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Mathematical Modeling of Tomato Terminal Velocity in Water

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Abstract: In this study, the terminal velocity of tomato (cv. *Rio grands*) was theoretically formulated and then determined experimentally using a water column. Some characters of tomato affecting its terminal velocity were determined using standard methods. The best model for terminal velocity of tomato as a function of water and tomato densities, shape factor and volume was modeled with determination coefficient of 0.84. Based on statistical analysis, fruit density created a considerable influence on terminal velocity while the parameter of fruit volume shape factor had small effect on terminal velocity. It can be concluded that in sorting systems, difference in terminal velocities of tomatoes could be addressed as a crucial factor for designing sorting systems.

Key words: Design sorting systems, hydro-sorting, terminal velocity, tomato, shape factor

INTRODUCTION

As world markets for fruit and produce become more sophisticated and technology continues to provide means to measure product quality, there is a corresponding market pull for produce with higher, or at least specified, quality levels. Electrical sizing mechanisms are too expensive and mechanical sizing mechanisms are slow to react (Tabatabaeefar and Rajabipour, 2005). Density, as a good indicator of fruit dry matter becomes an interesting tool for fruit quality sorting because of its inherently lower cost and simpler operation (Richardson *et al.*, 1997; Jordan *et al.*, 2000). Sorting products based on density is not new, patent for example, in the potato industry, extended from 1950s to the present day (Kunkel *et al.*, 1952; Wilson and Lindsay, 1969; Bajema, 2001). As found in the literature, some products (e.g., citrus and blueberries) have also been sorted by flotation techniques for quality or defects (Perry and Perkins, 1968; Patzlaff, 1980). In such sorting systems, fruits are placed in solutions like salt brine or alcohol-water. The specific gravity of the solution is adjusted to a value, which will differentiate between those fruit which are desirable and those, which are not. Problems to be overcome include detrimental changes in quality when alcohol-water solutions must be used, contamination of the solution by dirt which causes an accompanying change in solution density and prohibitive cost of mixing and maintaining the brine or alcohol-water solutions (Mohesenin, 1986). Hydrodynamic properties are very important characters in hydraulic transport and handling as well as hydraulic sorting of agricultural products. To provide basic data essential for development of equipment for sorting and sizing fruits needed to determine several properties of fruits such as: fruit density and terminal velocity of that (Matthews *et al.*, 1965; Dewey *et al.*, 1966)

Terminal velocity of fruits is a maximum velocity that each fruit can reach in specific medium (Mohesenin, 1986). According to Jordan and Clerk (2004), an approach to fruit sorting is to use the terminal velocity of fruit moving in a fluid that has a density above or below the target density. Fruit with different terminal velocities will reach different depths after flowing a fixed distance in a flume and may be separated by suitably placed dividers. This approach could use water as a sorting medium, which provides huge advantages in terms of the resulting low corrosion and disposal difficulties, and the fact that it does not need any density adjustment. Moreover, this approach allows purely mechanical setting of the separation threshold by adjusting the divider positions and no change in fluid density is required.

The main objective of this study was modeling of terminal velocity of tomato in water column to determine if there was a potential for terminal velocity methods as an practical approach used in sorting unit operation.

MATERIALS AND METHODS

Mathematical approach: The forces acting on tomato in water will be a gravitational force, downward, buoyancy force, upward, and drag force, opposite to the direction of motion, downward. The combination of these forces accelerates the tomato proportional to its mass (Crowe *et al.*, 2001):

$$ma = F_w + F_d - F_b \quad (1)$$

Hence:

$$ma = mg + 0.5\rho_w V^2 C_D A_P - \rho_w v g \quad (2)$$

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Where A_p is cross-sectional area of tomato (cm^2), which is perpendicular to the direction of motion, and C_D, is drag coefficient, which is a function of fruit velocity. Tomato has Reynolds number > 1 in water column (with means of 8124.11), therefore C_D can be modeled using Stokes law (Crowe *et al.*, 2001):

$$C_D = \frac{K\mu_w^n}{V^n D^n \rho_w^n} \tag{3}$$

where μ_w is the static viscosity of the water (Pa.s), K and n are constant factors, and D is the tomato diameter (m). On the other hand, dividing Eq. (2) by $m = v\rho_t$, gives:

$$a = g\left(1 - \frac{\rho_w}{\rho_t}\right) + 0.5\rho_w V^2 C_D A_P / (v\rho_t) \tag{4}$$

where ρ_t is tomato density (kgm^{-3}). For a spherical object, A/v can be computed directly as a function of the diameter. By separating A/v into two parts: a dimensionless shape factor (S_h), and size (Jordan and Clerk, 2004), the following relationship is obtained as

$$\frac{A_p}{v} = \frac{S_h}{size}\left[A_p \Big/ v^{\frac{2}{3}}\right] \Big/ \left[v^{\frac{1}{3}}\right] \tag{5}$$

and with diameter as:

$$D = e\left(\frac{6v}{\pi}\right)^{\frac{1}{3}} \tag{6}$$

where S_h is shape factor (dimensionless) and e is constant factor. Substituting C_D, A/v and D from Eq. (3), (5) and (6) into Eq. (4), gives:

$$a = g\left(1 - \frac{\rho_w}{\rho_t}\right) + K_1\left(\frac{\mu_w^n \rho_w^{(1-n)} V^{(2-n)} S_h}{v^{\left(\frac{n+1}{3}\right)} \rho_t}\right) \tag{7}$$

Then, setting acceleration to zero in Eq. (7), the terminal velocity (V_t) of the sample becomes:

$$V_t = K_2 \frac{(\rho_w - \rho_t)^{\left(\frac{1}{2-n}\right)} v^{\left(\frac{n+1}{3(2-n)}\right)}}{\mu_w^{\left(\frac{n}{2-n}\right)} \rho_w^{\left(\frac{1-n}{2-n}\right)} S_h^{\left(\frac{1}{2-n}\right)}} \tag{8}$$

The above equation can be generalized as reported by Kheiralipour (2008):

$$V_t = A(\rho_w - \rho_t)^b v^c S_h^d + E \tag{9}$$

where parameters of A, b, c, and d are curve fitting parameters and take appropriate values and parameter E is added to reducing errors.

Experiment methods: The 100 samples of tomato (cv. *Rio grand s*) were transferred to the laboratory in polyethylene bags to reduce water loss during transport, in May 2009 in the Biophysical laboratory and Biological laboratory of the University of Tehran. Samples were kept in cold storage at 4ºC. Volume and density of samples were determined by the water displacement method (Mohesenin, 1986). Tomato's mass was determined with an electronic balance with 0.01 g sensitivity (GF3000, A&D, Japan). Projected area of the tomato was determined from pictures of the samples taken by Area Measurement System-Delta Ten gland. A glued Plexiglas column was used with a height of 1200 mm and a cross-section of 350 ´350 mm as shown in Fig. 5. The column was constructed with a diameter at least five times more than that of the fruit (Vanoni, 1975). Each sample was placed on the bottom of the column with a nondestructive instrument and then released. In order to determine the terminal velocity, a digital camera, JVC (770) with 25 fps, recorded the moving of samples from releasing point to the top of water column, simultaneously. Subsequently, video to frame software was used to change video film to images in order to calculate terminal velocity of samples by knowing the fact that each picture takes 0.04 s.

Determined data were considered for modeling terminal velocity using SPSS, 15, Software. The model (Eq. 9) was optimized by adjusting various combinations of the five parameters to maximize the determination coefficient (R^2) and to minimize root mean square error (RMSE) and reduced chi-square (χ^2).

RESULTS AND DISCUSSION

Curve results: In order to quantify effectiveness of characters among differences between water and tomato densities, volume and shape factor of samples, they were individually plotted versus terminal velocity in Fig. 1-3, respectively. Figure 1, with more regular plots than those of Fig. 2 and 3, shows the higher effectiveness of differences between water and tomato densities on terminal velocity than those of volume and irregular shape factor.

Modeling results: The five models were tested, and results are summarized in Table 1. The effectiveness of all parameters including shape factor, volume, and water and tomato densities for determining terminal velocity is shown in model 1 with R^2, RMSE $\chi^2$and of 0.84, 0.013 and 0.0002, respectively.

$$V_t = 0.023(\rho_w - \rho_t)^{1.91} v^{-1.91} S_h^{-2.49} + 0.038$$
$$R^2 = 0.84$$

With deleting volume in model 2, shape factor in model 3, and both volume and shape factor in model 4, little change in R^2, RMSE and χ^2 was observed. This

Table 1: Comparison of terminal velocity models developed with different parameters and corresponding correlation factors.

Model	A	b	c	d	E	R^2	RMSE	χ^2
1	0.023	1.91	-1.91	2.49	0.038	0.84	0.13	0.0002
2	0.047	1.85	0.000	-0.056	0.037	0.83	0.018	0.0035
3	0.64	1.83	-0.029	0.000	0.037	0.83	0.015	0.0028
4	0.74	1.84	0.000	0.000	0.037	0.82	0.024	0.0049
5	0.074	0.000	-1.2	-2.25	0.34	0.59	0.269	0.0917

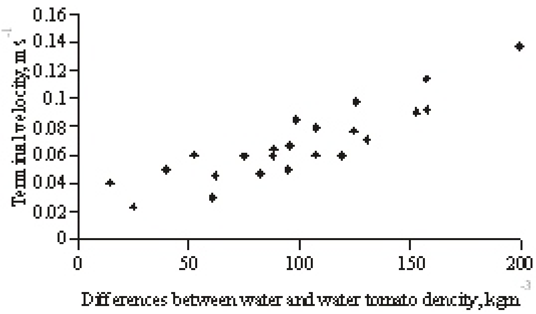

Fig. 1: Terminal velocity versus differences between water and tomato density for all experiments

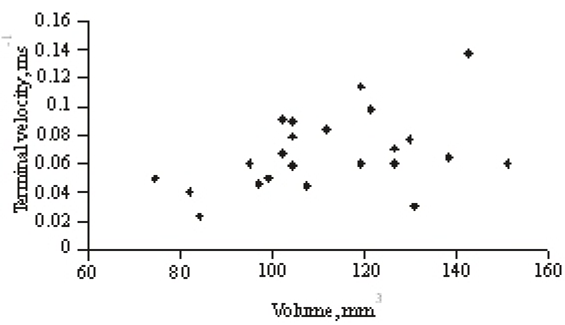

Fig. 2: Terminal velocity versus tomato volume for all experiments

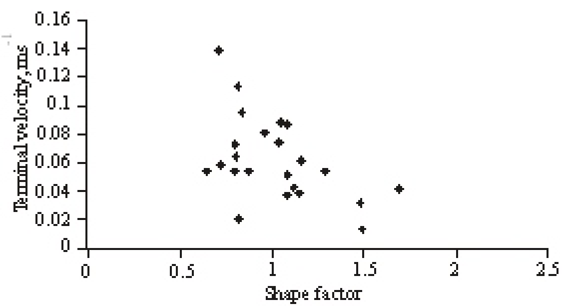

Fig. 3: Terminal velocity versus tomato shape factor for all experiments

shows that the effectiveness of volume and shape factor of tomato on terminal velocity was low. But by abstracting differences between water and tomato density in model 5, much change in R^2, RMSE and χ^2 was observed. From this model it can be seen that the most effective parameter on the terminal velocity of tomato (cv. *Rio grand s*) is density. According to Jordan and Clark

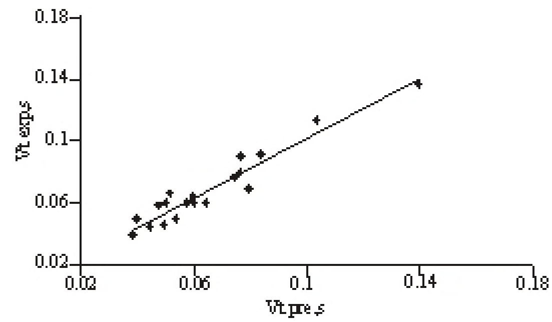

Fig. 4: Experimental versus predicted terminal velocity values using the best model

Fig. 5: Water column and camera setting to the right side

(2004), fruit density is a strong indicator of internal sugar status in kiwifruit, and this measurement minus the density of the supporting fluid has a major effect on drop velocity and thus on the transit time to reach the bottom of a fluid tank. Fruit shape also effect velocity but should not be of a magnitude to cause concern (Jordan and Clerk, 2004). Mirzaee *et al.* (2009) reported that apricot fruits with approximately constant volume can be sorted based on their densities. This is due to the fact that fruits with approximately constant volume and different densities have different terminal velocities and can be separated accordingly. The similar finding was reported by Mirzaee *et al.* (2008) for apricot fruit, Kheiralipour *et al.* (2009) for kiwi fruit.

Figure 4 shows experimental data versus predicted values using the best model (model 1). Data points are banded around a 45° straight line:

$$V_{t\,exp} = 0.974V_{tpre} + 0.006$$
$$R^2 = 0.93$$

It is clear that the selected model shows a good agreement between the predicted data and the experimental terminal velocity values.

CONCLUSIONS

From this study it can be concluded that:

- The best model for terminal velocity of tomato (cv. *Rio grand s*) as a function of water and tomato densities, shape factor and tomato's volume was obtained with R^2 of 0.84.
- Density of this tomato variety was the most effective parameter on its terminal velocity and samples with approximately constant volume can be sorted based on their densities.

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Nomenclature

D	tomato diameter, mm
A_p	Projected area, cm^2
v	Tomato volume, cm^3
m	mass, g
ρ_t	tomato density, kg/m^3
ρ_p	potato density, kg/m^3
S_h	Shape factor of tomato
g	Gravitational acceleration, m/s^2
ρ_w	Water density, kg/m^3
μ_w	Static viscosity of water, Pa.s
F_d	Drag force, N
F_b	Buoyancy force, N
F_w	Gravitational force, N
a	Acceleration, m/s^2
V	Velocity, m/s
V_t	Terminal velocity, m/s
T_d	Dropping time, s
T_r	Rising time, s
n	Constant factor
K	Constant factor
e	Constant factor
B	Constant factor
E, A, b, c, d	Curve fitting parameter
C_D	Drag coefficient, dimensionless
N_R	Reynolds number, *dimensionless*

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