) can be rewritten in terms of a normalized wavelength as:

$$R = \frac{\lambda^2}{\lambda^2 - \lambda_g^2} \quad (\text{Rostami and Makouei, 2008}) \quad (7)$$

$$2n \left(\frac{dn_{core}}{dT} \right) = KR + JR^2 \quad (8)$$

where, the constants K and J are related, respectively to the thermal expansion coefficient (α_t) and the energy gap temperature coefficient (dE_g/dT) according to the relations presented in Osama (2010a) and their values are given in Table 2 for silica glasses (Osama, 2010b). Integrating Eq. (8), we obtain:

$$n_{core} = \sqrt{(KR + JR^2)T} + n_o \quad (9)$$

where, n_o is constant equal the value of 1.455

The relation between the erbium concentration and the temperature can be obtained by substituting the Eq. (9) in (5). This is given by:

$$E_r(T) = E_{ro} \exp \left[-\frac{\alpha}{\beta} \left[1 - \frac{1}{2\Delta n} \left(\frac{(\sqrt{(KR + JR^2)T} + n_o)^2}{n_{clad}^2} - 1 \right) \right] \right]^{\gamma \delta} \quad (10)$$

The small signal gain coefficient (g) as a function of erbium concentration $E_r(n_{core})$ is given from Eq. (5) and (9), with the optimized values of α , β , δ and the relative inversion $D = 1$, which represents a complete medium inversion (Cheng and Xiao, 2005):

$$g = E_r(n_{core}) \frac{\{\sigma_e(\lambda_k)(1+D) - \sigma_a(\lambda_k)(1-D)\}}{2} \quad (11)$$

where, $\sigma_a(\lambda_k)$ and $\sigma_e(\lambda_k)$ are the absorption and emission cross sections of the laser transition at λ_k , respectively. The amplifier gain, G , can be calculated as:

$$G = \exp \left[\int_0^L (N_2(z)\sigma_s^{(e)} - N_1(z)\sigma_s^{(a)})\Gamma_s dz \right] \quad (12)$$

Equation (12) appears to suggest that the gain is a complex function of the shape of the inversion along the length of the fiber. In fact, the gain can be related simply to the average inversion. Define an average upper-state population \overline{N}_2 and similarly for the lower-state population \overline{N}_1 , which both are function of $E_r(n_{core})$ as Jander and Brockles (2004):

$$\overline{N}_2 = \frac{1}{L} \int_0^L N_2(z) dz \quad (13)$$

$$\overline{N}_1 = \frac{1}{L} \int_0^L N_1(z) dz \quad (14)$$

Equation (12) can then be simplified to:

$$G = \exp \left[(\overline{N}_2\sigma_s^{(e)} - \overline{N}_1\sigma_s^{(a)})\Gamma_s L \right] \quad (15)$$

This shows that, the signal gain after traversal of the fiber is dependent only on the average inversion of the erbium ions in the fiber and does not depend on the details of the shape of the inversion as a function of position along the fiber length.

RESULTS AND DISCUSSION

Table 1 lists some optimized results of the EDFA (Osama, 2010b) and the required radial distribution of

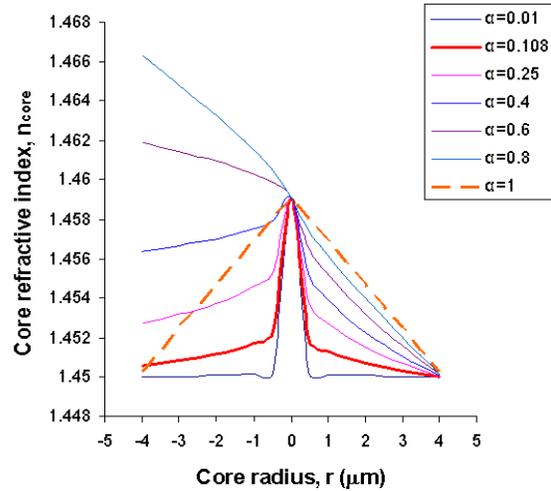


Fig. 1: Profiles of the core refractive index

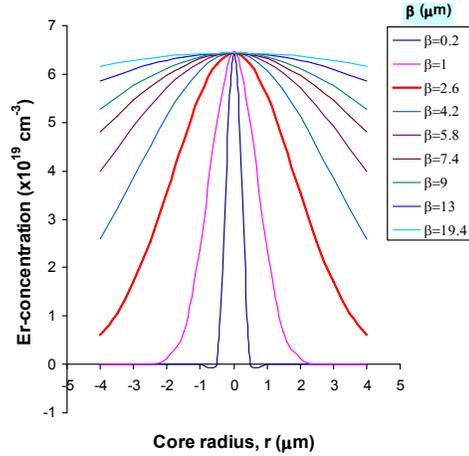


Fig. 2: Effect of β on Er-concentration

the core refractive index is shown in Fig. 1 which gives a symmetrical behavior of the core refractive index in both positive and negative sides of the core radius at $\alpha = 0.108$ (optimal value). The maximum value of $n_{core} = 1.459$ at which the erbium concentration are concentrated on the axis while it is decreases on the wings. For $\alpha > 0.108$, there is unsymmetrical behavior of the core refractive index in both the positive and negative parts.

Figure 2 shows the effect of the parameter β (describing erbium radial distribution, as defined in Eq. (1)) on the erbium concentration and consequently the gain bandwidth. Note that, the total erbium concentration, i.e., the areas under the profiles, is altered with various β values. As a comparison, we adopt a uniform erbium concentration while keeping the total concentration the same this with different distribution of erbium ions. The maximum value of erbium concentration is located at the axial core radius where $r = 0$ and this value decrease on the two side where the core radius increases. Figure 2 shows also

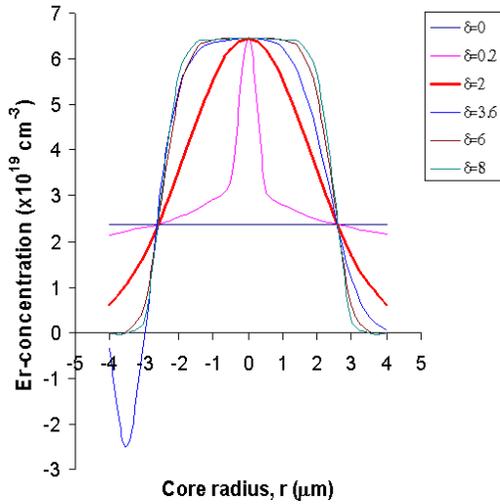


Fig. 3: Effect of δ on Er-concentration

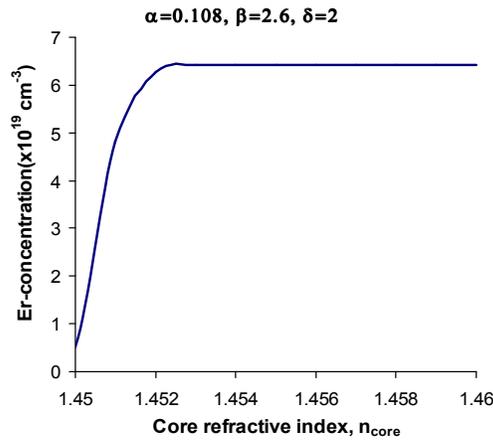


Fig. 4: The optimized values of α , β and δ on Er concentration with the effect of core refractive index

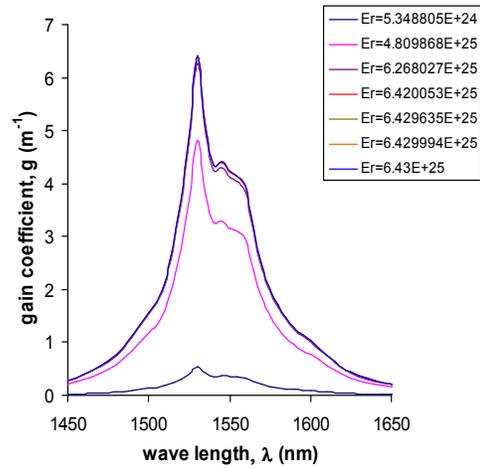
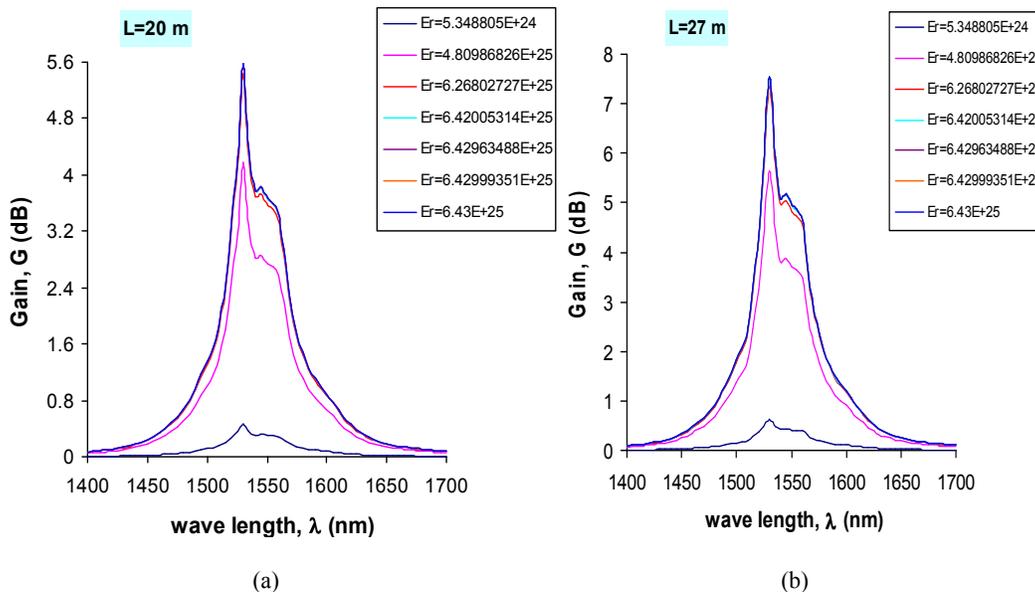


Fig. 5: Small signal gain coefficient of erbium doped alumino-germanosilicate glass fiber amplifier at different Er-concentration

that, the optimal value of β is 2.6 μm . This corresponds to the optimal distribution curve. At this value of β the distribution allows the signal to make total internal reflection inside the fiber.

Figure 3 shows the effect of the parameter δ on the erbium concentration profile. However, it is shown that for some values of δ , the erbium concentration takes a negative value which is not true. The optimal distribution take place when $\delta = 2$, which gives the same effect on the erbium concentration profile on the positive and negative sides of the core radius with maximum value of the erbium concentration = $6.43 \times 10^{19} / \text{cm}^3$ on the axis. The optimized values of α , β and δ of the EDFAs on the calculation of the erbium concentration as a function the core refractive index according to Eq. (5) is shown in Fig. 4.



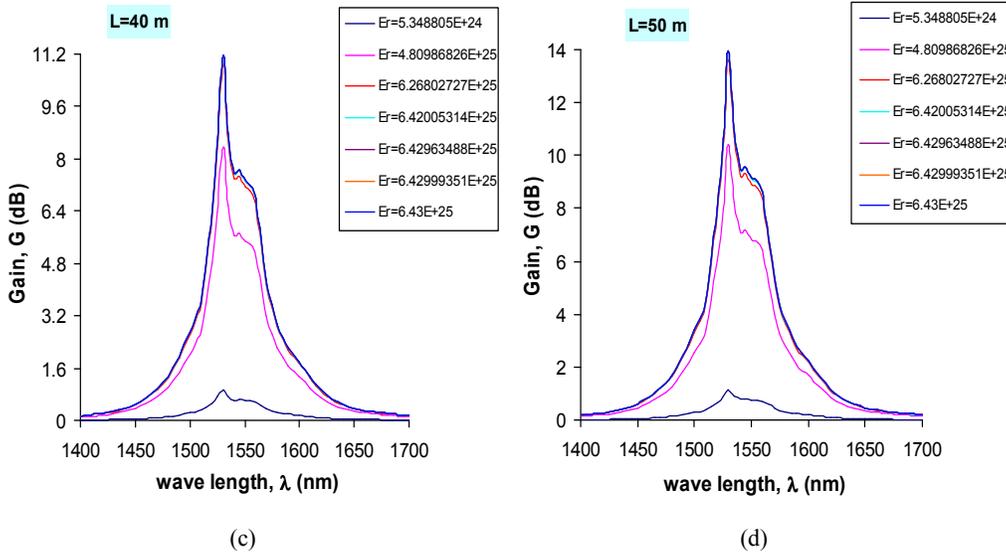


Fig. 6: Gain spectra for erbium doped alumino-germanosilicate glass fiber amplifier, for different values of Er-concentration at the labeled amplifier lengths as in a, b, c and d

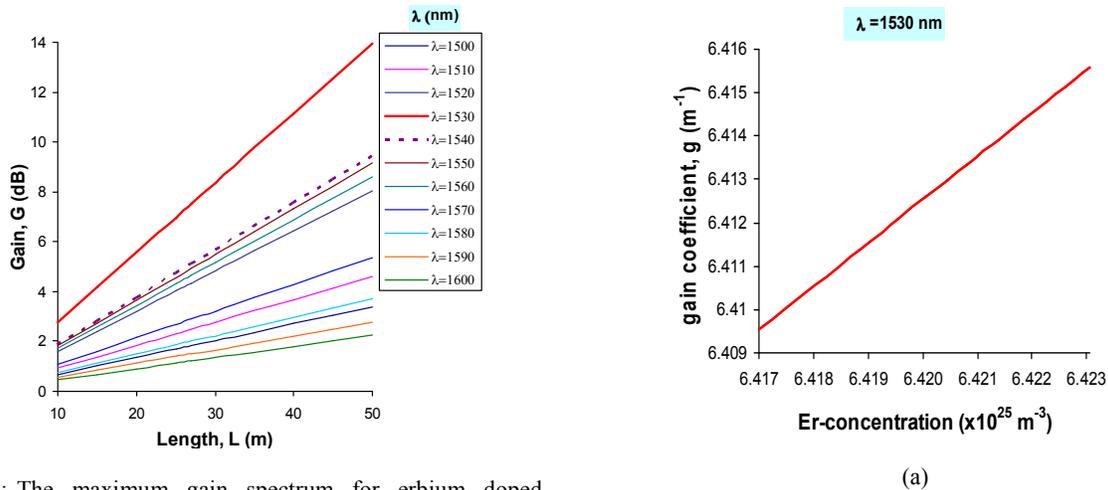


Fig. 7: The maximum gain spectrum for erbium doped alumino-germanosilicate glass fiber amplifier

The relation between the small signal gain coefficients (g) m^{-1} and the wavelength (λ) nm around the 1.5 μm laser transition of a typical Er^{+3} doped alumino-germanosilicate glass fiber amplifier at different values of erbium concentration, are illustrated in Fig. 5. Notice that when the erbium concentration increases the gain coefficient increases and show less broadening. Also the maximum gain coefficient (peak) at the wavelength 1530 nm, this peak is the reason for choosing the erbium ions doped silica fibers to be the most widely used devices, because they have a transition in the third telecommunication window, around 1530 nm.

The relation between the gain, G (dB), for erbium doped alumino-germano-silicate glass fiber amplifier and the wavelength with different values of the amplifier lengths at different erbium concentration are

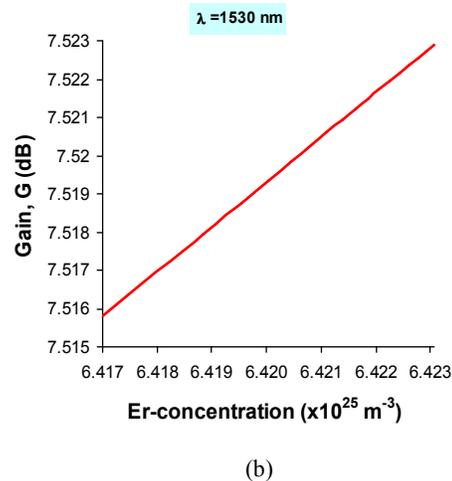


Fig. 8: The effect of Er-concentration on the gain coefficient and the gain at wavelength 1530 nm

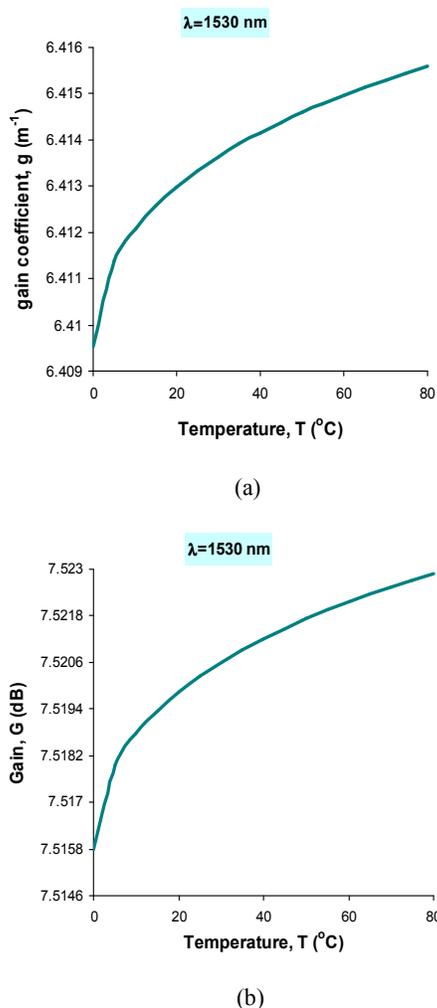


Fig. 9: The effect of temperature on the gain coefficient (a) and the gain at constant wavelength 1530 nm (b)

studied using Eq. (14). Figure 6 shows such relation at $L = 20, 27, 40, 50$ m in the range of the wavelength from 1400 to 1700 nm. It is found that, as the erbium concentration increases, the amplifier gain increases and the curve show less broadening, the maximum gain takes place at wavelength = 1530 nm.

The gain is also plotted with amplifier length at different values of wavelength and at $n_{\text{core}} = 1.46$. In Fig. 7, the relation is found to be linear between the gain and the length. This is clear from the relation of (G, L) . Also the maximum gain at wavelength = 1530 nm which is corresponding to the peak of the erbium ions doped silica fibers.

The effect of erbium concentration on the gain coefficient and the gain at the peak of the erbium ions are calculated according to the Eq. (10) and (14) is plotted in Fig. 8.

Figure 9 shows the temperature dependence of the small signal gain coefficient and the gain at constant

signal wavelength 1530 nm, according to the Eq. (9), (10) and (14). This show slightly increases in the values of gain coefficient $(g) \text{ m}^{-1}$ and of Gain $(G) \text{ dB}$ with Temperature $(T) \text{ }^\circ\text{C}$. The value of the signal wavelength was chosen in the gain window of EDFA at $\lambda = 1530$ nm.

CONCLUSION

Radial effects of the core graded-index and erbium doped concentration are studied with the optimized parameters values α, β and δ for an Erbium-Doped Fiber Amplifier (EDFA) in a two-level model. Also the small signal gain coefficient and the gain spectrum in the wavelength range (1400-1700) nm for different erbium concentrations and different amplifier lengths are studied. There is evidence to show that, the gain increases with the erbium concentration and the amplifier length. Where the relation between the gain and the amplifier length at different wavelengths is linear with the maximum gain at $\lambda = 1530$ nm. Also the temperature dependence of the small signal gain coefficient and the gain at the peak wavelength of EDFA was studied which shows, slightly increase in the values of both with temperature. The value of the signal wavelength was chosen in the gain window of EDFA at 1530 nm.

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