Asian Journal of Business Management 4(4): 373-375, 2012
ISSN: 2041-8752

Submitted: June 11, 202 Accepted: July 04, 2012 Published: September 25, 2012

The Chaotic General Economic Equilibrium Model and Monopoly

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Abstract: The basic aim of this study is to construct a relatively simple chaotic general economic equilibrium growth model that is capable of generating stable equilibrium, cycles, or chaos. An important example of general economic equilibrium is provided by monopolies. A key hypothesis of this study is based on the idea that the coefficient $\pi = b\, m_{RS}/m\, (\alpha-1)\, (1+1/e)\, m_{RT}$ plays a crucial role in explaining local stability of the general equilibrium output, where, b: The coefficient of the quadratic marginal-cost function, m: The coefficient of the inverse demand function, m_{RS}: The marginal rate of substitution, m_{RT}: Marginal rate of transformation, α: The coefficient of the monopoly price growth, e: The coefficient of the price elasticity of demand.

Keywords: Chaos, general equilibrium, monopoly, output

INTRODUCTION

Chaos theory reveals structure in unpredictable dynamic systems. It is important to construct deterministic, nonlinear general economic equilibrium growth model that elucidate irregular, unpredictable general economic equilibrium behavior.

Chaos theory can explain effectively unpredictable economic long time behavior arising in a deterministic dynamical system because of sensitivity to initial conditions. A deterministic dynamical system is perfectly predictable given perfect knowledge of the initial condition, and is in practice always predictable in the short term. The key to long-term unpredictability is a property known as sensitivity to (or sensitive dependence on) initial conditions.

Chaos theory started with Lorenz (1963) discovery of complex dynamics arising from 3 nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve cans exhibit very complex behavior. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983, 1997), Grandmont (1985), Goodwin (1990), Medio and Gallo (1993, 1995), Lorenz (1993) and Jablanović (2011, 2012), among many others.

The basic aim of this study is to provide a relatively simple chaotic general economic equlibrium output growth model that is capable of generating stable equilibriums, cycles, or chaos. This chaotic model has included the behavior of monopolies that have control over the prices they charge.

General economic equilibrium and monopoly: Marginal rate of substituon, m_{RS}, is the amount of a good A that a consumer is willing to give up in order to obtain one additional unit of good B. The marginal rates of substitution between 2 goods are equal to the price ratio. For our 2 goods, A, and B,

$$m_{RS} = P_A/P_B \quad (1)$$

where,
mRS : Marginal rate of substitution
P_A : The price of good A
P_B : The price of good B

Marginal rate of transformation is an amount of good A that must be given up to produce one additional unit of a good B. The marginal rate of transformation is the ratio of the marginal cost of producing good A, MC_A, to the marginal cost of producing good B, MC_B. A profit-maximizing monopoly always produces where Marginal Evenue (MR) equals Marginal Cost (MC). Then, the marginal rate of transformation is the ratio of the marginal revenue of the commodity A, MR_A, to the marginal revenue of the B, MR_B.

$$m_{RT} = MC_A/MC_B = MR_A/MR_B \quad (2)$$

where,
m_{RT} : Marginal rate of transformation
MC_A : The marginal cost of producing good A
MC_B : The marginal cost of producing good B
MR_A : The marginal revenue of the commodity A
MR_B : The marginal revenue of the commodity B

On the other hand, a monopoly's marginal revenue of the commodity A is:

$$MR_{A,t} = P_{A,t}\, [1+(1/e)] \quad (3)$$

where,
MR_A : The marginal revenue of the commodity A
P : Monopoly price
e : The coefficient of the price elasticity of demand

In accordance with Eq. (1)-(3), we obtain:

$$MR_{A,t} = (m_{RT}/m_{RS})\, P_{A,t}\, (1+1/e) \quad (4)$$

Further, it is supposed that:

$$P_{A,\,t+1} = P_{A.t} + \alpha\, P_{A,\,t+1} \quad (5)$$

or

$$(1-\alpha)\, P_{A,\,t+1} = P_{A.t} \quad (6)$$

where,
P : Monopoly price
α : The coefficient of the price growth

In the chaotic model of a profit-maximizing monopoly, take the inverse demand function:

$$P_t = n - mQ_t \quad (7)$$

where,
P : Monopoly price
Q : Monopoly output
n, m : Coefficients of the inverse demand function

Further, suppose the quadratic marginal-cost function for a monopoly is:

$$MC_{A,t} = a + bQ_t + cQ_t^2 \quad (8)$$

where
MC : Marginal cost
Q : Monopoly output
a, b, c : Coefficients of the quadratic marginal-cost function

A monopoly maximizes profit by choosing the quantity at which Marginal Revenue (MR) equals Marginal Cost (MC), or:

$$MR_{A,t} = MC_{A,t} \quad (9)$$

It is supposed that a = 0 and n = 0. Finally, substitution (6), (7), (8) and (9) in (4) gives:

$$Q_{A,t+1} = [b\, m_{RS}/m\, (\alpha-1)\, (1+1/e)\, m_{RT}]\, Q_{A,\,t} - [cm_{RS}/m\, (\alpha-1)\, (1+1/e)\, m_{RT}]\, Q_{A,t}^2 \quad (10)$$

Further, it is assumed that the equilibrium output, $Q_{A,\,t}$, is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the equilibrium output. Growth rate depends on the current size of the equilibrium output, $Q_{A,t}$, relative to its maximal size in its time series $Q_{A,tm}$. We introduce $q_{A,t}$ as $q_{A,t} = Q_{A,t}/Q_{A,tm}$. Thus $q_{A,t}$ range between 0 and 1. Again we index $q_{A,t}$ by t, i.e., write $q_{A,t}$ to refer to the size at time steps t = 0, 1, 2, 3... Now, growth rate of the equilibrium output is measured as:

$$q_{A,\,t+1} = [b\, m_{RS}/m\, (\alpha-1)\, (1+1/e)\, m_{RT}]\, q_{A,t} - [c\, m_{RS}/m\, (\alpha-1)\, (1+1/e)\, m_{RT}]\, q_{A,t}^2 \quad (11)$$

This model given by Eq. (11) is called the logistic model. For most choices of b, c, m, e, m_{RS}, m_{RT}, and α there is no explicit solution for (11). Namely, knowing b, c, m, e, m_{RS}, m_{RT}, and α and measuring $q_{A,0}$ would not suffice to predict $q_{A,\,t}$ for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect-the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference Eq. (11) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every 2 or more periods, and even chaos, in which there is no apparent regularity in the behavior of $q_{A,t}$. These difference Eq. (11) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point $q_{A,0}$ the solution is highly sensitive to variations of the parameters b, c, m, e, m_{RS} , m_{RT}, and α; secondly, given the parameters b, c, m, e, m_{RS}, m_{RT}, and α the solution is highly sensitive to variations of the initial point $q_{A,0}$. In both cases the 2 solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

The logistic equation: The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation:

$$z_{t+1} = \pi\, z_t\, (1 - z_t),\ \pi \in [0, 4],\ z_t \in [0, 1] \quad (12)$$

is equivalent to the iteration of growth model (11) when we use the following identification:

$$z_t = c\, (\alpha - 1)\, /b\, (1 - \alpha)\, q_{A,t}$$

and

$$\pi = b\, m_{RS}/m\, (\alpha - 1)\, (1+1/e)\, m_{RS} \quad (13)$$

Using (11) and (13) we obtain:

$$z_{t+1} = [c(\alpha - 1) /b (1 - \alpha)]q_{A,t+1}$$
$$= [c(\alpha - 1)/b(1 - \alpha)]\{[b\, m_{RS} /m(\alpha - 1)(1+1/e)\, m_{RT}]q_{A,t}-[c\, m_{RS}/m(1-\alpha)(1+1/e)m_{RT}]\, q_{A,t}\}$$
$$= [c\, m_{RS}/m(1-\alpha)(1+1/e)\, m_{RT}]q_{A,t}-[c2(\alpha-1)m_{RS}/b\, m(1-\alpha)2(1+1/e)m_{RT}]q_{A,t}^2$$

On the other hand, using (12), and (13) we obtain:

$$z_{t+1} = \pi z_t (1 - z_t) = [b\, m_{RS}\, m(\alpha-1)(1+1/e)m_{RT}]\, [c(1-\alpha)(1-\alpha)]q_{A,t}\, \{1-[c(\alpha-1)/b(1-\alpha)]q_{A,t}\}$$
$$= [c\, m_{RS}/m(1-\alpha)(1+1/e)m_{RT}]q_{A,t} - [c^2 (\alpha-1)m_{RS}/m(\alpha-1)2(1+1/e)]q_{A,t} - [c^2 (\alpha-1)m_{RS}/b\, m(1-\alpha)2(1+1/e)m_{RT}]q_{A,t}^2$$

Thus we have that iterating:

$$q_{A,t+1} = [b\, m_{RS}/m(\alpha-1)(1+1/e)m_{RT}\, q_{A,t} - [c\, m_{RS}/m(\alpha-1)(1+1/e)m_{RT}]\, q_{A,t}^2$$

is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using (13).

It is important because the dynamic properties of the logistic Eq. (12) have been widely analyzed (Li and Yorke, 1975; May, 1976).

It is obtained that:

- For parameter values $0<\pi<1$ all solutions will converge to $z = 0$
- For $1<\pi<3$, 57 there exist fixed points the number of which depends on π
- For $1<\pi<2$ all solutions monotonically increase to $z = (\pi - 1) /\pi$
- For $2<\pi<3$ fluctuations will converge to $z = (\pi - 1) /\pi$
- For $3<\pi<4$ all solutions will continuously fluctuate
- For 3, $57<\pi<4$ the solution become "chaotic" which means that there exist totally a periodic solution or periodic solutions with a very large, complicated period. This means that the path of zt fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

CONCLUSION

This study suggests conclusion for the use of the simple chaotic general equilibrium growth model in predicting the fluctuations of the equilibrium output. This chaotic model has included the behavior of monopolies that have control over the prices they charge. The model (11) has to rely on specified parameters b, c, m, e, m_{RS}, m_{RT}, and α and initial value of the equilibrium output, $q_{A,0}$. But even slight deviations from the values of parameters b, c, m, e, m_{RS}, m_{RT}, and α and initial value of the equilibrium output show the difficulty of predicting a long-term behavior of the general equilibrium output.

A key hypothesis of this study is based on the idea that the coefficient $\pi = b\, m_{RS}\, m\, (\alpha - 1)\, (1+1/e)\, m_{RT}$ plays a crucial role in explaining local stability of the general equilibrium output, where, b: The coefficient of the quadratic marginal-cost function, m: The coefficient of the inverse demand function, m_{RS}: The marginal rate of substitution, m_{RT}: Marginal rate of transformation, α: The coefficient of the monopoly price growth, e: The coefficient of the price elasticity of demand.

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