Research Journal of Applied Sciences, Engineering and Technology 8(23): 2343-2349, 2014
DOI:10.19026/rjaset.8.1238
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: September 22, 2014 Accepted: November 10, 2014 Published: December 20, 2014

Research Article

Discontinuity of Gas-dynamic Variables in the Center of the Compression Wave

Pavel Viktorovich Bulat and Mikhail Pavlovich Bulat
Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics,
Kronverksky pr. 49, Saint-Petersburg 197101, Russia

Abstract: The purpose of research-the study of the flow in the center of the centered isentropic compression waves. Gas-dynamic discontinuities cover shocks, shockwaves, interfaces and sliding surfaces and also the center of the centered compression wave one-dimensional and two-dimensional. For a long time there has been no analysis of the shockwave structures arising in the center of compression waves. At the same time, the problem of development of supersonic and hypersonic air inlets demands to consider the process of the stream isentropic compression. This problem is connected (three-dimensional case) to the problem of arising inside the streams of hinged shocks as opposite to the usual discontinuities not resulted by interaction of supersonic streams, waves and discontinuities, but like from nowhere. This study sets the problem for study in the terms of the developed theory of the interference of gas-dynamic discontinuities of the area of existing solutions for the structures of possible types. We have obtained the relations describing the parameters in the center of the compression wave. We have considered the neutral polar of neither compression meeting the case when in the center of the compression wave there neither shocks nor depression waves. The analysis of properties of the centered compression wave adds to the theory of stationary gas-dynamic discontinuities. We have specified the borders of the shock structure existence area optimal for development of supersonic diffusers.

Keywords: Centered compression wave, compression polar, isentropic compression, shockwave structure, shockwave, shock

INTRODUCTION

Objects of study are centered isentropic compression waves, mathematical model of the boundaries of the existence of different shock-wave structures in the center of the wave, as well as application of the theory developed for the design of isentropic air intakes.

If to assign the form of a concave surface according to the equation of the streamline in the Prandtl-Mayer plane wave, when it is covered with a supersonic stream of the compression wave (characteristics) of the Centered Compression Wave (CCW) ω_σ cross in the same point (point A in Fig. 1) (Uskov and Bulat, 2012; Bulat and Bulat, 2013; Uskov and Chernishev, 2006a). The shockwave structure forms with the basic shock σ by the limit intensity and with reflected gas-dynamic discontinuity R, which can be a shock, a centered depression wave or a weak discontinuity of the second order (discontinuity characteristics when not the values of gas-dynamic variables but their derivatives of discontinuity characteristic endure a discontinuity).

Until recently, the analysis of Such Shockwave Structures (SWS) has not been completed and classification of discontinuities and SWS arising as the result of their interference (Uskov *et al*., 1995), did not contain the information on the centered compression waves. This study`s objective is the gap recovery.

Stokes (1848) inserted the concept of discontinuity in the field of continuous medium stream and received two conditions for density ρ and gas velocity *w* on the continuity sides, following the law of mass conservation and the law of momentum. The discontinuities considered by Stokes (1848), according to the modern classification, are called as normal, therefore gas travels through their surface. The normal strong gas-dynamic discontinuities serve as a model for the shockwaves which were named by Riemann (1860). In the Russian literature, stationary waves are often called as shocks and shockwaves mean only running waves. Rankine in 1869-1870 (Rankine, 1869, 1870a, b) received the equation adding the system of the Stokes equations. He specified the link between the parameters on the shockwave sides having considered continuously changing inside medium conditions with equilibrium heat exchange. The total amount of the heat received by the medium he specified as equal to zero. Using the relations of the equilibrium thermodynamics and the formula in the Stokes` work, Rankine received

Corresponding Author: Pavel Viktorovich Bulat, Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics, Kronverksky pr. 49, Saint-Petersburg 197101, Russia

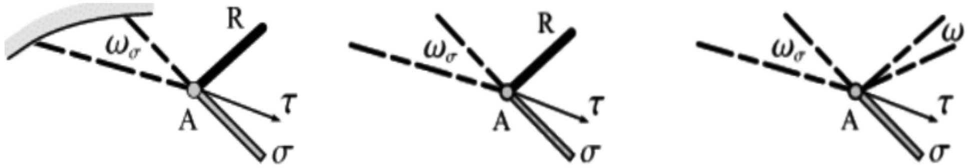

Fig. 1: Centered compression wave; a): Common case; b): Reflected discontinuity-shock; c): Reflected discontinuity-centered depression wave

the expressions for the velocity of distribution of normal discontinuity according to the stationary medium D and the following stream velocity u through the known pressures before the discontinuity and after it and specific volume before the discontinuity.

The third condition on the normal discontinuity as consequence of the law of conservation of energy over consideration of the gas condition inside of the shockwave, for the first time was received by Hugoniot in 1887–1889 (Hugoniot, 1889). This condition coincides with the earlier condition of Rankine, but for its derivation Hugoniot needed no extra suppositions.

Other types of discontinuity are contact discontinuities. Gas cannot travel through their surface. The contact discontinuity concept in 1868 was offered by Von Helmholtz (1868), who, in the series of his works, considered the stationary vortex streams of the non viscous medium. Helmholtz laid down the conditions of the dynamic compatibility of streams (CDC) on the contact discontinuity: the equity of static pressure on the sides of its surface and equity of normal to its surface components of the gas stream velocities.

The detailed analysis of gas-dynamic waves (isentropic waves of depression and compression) and angle shocks arising in the plane stationary streams of nonviscous low-conductivity perfect gas, was published in 1908 by Mayer (1908). Starting from the Mayer`s work, as the basic parameter characterizing the gas-dynamic discontinuity, they consider its intensity, i.e., relation of static pressure $J = P_2/P_1$ on its sides.

In an explicit form, the centers of centered waves of depression and compression were considered as discontinuities only in the works of the scientific school of Uskov and Bulat (2012) and Bulat *et al*. (1993) and M.V. Chernyshev (Uskov and Chernishev, 2006b, 2008) in 2006-2008 analyzed travelling of the under-expanded stream around the edge of Laval supersonic nozzle, where gas-dynamic parameters endure discontinuity. Further, P.V. Bulat carried out a short analysis of the centered compression wave (Uskov and Bulat, 2012; Bulat and Bulat, 2013).

Let`s consider the domains of existence of different SWS appearing in the CCW center.

MATHEMATICAL MODEL OF THE SHOCKWAVE STRUCTURE

It is known that inside the compression wave the stream parameters are described with the Prandtl-Mayer solution for the plane centered wave:

$$\omega_1 + \vartheta_1 = \omega_\infty + \vartheta_\infty$$

where,

ω = The Prandtl-Mayer function

ϑ = The angle of velocity vector

Then, inserting the concept of $J_\omega = P_1/P_\infty$ the compression wave intensity can be written:

$$\omega_\infty - \omega\left(\sqrt{\frac{2}{\gamma-1}\left(1+\frac{\gamma-1}{2}M^2\right)J_\omega^{-\frac{\gamma-1}{\gamma}}-1}\right) = \vartheta_1 - \vartheta_\omega$$

The angle of the stream turn β is specified by the following functional dependences: in the center of isentropic compression wave:

$$\beta = \omega\left\{\sqrt{\frac{2}{\gamma-1}\left(1+\frac{\gamma-1}{2}M^2\right)J_\omega^{-\frac{\gamma-1}{\gamma}}-1}\right\} - \omega_\infty$$

On the shock:

$$tg\beta = \sqrt{\frac{J_m - J}{J+\varepsilon}}\frac{(1-\varepsilon)(J-1)}{(J_m+\varepsilon)-(1-\varepsilon)(J-1)}, \varepsilon = \frac{\gamma-1}{\gamma+1}$$

where,

$$J_m = (1+\varepsilon)M^2 - \varepsilon$$

The curves described by these equations we will call as compression polar and shock polar correspondingly. Points on the compression polar show the relation of pressure after CCW to the pressure in the undisturbed stream and the stream turn angle in the center of the compression wave.

At the origin ($\Lambda = 0$, $\beta = 0$, where $\Lambda = lnJ$) the compression polar and the shock polar have the order of contact not less than the second one. This property can be simply expressed as follows:

$$\beta_\delta^{(i)} = \beta_\omega^{(i)} + \Delta_i \beta^{(1)}$$

where,

$$\Delta^1 = \Delta^2 = 0, \beta^{(1)} = \frac{d\beta}{dJ}_{j=1} = \frac{\left(M^2-1\right)^{1/2}}{\gamma M^2}$$

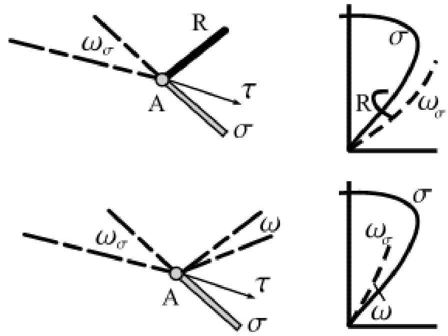

Fig. 2: CCW with reflected discontinuity shock and depression wave

The difference in values of the higher derivatives (i>2) of these curves at J = 1 depends on Δ_i. Leaving out elementary computation, one can write for Δ_i at i = 3.4:

$$\Delta_3 = \frac{\left(\left(M^2-1\right)^{-1}-1\right)^2-4\varepsilon}{4(1+\varepsilon)^2}$$

$$\Delta_4 = \frac{A}{M^2}-\frac{B}{M^4}-\frac{C}{M^6}+\frac{D}{M^2-1}+\frac{E}{(M^2-1)^2}-\frac{F}{(M^2-1)^3}+G$$

where,

$$A = \frac{1}{2\gamma}\left(\frac{14}{\gamma}+\frac{11}{\gamma^2}-1\right),\ B = \frac{12}{\gamma},\ C = \frac{16}{\gamma^3},$$

$$D = \frac{1}{8}\left(17-\frac{15}{\gamma}-\frac{53}{\gamma^2}-\frac{7}{\gamma^3}\right),$$

$$E = \frac{1}{8}\left(-4-\frac{13}{\gamma}+\frac{11}{8\gamma^2}-\frac{3}{\gamma^3}-\frac{3}{\gamma^4}\right)$$

$$F = \frac{1}{8}\left(-13-\frac{7}{\gamma}-\frac{25}{\gamma^2}-\frac{19}{\gamma^3}\right),$$

$$G = \frac{1}{8}\left(23+\frac{1}{\gamma}+\frac{17}{\gamma^2}+\frac{7}{\gamma^3}\right)$$

At γ = *const* dependence $\Delta_3(M)$ is nonmonotonic. It has the roots at the Mach number values equal to:

$$M_{f_{1,2}} = \sqrt{\frac{2}{5-3\gamma}\left(3-\gamma\pm\sqrt{\gamma^2-1}\right)}$$

And minimal at $M = \sqrt{2}$ for any γ. At $M = M_{f1,2}$ the polers (of compression and shock) at the origin have the third order of contact. Function Δ_4(M) has no real roots. Product $\Delta_3\beta_0^{(1)}$ has extremums at $M = \sqrt{2}$ and $M = M_\Delta$:

$$M\Delta = \sqrt{2}\left\{\frac{2(1-\varepsilon)+\sqrt{3(1-\varepsilon)}}{1-4\varepsilon}\right\}^{1/2}$$

$\Delta_3\beta_0^{(1)}$ tends to ∞ at $M\to 1$ *and* to 0 at $M\to\infty$. The product roots coincide with the roots Δ_3.

The type of reflected discontinuity depends on the polars mutual location (Fig. 2).

As it follows from the abovementioned results, the compression polar can go at the origin both inside the shock polar ($M < M_{f_1}$ and $M > M_{f_2}$) and outside ($M_{f_1} < M < M_{f_2}$) (Fig. 3).

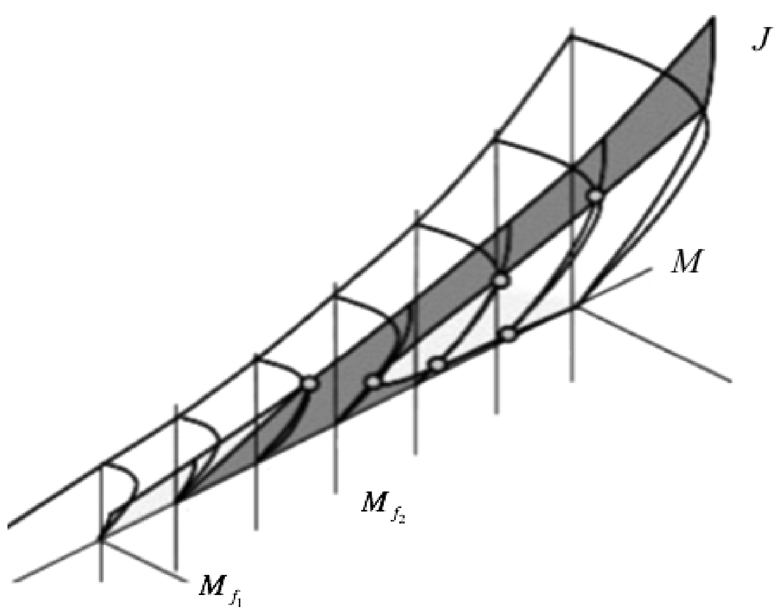

Fig. 3: Transformation of polars according to change of the Mach number the compression polar is toned, the shockwave is transparent

RESULTS AND ANALYSIS

NEUTRAL SWS: For some values of M and γ, the compression polar and the shock polar can cross each other. At the crossing point there is equation of the intensities ($J_\omega = J_\sigma$) of the shock σ and CCW, as well as the stream turn angles on these discontinuities. And consequently, the condition of co linearity of velocity vectors on the tangent discontinuity is carried out according to degeneration of the reflected discontinuity R into the characteristic. Let`s call such SWS as neutral, intensity CCW in the polar crossing point we designate $J_н$ and relevant curve $J_н$ (M,γ) we call as the neutral compression polar. The typical neutral SWS is shown in Fig. 4. The compression polar can cross the shock polar distributing at the origin inside it and outside it.

The neutral polar has two branches (Fig. 5). As the isentropic compression wave cannot break the stream up to the velocity lower than the sonic speed, its existence domain is limited with upper sound line $J_{s\omega}$. For comparison Fig. 5 shows the shock sound intensity J_s.M_2 is a crossing point of the diagram left branch$J_н(M)$ with the sound line of the compression polar $J_{s\omega}$. M_3 is a fold point of neutral polar $J_н(M)$. Point

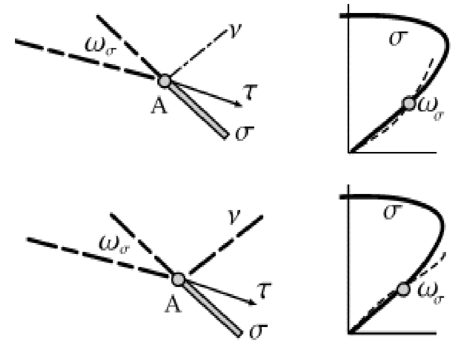

Fig. 4: Neutral SWS

"s" of crossing $J_н$ and J_s meets the case when the neutral polar crosses the shock polar in the sound point.

Dependence $J_н$ (M, γ) is shown in Fig. 6. Here, the surface J_H is in dark color and J_s is semi-transparent. The plane γ-M demonstrates diagrams of the characteristic Mach numbers M_{Hi}. One can wee that at γ = 1.67 M_{H4} tends to infinity. M_{H3} (projected fold line of surface J_H (M, γ)) blends at γ = 1.1 with M_{H4}. So, at γ = 1.1 there is a feature like a "fold". The resulting picture resembles the domain of existence of characteristic points for regular interaction of follow shocks (Uskov and Starykh, 1990).

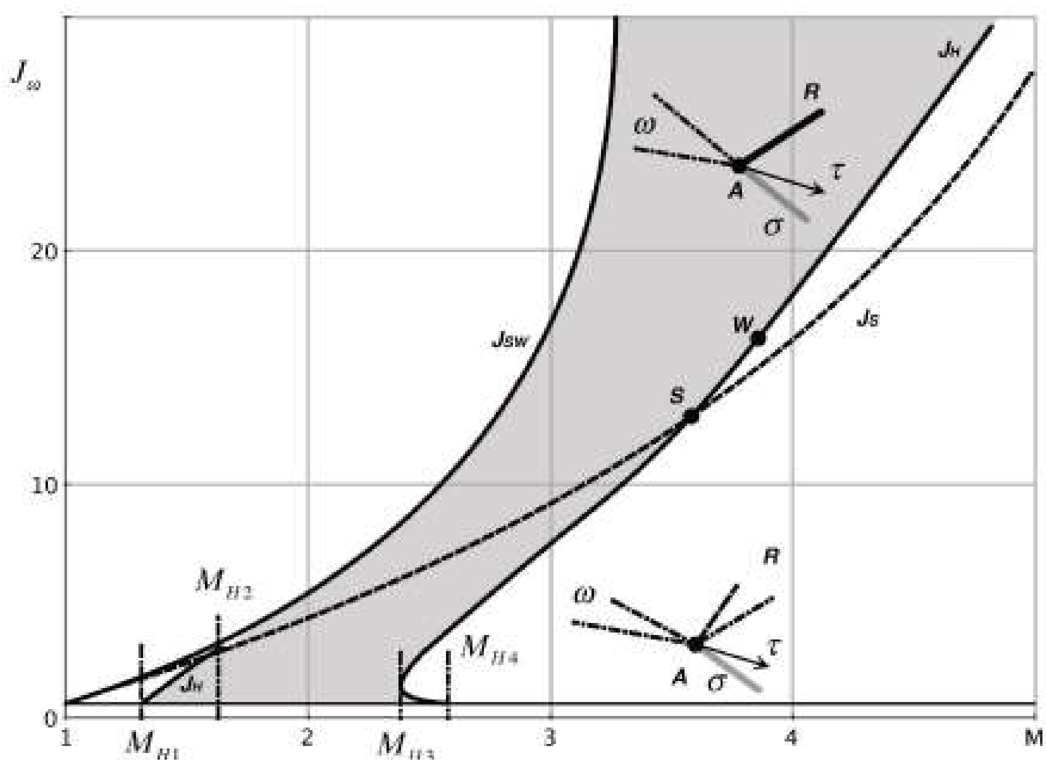

Fig. 5: Neutral polar and domain of CCW existence with reflected discontinuity-shock (shaded) J_ω is the intensity of the centered compression wave, $J_{s\omega}$ is the sound intensity of the centered compression wave, J_s is the sound intensity of shock, $M_{1\text{-}4}$ are the special Mach numbers, M_w is the Mach number limiting the domain of existence of shockwave structures with reflected discontinuity-shock

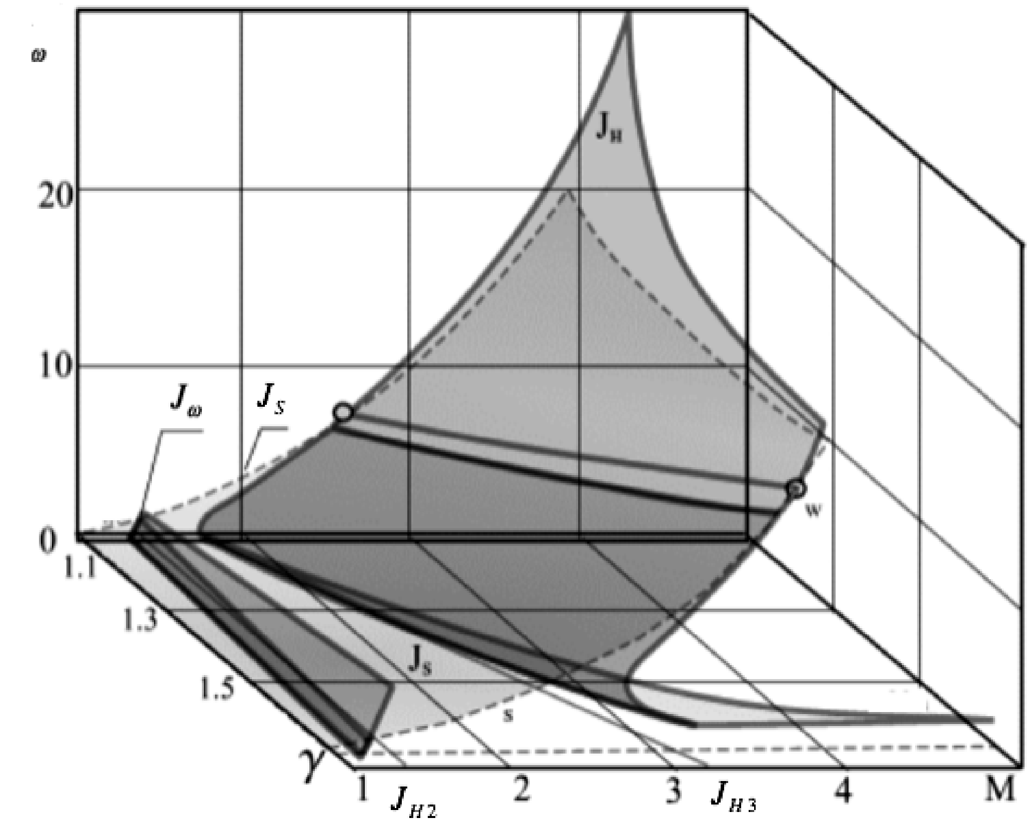

Fig. 6: Neutral polar at different adiabatic indexes

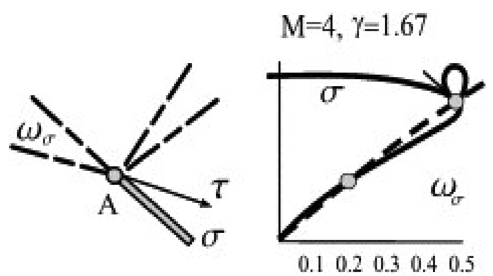

Fig. 7: Polars meeting the "special Mach number M_w. At $M>M_w$ SWS with reflected discontinuity-a shock cannot exist

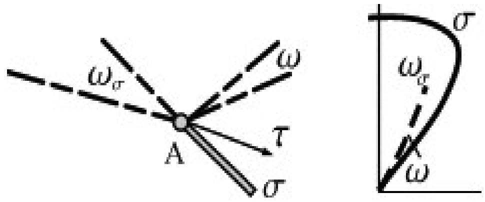

Fig. 8: $M<M_{H1}$

Domains of existence of stable shockwave structure in the centered compression wave: The neutral polar restricts the domains of SWS existence with different type of reflected discontinuity. Two branches of the neutral polar and sound line limit the domain of the SWS existence on three sides with reflected discontinuity-the depression wave (in Fig. 5 this area is shaded). In Fig. 5 and 6 the special intensity CCW and appropriate Mach number M_w is marked with index "w". For any adiabatic index starting with the Mach number M_w, the SWS containing the reflected shock cannot exist (polars have no crossing points, Fig. 7). In the domain of the Mach numbers, large M_w, the reflected shock is always a depression wave for any values of intensity CCW except for $J_н$. The special Mach numbers, appropriate intensities CCW and the stream turn angles are given in Table 1.

Let`s consider how the mutual location of the shock polar and the compression polar change following increasing the Mach number of undisturbed stream. For the Mach number $M<M_{H1}$, the compression polar is inside the shock polar in full, consequently, in this range of Mach numbers only *SWS* is possible with reflected discontinuity-depression wave (Fig. 8). The compression polar for the Mach numbers M higher than M_{H1} near the origin distributes beyond the shock polar crossing it upper. Here we have as a reflected discontinuity a shock or a depression wave, which is specified by intensity CCW (Fig. 9). These two cases is separated with the neutral configuration, i.e., for the wave intensity more than $J_н$the reflected discontinuity is a compression wave and for the wave intensities less than $J_н$ the reflected discontinuity is a shock.

Table 1: Special Mach numbers, appropriate intensities CCW and the stream turn angles

γ	M_{H1}	M_{H2}	M_{H3}	M_{H4}	M_w	J_w	$\beta_{\sigma\omega}$
1.10	1.302	1.486	-	1.666	2.599	6.345	0.698
1.25	1.265	1.484	1.940	2.000	2.990	8.713	0.669
1.40	1.245	1.478	2.230	2.539	3.483	12.18	0.635
1.67	1.225	1.746	2.857	-	4.670	22.76	-

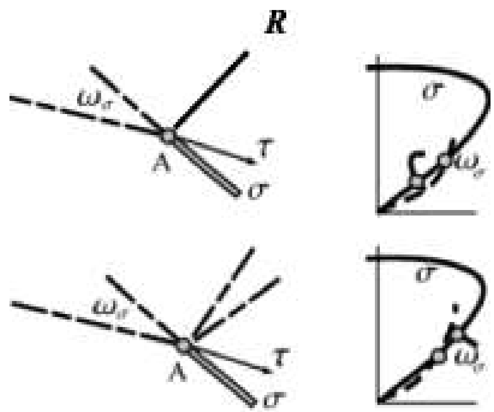

Fig. 9: $M_{H1}<M<M_{H2}$

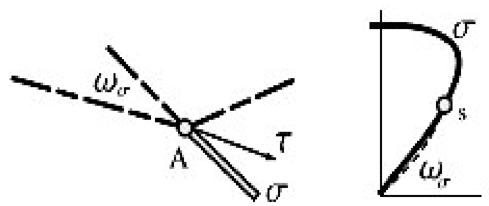

Fig. 10: $M = M_{H2}$

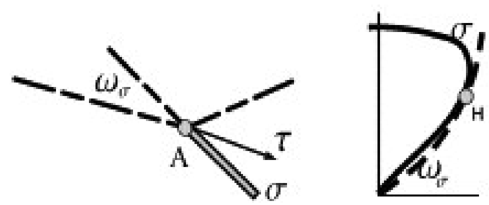

Fig. 11: $M = M_{H3}$

Following the increasing of M, the value $J_н$ increases and for M_{H2} reaches the sound intensity of the compression wave. For this Mach number the compression polar is beyond the shock polar completely touching it with “upper edge” (point s in Fig. 10).

In the range of Mach numbers M_{H2}-M_{H3}, the compression polar goes beyond the shock polar and does not cross it, correspondingly, characteristic *SWS* in this range cannot arise. For the Mach number M equal to M_{H3} the compression polar contacts with the shock polar. In Fig. 11 the contact point is marked with a circle. The circle meets $J_н$ and neutral SWS. For all other intensities, the reflected discontinuity is a shock.

Following the decrease of γ, the compression wave intensity, appropriate the contact point, decreases and for $\gamma = 1.1$ turns into 1. This γ meets the third order of the polar contact. At any γ, the contact point lies lower than the sound point on the shock polar.

For the Mach numbers over M_{H3} and less than M_{H4}, the polars cross in two points. Between the crossing points the compression polar goes inside the shock polar. Following the increasing M the upper point moves onto the strong branch of the shock polar and the intensity appropriate to the lower point decreases and turns into 1 for $M = M_{H4}$. For higher M the polars have only one crossing point. In gas with $\gamma = 1.67$-M_{H4} turns into infinity, i.e., a range where the compression polar crosses the shock polar inside and absent in one point.

CONCLUSION

We have considered the centered compression wave. We have studied the domains of different SWS existence arising in the center of CCW. We have studied the compression polars (dependence of the CCW intensity on the stream turn angle). It is shown that for the intensities near 1 the compression polar has the second order of contact with the shock polar and for the special numbers M_f-the third order. Depending on the Mach number, the compression polar can distribute both inside the shock polar and outside it, it can cross in one or two points. There are contact points of these curves. The SWS meeting the polars crossing contains a discontinuity characteristic as the reflected discontinuity. Such a polar and corresponding SWS are called as neutral. The neutral polar separates the domains of existence of SWS with the reflected shock and depression wave.

ACKNOWLEDGMENT

This study was prepared as part of the "1000 laboratories" program with the support of Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics (University ITMO)and with the financial support of the Ministry of Education and Science of the Russian Federation (the Agreement #14.575.21.0057).

REFERENCES

Bulat, M.P. and P.V. Bulat, 2013. The analysis centric isentropic compression waves. World Appl. Sci. J., 27(8): 1023-1026.

Bulat, P.V., O.N. Zasukhin and V.N. Uskov, 1993. Formation of a Jet during the Laval Nozzle Smooth Start. In: Dulov, V.G. (Ed.), ICH: Gas Flow in Channels and Jets. Publishing House SPbGU, SPb, pp: 3-22.

Hugoniot, H., 1889. Propagation du mouvementdans les corps. Chapitre V. Sur les discontinuit́es qui se manifestentdans la propagation du movement. J. l'É colePolytechnique, cahier LVIII, pp: 68-125.

Mayer, T., 1908. Uber zweidimensionale Bewegungsvorgange in einem Gas, das mit Uberschallgeschwindigkeit strömt. Ph.D. Thesis, Georg-August Uni-versitat, Gottingen.

Rankine, W.J.M., 1869. On the thermodynamic theory of waves of finite longitudinal disturbance. P. R. Soc. London, 18(115): 80-83.

Rankine, W.J.M., 1870a. On the thermodynamic theory of waves of finite longitudinal disturbance. Philos. Mag., 4(39): 306-309.

Rankine, W.J.M., 1870b. On the thermodynamic theory of waves of finite longitudinal disturbance. Philos. T. R. Soc. Lond., 160(1870): 277-288.

Riemann, B., 1860. Ueber die Fortpflanzung ebener Luftwellen von endlicher Schwingungsweite. Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen, 8: S. 43.

Stokes, G.G., 1848. On a difficulty in the theory of sound. Philos. Mag., 3(33): 349-356.

Uskov, V.N. and A.L. Starykh, 1990. Analysis of the areas of solutions for the equation of stationary gas-dynamic discontinuities interference. Sb. Non-stationary Gas Streams with Shockwaves, pp: 359-372.

Uskov, V.N., A.L. Adrianov and A.L. Starykh, 1995. Interference of Stationary Gas-dynamic Discontinuities. Novosibirsk: VO Nauka, pp: 180.

Uskov, V.N. and M.V. Chernishev, 2006a. Differential characteristics of the flow field in a plane over expanded jet in the vicinity of the nozzle lip. J. Appl. Mech. Tech. Phy., 47(3): 366-367.

Uskov, V.N. and M.V. Chernishev, 2006b. Differential characteristics of the flow-fields of a plane over-expanded jet around the nozzle edge. Appl. Mech. Tech. Phys., 47(3): 72-83.

Uskov, V.N. and M.V. Chernishev, 2008. Differential parametrs of incident shock near the nozzle Lip in the over-expanded jet. Proceeding of 18th International Shock Interaction Symposium, pp: 109-112.

Uskov, V.N. and P.V. Bulat, 2012. About the problem of ideal diffuser for compression of supersonic stream. J. Basic Res., 6(1): 178-184.

Von Helmholtz, H.L.F., 1868. Monatsberichte der Preußischen Academie der Wisszu Berlin, Bd. 23: S. 215-228. [Russian translation: H/Helmholtz Fundamental Vortex Theory, translated from German, edited by Chaplygin, S.A., Moscow, Izhevsk, 2002, pp: 41-51].