Mathematical Model on Thin Layer Drying of Finger Millet (*Elue sine coracana*)

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Abstract: Thin layer drying characteristics of the finger millet (*Elue sine coracana*) samples with an Initial Moisture Content (IMC) of 38.5%, on dry basis (db) were studied. The drying experiments were carried out in a conventional tray dryer at temperatures of 50, 70 and 80°C. The drying data were fitted to nine thin layer models and a thin layer model for the finger millet was developed by regressing the coefficients of the best fit. The logarithmic model was found to satisfactorily describe the drying kinetics of the millet. The drying constants were found to vary linearly with temperature. Also, effective diffusivity was evaluated by using Fick’s law, which varied from $1.526 \times 10^{-10}$ to $2.85 \times 10^{-10}$ m²/s. Temperature dependence of diffusivity was found by Arrhenius type of relationship and the activation energy for the diffusion of the moisture associated with the millet was found to be 35.37 kJ/gmol.

Key words: Activation energy, drying constants, effective diffusivity, finger millet, mathematical model, moisture diffusion, thin layer drying characteristics

INTRODUCTION

Finger millet under study is an annual cereal plant. It is widely grown in the arid areas of Africa and Asia. It is the main food grain for many people, especially in the dry areas of India and Sri Lanka and is used in the form of flour. The products made of the finger millet flour are easily digestible and can be recommended for all age groups. Malted flour is used as staple food in many parts of India.

Drying involves simultaneous heat and mass transfer. The main objective of drying is to reduce the moisture content and to increase the shelf life. Several mathematical models have been developed describing the drying process. However only thin layer drying models are widely used for the designing of the dryers. Some of the thin layer models reported were for drying of Toria seeds (Rangroo and Rao, 1992), Dates (Bakri and Hobani, 2000), chili pepper (Toyosi and Adeladun, 2010), two varieties of millet samples (Ojendiran and Raji, 2010), Sri Lankan paddy (Syamali et al., 2009), Sesame seeds (Khazaei and Daneshmandi, 2007), Amaranth grain (Ronoh et al., 2010), parboiled wheat (Debabandya and Srinivas, 2005).

The objective of the present study was to develop the thin layer model and determination of effective diffusion coefficients for drying of finger millets using tray dryer, which were not reported earlier. Also, the temperature dependence of diffusivity was assessed by Arrhenius type equation. Activation energy required for moisture diffusion was also calculated.

MATERIALS AND METHODS

The experimental studies were carried out in laboratory of department of Chemical Engineering, B.V.Raju Institute of Technology of Jawaharlal Nehru Technological University, Hyderabad, India in August’2010.

Sample preparation: Finger millet samples were procured from the local market. Initial moisture content of the samples was determined by the standard oven method (AOAC, 1960), which was found to be 11.23% (db). Grain samples of known weight (~ 400 g) were soaked for 3 hrs. at 45°C in a constant temperature water bath, the excess water was drained and the surface moisture was removed. Further, the samples were stored in self sealed covers for equilibration for 24 h. there by increasing the moisture content to 38.5% (db)

Experimental apparatus and procedure: Conventional laboratory tray dryer was used for the study, which consisted of a heater, digital temperature controller and a blower. The dimensions of the drying chamber were 0.2*0.2*0.3 m.

Finger millet samples of around 100g were dried in a laboratory tray drier at temperatures of 50, 70 and
Table 1: Thin layer mathematical models

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Model name</th>
<th>Model equation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Page</td>
<td>MR = exp(-kt^n)</td>
<td>Page (1949)</td>
</tr>
<tr>
<td>2.</td>
<td>Modified page</td>
<td>MR = exp(-kt)^n</td>
<td>White et al. (1981)</td>
</tr>
<tr>
<td>3.</td>
<td>Modified page II</td>
<td>MR = exp(-c(t/L^2))^n</td>
<td>Diamante and Munro (1991)</td>
</tr>
<tr>
<td>4.</td>
<td>Lewis</td>
<td>MR = exp(-kt)</td>
<td>Bruce (1985)</td>
</tr>
<tr>
<td>7.</td>
<td>Two term model</td>
<td>MR = aexp(-k(t)+bexp(-k,t))</td>
<td>Henderson (1974)</td>
</tr>
<tr>
<td>8.</td>
<td>Simplified Fick’s diffusion</td>
<td>MR = aexp(-c(u^2))</td>
<td>Diamante and Munro (1991)</td>
</tr>
<tr>
<td>9.</td>
<td>Wang and Singh</td>
<td>MR = 1+a1t+b1t^2</td>
<td>Wang and Singh (1978)</td>
</tr>
</tbody>
</table>

80°C. The air velocity was 2.2 -2.4 m/s, which was measured using the hot air anemometer. Moisture loss was recorded using a digital balance with an accuracy of ±0.01g. Drying studies were continued till the equilibrium was reached. The data was collected in triplicates.

**Mathematical modeling:** Drying characteristics can be investigated by effectively modeling the drying behavior. Hence, the experimental data of the finger millet at three different temperatures were fitted to nine thin layer drying models, presented in Table 1. The dimensionless moisture ratio in these models was given by equation \( MR = \frac{M - M_e}{M_0 - M_e} \) where \( M \) is the moisture content at any time, \( M_0 \) is the initial moisture content and \( M_e \) is the equilibrium moisture content. The values of \( M_e \) may be relatively small compared to \( M \) and \( M_0 \), so the equation can be simplified to \( MR = \frac{M}{M_0} \) (Akgun and Doymaz, 2005; Togrul and Pehlivan, 2002; Thakor et al., 1999).

The non-linear regression analysis in the present study was performed using the software MATLAB 7.0. The goodness of the fit of the tested mathematical models to the experimental data was evaluated with the correlation coefficient \( R^2 \), the reduced chi square \( (\chi^2)^2 \) and the root mean square error (RMSE). The best fit was that which results in higher \( R^2 \) and the lowest \( (\chi^2)^2 \) and RMSE. (Yaldiz and Ertekin, 2004; Gunhan et al., 2005; Ozdemir and Devres, 1999). The reduced \( (\chi^2)^2 \) and RMSE were evaluated as:

\[
\chi^2 = \sum_{i=1}^{n} \left( \frac{MR_{exp,i} - MR_{pre,i}}{N - z} \right)^2
\]

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left( MR_{pre,i} - MR_{exp,i} \right)^2}{N}}
\]

where, \( MR_{exp,i} \) is the \( i \)th experimentally observed moisture ratio, \( MR_{pre,i} \) is the \( i \)th predicted moisture ratio, \( N \) is the number of observations and \( Z \), the number of constants in models (Akpinar, 2006).

**Evaluation of effective diffusivities:** The drying characteristics in the falling rate period can be described by using Fick’s diffusion equation (Crank, 1975), Eq. (3) can be used to evaluate effective diffusivity of spherical particles

\[
MR = \frac{6}{\Pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left( -\frac{n^2 D \Pi^2 t}{r^2} \right)
\]

where, \( D \) is the effective diffusivity and \( R \) is the radius of the grain. For long drying times Eq. (3) can be further simplified to first term of the series (Tutuncu and Labuza, 1996). Thus Eq. (3) can be written in the logarithmic form as Eq. (4):

\[
\ln MR = \ln \frac{6}{\Pi^2} - \frac{\Pi^2 D t}{r^2}
\]

Diffusion coefficients were determined by plotting \( \ln MR \) verses drying time, \( t \). The plot yields a straight line with the slope of \( \frac{\Pi^2 D}{r^2} \) from which the effective diffusivity was evaluated.

**Temperature dependence of diffusivity and calculation of activation energy:** Temperature dependence of diffusivity was studied by the Arrhenius type relationship, since temperature plays a significant role in the process of drying (Ozdemir and Derves, 1999). The relation is given by equation

\[
D = D_0 \exp (-\frac{E_a}{RT})
\]

where, \( D_0 \) is the pre-exponential factor (m^2/s), \( E_a \) is the activation energy required for moisture diffusion (kJ/gmol), \( R \) is the universal gas constant.

**RESULTS AND DISCUSSION**

The effect of drying temperature at a particular air velocity (2.2-2.4 m/s) on drying curves was presented in Fig. 1, which indicated that the moisture ratio decreased with the increased drying time. Further it can be observed from the drying curves that the temperature significantly affects the rate of drying and the total drying process was found to be occurred in the falling rate period only. This indicated that the process describing the drying behavior of the finger millet was diffusion governed (Singh and Sodhi, 2000).
The moisture ratio calculated from the drying data at different temperatures was fitted to the thin layer models given in Table 1. The statistical regression results of different models, including the drying model coefficients, were listed in Table 2. The $R^2$ values were greater than 0.97 for the different models except for Wang and Singh model. Further, it is assumed that the model which has highest $R^2$ and the lowest $\chi^2$ and RMSE could be considered as the best fit. Consequently logarithmic model was selected to predict the drying characteristics in the present study.

Hence, the effect of temperature on the drying constants of the Logarithmic model was taken into account by developing the relation between these constants and the drying temperature. The regression equations relating the constants of the selected model and the drying temperature are the following:

$$\text{MR} = a \exp(-kt) + c$$  where $a$, $k$, and $c$ are constants

$$a = 1.188 - 0.003546T,$$  \hspace{1cm}  $R^2 = 0.9991$

$$k = -0.0213 + 0.00064T,$$  \hspace{1cm}  $R^2 = 0.9923$

$$c = -0.01847 + 0.001162T,$$  \hspace{1cm}  $R^2 = 0.9343$

Thus, the thin layer model for the finger millet was

$$\text{MR} = (1.188 - 0.003546T) \exp\left[\left(-0.0213 + 0.00064T\right)t\right] + (-0.01847 + 0.001162T)$$

Figure 2 gives the experimental MR verses $t$ curve as compared to that of predicted by the logarithmic model at the three temperatures and it may be observed from the figure that there is good matching between experimental values and predicted values.

Effective diffusivities of the finger millet at three different temperatures was evaluated by plotting $\ln \text{MR}$ vs $t$ (Fig. 3) and the data was presented in Table 3. The values varied from $1.526 \times 10^{-10}$ to $2.851 \times 10^{-10}$ m$^2$/s, and as expected the diffusion coefficient has increased.
Fig. 2: Drying curves for the experimental data and that predicted based on the logarithmic model

Fig. 3: lnM vs drying time (min)

Fig. 4: Variation of effective diffusivity as function of temperature

with the increasing drying temperature. The diffusion coefficients of the finger millet were in accordance with the range reported in literature, $10^{-9}$ to $10^{-11}$ m$^2$/s (Madamba et al., 1996).

Further, a plot (Fig. 4.) was made between $1/T$ verses lnD and it may be observed from Fig. 4 that a linear relationship is obtained. Therefore, the diffusion kinetics follows Arrhenius type relationship. The slope of this line gives the value of $Ea/R$, from which the activation energy was evaluated as 35.37 kJ/gmol.

CONCLUSION

The conclusions drawn from the present study were that the Logarithmic model gave an excellent fit for the drying data of the finger millet. The effective diffusivities increased with the drying temperature and varied from $1.526 \times 10^{-10}$ to $2.851 \times 10^{-10}$ m$^2$/s. The temperature dependence of diffusivity follows Arrhenius type of relationship and the activation energy for the diffusion of moisture was found to be 35.37 kJ/gmol.

ACKNOWLEDGMENT

One of the authors (G.B. Radhika) acknowledge the Management, Principal and the Head, Department of Chemical Engineering, B.V. Raju Institute of Technology for providing the necessary facilities to carry out the work.

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**AUTHOR’S CONTRIBUTION**

G.B. Radhika: Main worker for the paper as a part of her Ph.D. work, collected the experimental data, and processed the data, prepared the manuscript. S.V. Satyanarayana: Co-supervisor for the Ph.D. work, assisted in data processing and drafting the manuscript. D.G. Rao: Supervisor for the Ph.D. work, mainly conceived the project, assisted in data processing, redrafted the manuscript with editorial corrections.