

Research Article

Mach Reflection of a Shock Wave from the Symmetry Axis of the Supersonic Nonisobaric Jet

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Abstract: The purpose of the study is to determine the conditions of Mach disk (direct shock wave) in a supersonic nonisobaric jet. Brief details on shock waves triple configurations, Mach reflection of a shock wave from a rigid wall and the axis of symmetry are given. Various known semi-empirical model of calculating the Mach disc diameter in a supersonic jet and its removal from the nozzle are discussed. It is shown that the Mach disk in the jet is corresponded by the so-called stationary Mach shock waves triple configuration, in which the intensity of the main shock (Mach disk) is the maximum possible for a given Mach number. The theoretical and calculative-experimental validation of the Mach disk formation model in a supersonic non-isobaric jet is provided. The results of calculation are in satisfactory match with experiment.

Keywords: A stationary mach configuration, irregular reflection of shock, mach disk, mach stem, the regular reflection of shock, the triple point, triple configuration of shock waves

INTRODUCTION

Main purpose of the study is to determine the conditions of Mach disk (direct shock wave) in a supersonic nonisobaric jet, development and experimental verification of the method of calculating Mach disk offset from the nozzle, as well as disks diameter and intensity. Supersonic jet has a periodic structure consisting of repeating barrel-shaped cells. If an incalculable jet, which is defined as the ratio between the pressure on a supersonic nozzle section and the pressure in the environment, differs significantly from one, the straight shocks in the shock-wave structure of the jet became clearly and have been called the Mach disk. Mach disk only has significant dimensions in the first barrel of the jet and sometimes the second. However, its presence strongly influences the geometry of the jet down the flow (Fig. 1). In jets of rocket engines in areas beyond the Mach disk the reaction of combustion and dissociation occurs, which leads to a significant increase in temperature, pressure and radiant flux. Despite the practical importance of the task, the complete theory of Mach disk formation in a supersonic jet is still missing. Below we present the necessary information about the triple configuration of shock waves, the description of "stationary Mach configuration" model and a basis for its application to the problem of calculating the Mach disk in a supersonic jet of an ideal gas.

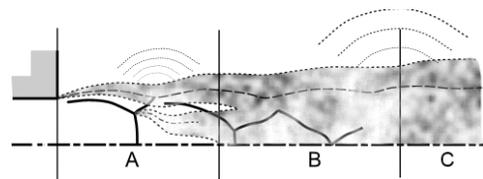


Fig. 1: The typical parts of supersonic turbulent je
A: Starting part; B: Transition region; C: The main part

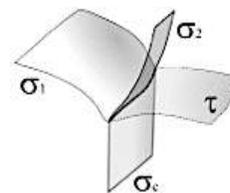


Fig. 2: Triple configuration of shock waves

Brief information about the theory of triple configurations jumps: Triple Configuration (TC) of shock waves-is the shock wave structure consisting of the three fronts of shocks σ , intersecting at some line and the surface of tangential discontinuity τ , coming from this line (Fig. 2), σ_1 - is called incoming shock, σ_2 -outgoing, σ_s - main shock wave.

Triple configurations occur in irregular reflection of a shock wave from a solid surface and the axis of symmetry in axisymmetric flows, in some problems of

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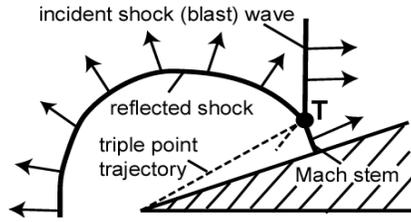


Fig. 3: Formation of triple configuration the shock wave climb on the wedge
T: Triple point; Arrows: The direction of shock waves movement

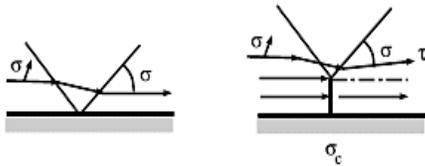


Fig. 4: Regular (left) and irregular (right) shock reflection from the wall (σ_c : mach stem)

the interaction of colliding jumps, as well as the interaction of overtaking jumps. The concept of Triple Configurations (TC) was introduced in the problem of interaction of the shock wave with the wedge (Fig. 3).

As climbing wave spreads along the wedge, its intensity increases and the surface curves, while remaining perpendicular to the surface of the wedge. At a certain point, the shock wave splits forming the Mach stem.

Later, the stationary case of the shock reflection from a solid wall was researched. With a small shock intensity falling on the wall, it is reflected in a regular manner (Fig. 4). At some point of intensity, the reflected shock wave cannot turn the flow in such way, that the velocity vector behind it would be parallel to the wall, because the limit rotation angle of flow at the shock, calculated using the Mach number behind the first shock, is lesser than the inclination angle of the velocity vector to the surface of the wall behind the first shock. Then the reflection occurs irregularly with the formation of the Mach stem.

In contrast to the intensity of the non-stationary case the Mach stem intensity is always equal to the maximum possible for a given Mach number $J_m = (1 + \varepsilon)M^2 - \varepsilon$, $\varepsilon = \frac{\gamma-1}{\gamma+1}$, γ -the adiabatic index. It is convenient to study triple configuration on the plane shock polars $\ln J - \beta$ (logarithm of the shock intensity, equal to the ratio between pressure behind the shock and pressure in front of the shock, the rotation angle of the flow at the shock), nicknamed “heart-shaped curves” for the characteristic form. Detailed analysis of the heart-shaped curves, singular points, the regions of TC existence is given in work of Uskov *et al.* (1995).

Mathematical model underlying the method of calculation of steady and unsteady gas-dynamic

discontinuities described by Uskov and Mostovkykh (2010). The scientific team led by V.N. Uskov consistently developed theory of extreme shock-wave structures. First theory of interference stationary gasdynamic discontinuities was generalized by Uskov *et al.* (1995) to the case of second-order discontinuities. They investigated the dependence of flow irregularities behind the shock wave from the curvature of the shock wave and flow parameters before this shock wave.

Then the main theory has been complemented in the studies of Uskov and Omelchenko by the theory of one-dimensional traveling waves interference (Omelchenko and Uskov, 1995) and the interaction of oblique unsteady waves (Omelchenko and Uskov, 2002). Then it was extended to the case of interaction of simple rarefaction waves (Meshkov *et al.*, 2002). In parallel, a graduate student of V.N. Uskov Tao Gang developed a theory of optimal triple configurations of shock waves in a uniform flow (Tao and Uskov, 2000) and then in a non-uniform (Tao, 2000). And finally in the studies of Uskov and Mostovkykh (2008), M.V. Chernyshov (Uskov and Chernyshov, 2006) and P.S. Mostovkykh (Uskov and Mostovkykh, 2010) it was generalized to the case of triple configurations of shock waves in an unsteady and non-uniform gas flow. Later, the theory has been generalized to the case of perfect gas (Uskov and Mostovkykh, 2011). Bulat *et al.* (2012a) developed vibration theory of shock-wave structures (Uskov *et al.*, 2002, 2010; Uskov and Mostovkykh, 2012) in their interaction with obstacles. Significant contribution to the experimental verification of the theory, as well as the development of practical devices and their implementation was made by Zasukhin *et al.* (2012).

Classification of triple configurations shock waves: According to the classification (Uskov *et al.*, 1995) TC-1, TC-2 and TC-3 triple configuration are distinguished.

Triple configuration TK-1: Arise in the interaction of colliding shocks in different directions, for example, the supersonic internal compression inlets. TC-1 is corresponded by the intersection point of the polar left branches (Fig. 5). As the intensity of the incoming jump 1 increases, the intersection point moves toward the top of the main polar, until it reaches it, this moment corresponds to the Stationary Mach Configuration (SMC), which separates the existence regions of TC-1 and TC-2.

TC-1 can exist only at Mach numbers $M > M_T$:

$$M_T = \sqrt{\frac{3+\gamma}{2}}$$

At lower Mach numbers the polars do not intersect. At Mach number equal to the M_T behind the shock, the maximum static pressure at a given adiabatic index γ (for air $\gamma = 1.4$) is achieved.

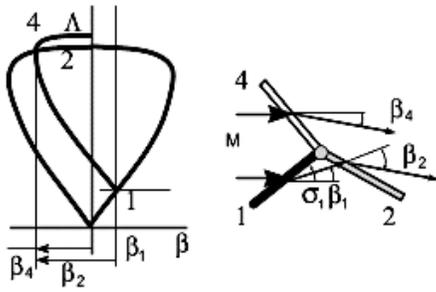


Fig. 5: Triple configuration TC-1

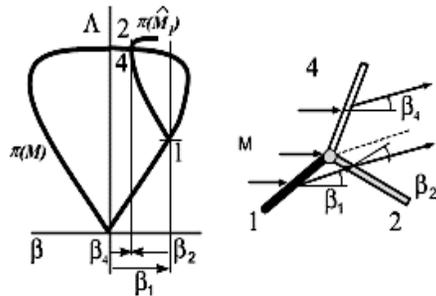


Fig. 6: Triple configuration TC-2

Triple configuration TC-2: Occur at irregular reflection of the shock wave from the wall. Difference from TC-1 is that in TC-2 the direction of flow deflection at the main shock (Mach stem) is reversed (Fig. 6). As the intensity of the shock J1 increases, the main shock 4 curves. During the reflection from the shock 1 from the wall, the Mach stem curves approaching the wall orthogonally.

Triple configurations TK-3: Arise during the interaction of catch-up shocks in one direction. Their research is of practical importance for the design of supersonic air intakes of external compression.

MATERIALS AND METHODS

Models of mach disk formation-history of research:

Let us focus more detailed on the question of the Mach disk position in the jet. Mach disk has notable sizes only in the first and, sometimes the second barrel. However, its presence strongly influences the geometry of the jet down the flow.

The impossibility of regular shock reflection off the symmetry axis without forming Mach disk was first pointed out in the study of Melnikov (1962). Indeed, on the axis of symmetry the conditions of the velocity vector's angle of inclination equality to zero and the flow line curvature behind the reflected shock must be fulfilled, but at $y = 0$ is impossible. When the falling shock to approaches the axis of symmetry, its curvature K_σ tends to infinity, as $K_\sigma \sim y^{-1}$ and therefore the

conditions for the formation of the Mach disk in the incalculable jet are always created.

Failures in creating methods for calculating jets usually significantly associated with an inability to find the position of the Mach disc. A detailed analysis of this issue is made in study of Avduevsky *et al.* (1989), where various hypotheses of the transition from regular shock reflection from the axis to an irregular (Mach) is performed. The most famous among them is the so-called Abbeth model (Abbett, 1971), often referred to in the known studies of Dash *et al.* (1985), Dash and Sinha (1985) and Dash and Thorpe (1981), which are devoted to the creation of methods for calculating the flame of the solid fuel tactical missile engine, as well as different assumptions about the magnitude of the pressure behind the reflected shock at the triple point. Last will not be considered, because today it is known that they are not applicable in the general case.

Abbeth procedure establishes a certain analogy between the Mach disk in the jet and Launch Shock Wave (LSW) that occurs at the Laval nozzle' launch. Initially it was assumed that the pressure in the minimum cross section of the flow behind the Mach disk is equal to atmospheric. Later it was found that this assumption is only approximately true in highly under expanded jets, in which the turbulent area begins immediately after the first barrel section. In low-incalculable jets the Abbeth hypothesis leads to very large errors.

Abbeth hypothesis was upgraded by Dash and Thorpe (1981), suggesting that in the critical section of the Mach disk the "throat flow" condition fulfills, i.e., the flow velocity becomes equal to the local speed of sound. If the triple point position is set on the falling shock, the initial conditions for the calculation of the flow, limited disk Mach and tangential discontinuity is thereby defined. Considering this flow a one-dimensional, we can perform the calculation in the same way as in the Laval nozzle. If the calculation results in the minimum cross section of the flow behind the Mach disk show that the speed is equal to the local speed of sound, then in the procedure of Abbeth-Dash it is considered that the position of the jet Mach disk is selected correctly.

Mach disk in the jet-stationary mach configuration:

Among the other models the criterion, experimentally described in study of Uskov and Mostovkykh (2012) is well confirmed. According to this criterion, the formation of Mach disk occurs when the intensity of the falling shock value approaches $J = J_0$ value, corresponding to Stationary Mach Configuration (SMC). Such configurations of shock waves were studied in detail in along with the problem of shock reflection from a solid wall (Silnikov *et al.*, 2014; Uskov and Chermyshev, 2008). An indirect confirmation of the criterion J_0 is the solution of first-

order problem about configurations of triple shock waves. If we are to perform a formal calculation of triple configuration at any point of hanging (falling on the symmetry axis) shock to produce a formal settlement jump triple configuration of shock waves, then at intensities $J < J_0$ outgoing tangential discontinuity τ has positive curvature. At the point of shock, where $J = J_0$, curvature τ becomes negative (Uskov and Mostovkykh, 2010), which corresponds to the empirical understanding of a tangential discontinuity form.

Let us discuss the triple configurations TC-2 more detailed, because they are directly related to the problem of calculating the Mach disk in a supersonic jet. Triple configuration TC-1 and TC-2 shares stationary Mach configuration, which is corresponded by the case when the secondary shock polar intersects the main polar at its apex. In the SMC (Fig. 7), the main shock wave is straightforward. Characteristic intensity J_0 is obtained by solving the cubic equation, corresponding to the polars intersection at the apex of the main polar (Fig. 7):

$$\begin{aligned} \sum_{n=0}^3 A_n z_0^n &= 0, z_0 = J_0, \\ A_3 &= 1 - \varepsilon, A_2 = m\varepsilon^2 - \varepsilon(M^2 + 1) - \\ &(M^2 - 2), m = (1 - \varepsilon)(M^2 - 1) - 1, \\ A_1 &= m\varepsilon(1 + \varepsilon)(M^2 + 1) - (1 + \varepsilon)(2M^2 - \\ &1), A_0 = m1 + \varepsilon 2M^2 \end{aligned}$$

For some typical Mach number M_{OR} in SMC of the polar, corresponding to Mach number behind the incoming shock 1, crosses the top of the apex of the main polar in the point of limit rotation angle 4 (Fig. 8).

Hence the transcendental equation for a special number M_{OR} :

$$\frac{4A_1 \left(\frac{A_1 - 3A_2}{A_1} \right)^2}{9 - \frac{A_1 A_2}{A_0}} = 9 - \frac{A_1 A_2}{A_0} - 4 \left(3 - \frac{A_2^2}{A_1} \right)$$

Coefficients A_i have the same value, as in the preceding equation.

At Mach numbers, smaller than M_{OR} the polar intersects the vertical axis and lies inside the main polar, therefore the intensity J_r , corresponded by polar's tangency to the axis (Fig. 9), is sometimes (wrongly) assumed as a transition criterion for the Mach. This allows to find the value of this intensity by solving the following equation:

$$\begin{aligned} \sum_{n=0}^3 A_n x_R^n &= 0, x_R = \frac{(1+\varepsilon)M^2}{(J_R+\varepsilon)}, \\ A_0 &= -(1 - \varepsilon)^2 L^4, L = \frac{J_R - \varepsilon}{J_R + \varepsilon}, \\ A_1 &= 2(1 - \varepsilon)(3 - \varepsilon)L^2 - 4(1 - 3\varepsilon)(1 - \varepsilon)L^3 + \\ &(1 + \varepsilon)^4 L^4, \\ A_2 &= 2L^2(1 - 2\varepsilon - \varepsilon^2) - 4L - 1, \\ A_3 &= 1 \end{aligned}$$

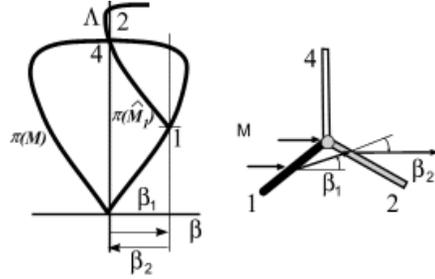


Fig. 7: Stationary Mach Configuration (SMC)

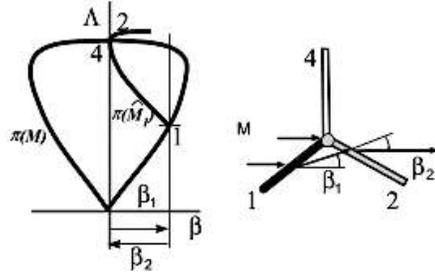


Fig. 8: The special configuration, corresponding to M_{OR}

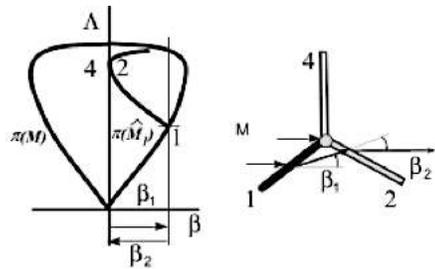


Fig. 9: TC, corresponding to $J = J_R, M < M_{OR}$

However, in the stationary case is not experimentally confirmed. This fact is denied as well by the developed theory of shock waves restructuring (Bulat *et al.*, 2012b). In axisymmetric ideal gas jet the triple point of hanging shock can be formally positioned in different sections. Triple configuration should contain the shocks with the lowest possible intensity (in accordance with the Lagrange mechanics principle of least action). At Mach number greater than M_{OR} the formation of triple configuration with Mach disk at the shock in the point with lesser intensity than J_0 impossible due to Baryshnikov theorem (Arnold, 1976), which states that with small change in the parameter the wave configuration (addition to the shock wave) must remain topologically the same. Indeed, the intensity of hanging shock in this problem is a parameter. If the restructuring took place earlier, at $J < J_0$, then with further change of parameter J at the moment when $J = J_0$ the configuration of shocks would topologically change because curvature τ would become negative.

If the Mach number than before the shock is less M_{OR} then $J_R < J_0$. It would seem that the equation of

hanging shock intensity J_r must be taken as a criterion of Mach disk formation. But again the theorem of Bogaevsky is violated, since with the small change in a parameter (hanging shock intensity) upwards, the point of polar's intersection with the shock moves on a strong branch of the main shock polar, i.e., decision is non-contractible (topologically changes) in the neighborhood of shock-wave structure adjustment point.

Model of stationary Mach configuration allows, with satisfactory for practice accuracy, to predict the position of the Mach disk in the supersonic jet. Positioning the Mach disk during calculation at the point of the hanging shock, in which its intensity $J = J_0$, it is possible determine the diameter of the Mach disk and its distance from the nozzle cut. Calculation results showed a good match with experiment.

At Mach numbers of $M < M_T$ stationary solution for the shock wave reflection from the jet axis does not exist. The assumption that at Mach numbers of $M_T < M < M_{OR}$ the Mach disk is formed in such point of the hanging shock, where its intensity is equal to J_R , is disproved by both theory and experiment.

RESULTS AND DISCUSSION

Testing the hypothesis of the SMC for the case of a mach disk formation in a supersonic overexpanded jet: For the practical justification of the model

systematic calculations of overexpanded jets, flowing out from conical and profiled nozzle were performed. Choice of overexpanded jets for analysis was due to the simplicity of the flow before the shock in such jets, representing a continuation of the flow at the nozzle. In the underexpanded jets the flow structure before the shock is much more complex, contains a set of compression waves and weak discontinuities, which complicates the calculation of the SWS geometry. Formation of the falling shock wave in overexpanded jet was performed by solving a system of ordinary differential equations describing its geometry (Bulat *et al.*, 1993), at each point of the shock; its intensity was compared with $J_0(M)$. The point in which, at given accuracy, the condition $J = J_0$ was true were considered the birthplace of the Mach disk. Figure 10 shows a comparison of the Mach disk diameter numerical calculations results and its distance from the nozzle (the ideal gas model). On the Fig. 10 the results of numerical calculations are shown as shaded areas corresponding to the "smearing" of the shocks on few difference cells.

The calculation results are in good match with numerical methods as well as with the concept of Mach disk diameter's dependence on the basic parameters of the supersonic jet, based on experimental studies.

For a more thorough test of the J_0 model a series of experiments on the nozzles with half-angle from 8 to

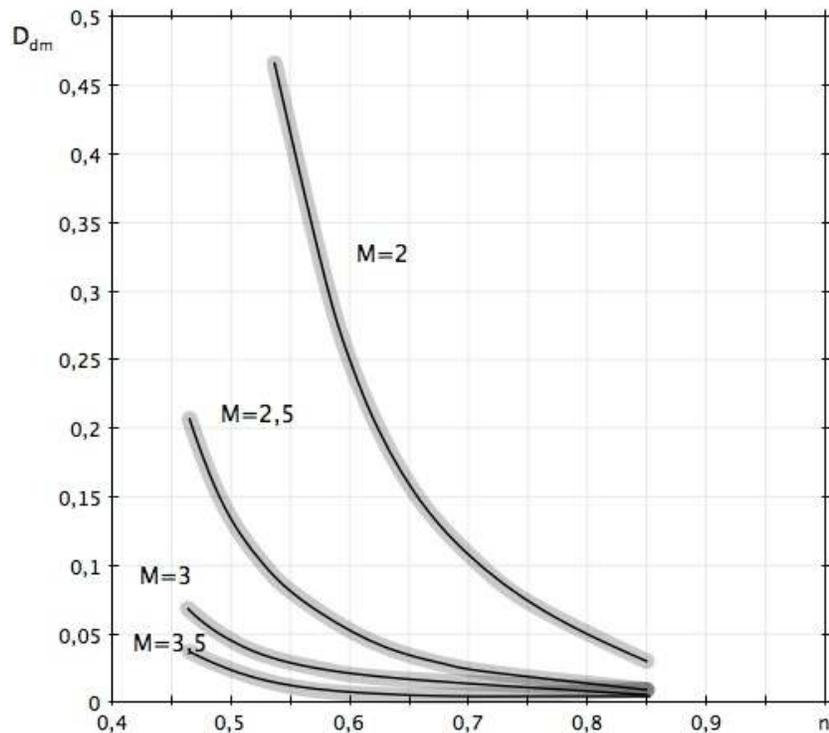


Fig. 10: Mach disk diameter's dependence on the mach number M and incalculability n in the over expanded jet, flowing from profiled nozzle

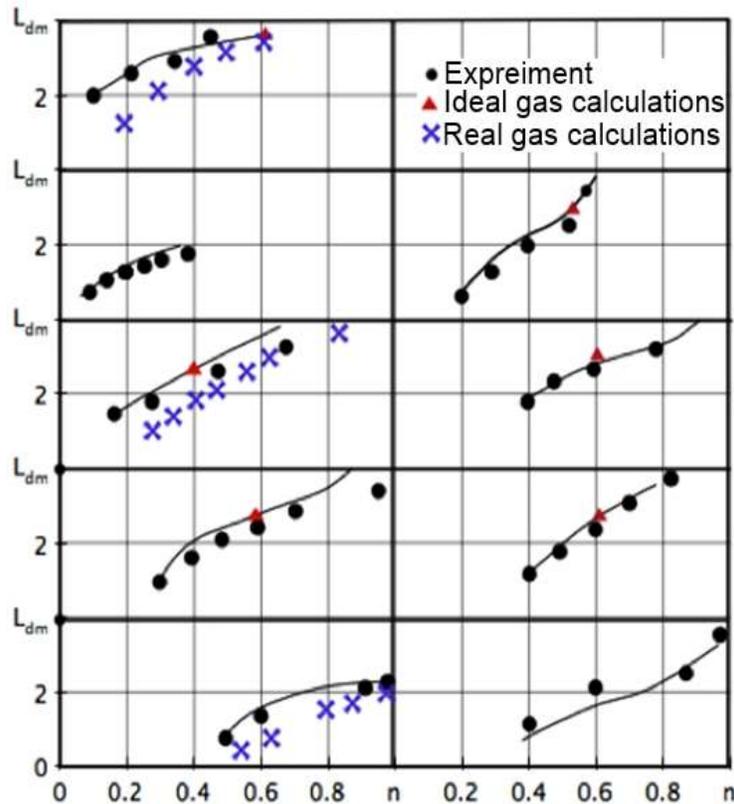


Fig. 11: Calculations of mach disc distance in jets, flowing out from a conical nozzle

15° were performed. In the calculations the gas in the nozzle to was by the flow from a point source.

It is known that the greater the angle of a nozzle is, the more is the difference between the flow at the nozzle and model of the flow from the source, so the checking numerical calculation were performed using sst-model of turbulence without the resolution of the boundary layer on the nozzle walls. Experimental data thus must be positioned between two calculated curves. Calculations (Fig. 11) showed a very good match of the results.

Testing the hypothesis of the SMC for the case of a mach disc formation in a supersonic underexpanded jet:

In order to correctly verify compliance of the calculation method with the experimental data it is necessary to minimize the influence of viscous effects the vicinity of the nozzle edge that distort the flow pattern during experiment. This can be achieved using such installation, in which the jet flows from a sonic nozzle with a bottom screen. Inside such nozzle, the boundary layer is small and the ejection flow at its edge is absent. Such experiment was conducted in the Dnepropetrovsk University (Belyaev and Karteshkin, 1982). Figure 12 and 13 show the dependence of distance L on the nozzle cut characteristic sections of the jet, as well as the dependence of diameter D of the jet's first barrel corresponding element on the jet's

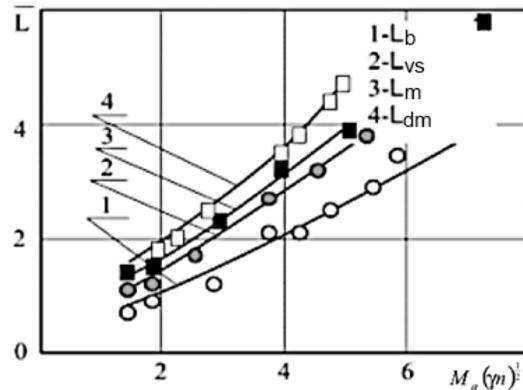


Fig. 12: Comparison of distances of the under expanded jet first barrel main sections, obtained by calculations with the experimental data (Belyaev and Karteshkin, 1982)

L_b : The distance from the nozzle to the point, where the reflected shock intersects with the boundary of the jet; L_{vs} : Distance from the nozzle cut to a section with a maximum diameter of hanging shock; L_m : The distance from the nozzle to the cross section with a maximum diameter of the jet boundary; L_{dm} : The distance from the nozzle exit to mach disk; n: Incalculability of the jet

automodel parameter. The symbols mark the experimental data, solid-calculative.

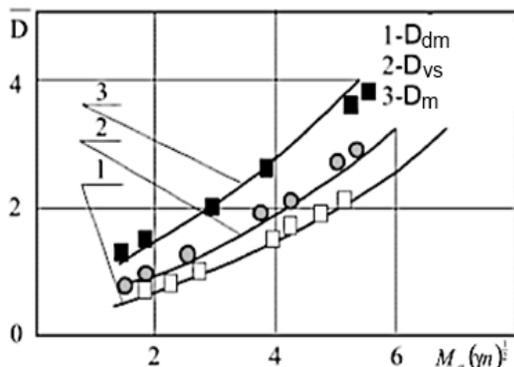


Fig. 13: Comparison of the calculation results of the first barrel main elements diameter with the experiment (Belyaev and Karteshkin, 1982)
 D_{vs} : Maximum diameter of hovering shock; D_m : Maximum diameter of the jet boundary; D_{dm} : The diameter of the mach disk

The triple point on a hanging shock was chosen in section, in which the hanging shock's intensity is equal to J_0 . There is not only a qualitative but also a satisfactory quantitative match with experiment.

CONCLUSION

Triple configuration of shock waves are briefly considered. It is shown that the Mach disk in a jet is a special case of triple configurations. The most well-known semi-empirical models, which allow to approximately calculate the diameter of the Mach disk and its distance from the nozzle are listed. The stationary Mach configuration, as the Mach disk model in the jet, is discussed. Checking calculations and comparisons with the results of full-scale and numerical experiment are performed.

Described studies are the next step in the development of the theory of shock waves. These models, results of calculations and experiments for the first time allowed to unequivocally prove that the Mach disk in nonisobaric supersonic jet is stationary Mach configuration.

ACKNOWLEDGMENT

This study was prepared as part of the "1000 laboratories" program with the support of Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics.

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