

## Shape Design of the Pan in Bread Baking Oven

<sup>1</sup>Lucheng Xie, <sup>2</sup>Xianlu He, <sup>2</sup>Heng Liu and <sup>2</sup>Chunzhi Yang

<sup>1</sup>Department of Life Science,

<sup>2</sup>Department of Mathematical and Computational Science, Huainan Normal University,  
Huainan, 232038, China

**Abstract:** In this study, we analyze diagrams of heat distribution around the baking pan's outer edges. By using Fourier's law, the model of heat distribution is developed. Models of instantaneous heat flux density on the pan are constructed for pans of different shapes-from rectangular to circular and other shapes in between. Then, we utilize two arcs to substitute the two parallel lines of the rectangle, creating a track-shaped pan and discovered that in this design, there is good performance in the baking process and heat is distributed evenly over the entire outer edge of the pan. Finally, simulation results are presented to show the effectiveness of the proposed method.

**Keywords:** Baking oven, heat transfer, shape of the baking pan design

### INTRODUCTION

As one kind of the oldest and most popular food processing techniques, baking oven has been under investigation by many researchers to improve the energy efficiency of the process and the food product quality (Savoie *et al.*, 1992; Sablani *et al.*, 1998; Lostie *et al.*, 2002; Sakin *et al.*, 2007a, b, 2009). In a baking oven, the hot air flows over the baking oven by natural convection, the radiation from the oven heating surfaces, the convection from the air and the conduction heat transfer across contact area between product and the oven surface. The moisture in the food simultaneously diffuses toward the each surface, then, it transfers from the surface and the product loses moisture with continuous movement of the oven air. These are the simultaneous momentum, heat and moisture transfer mechanisms within a baking product (Tong and Lund, 1990; Ozilgen and Heil, 1994) and between the product and its environment (Broyart and Trystram, 2002), which, theoretically, have been well known.

The study of the baking oven requires the following analyses: heat utilization and heat transfer. During a standard cooking procedure, a large proportion of the energy supply to the oven is absorbed by the structure and lost in the around environment (Ploteau *et al.*, 2012). In the context of energy efficiency, we should reduce consumption by adjusting the thermal capacity of the oven and the air temperature levels and optimize radiation whilst maintaining the quality of the product.

When baking in a rectangular pan heat is concentrated in the four corners and the product gets

overcooked at the corners (and to a lesser extent at the edges). In a round pan the heat is distributed evenly over the entire outer edge and the product is not overcooked at the edges. However, since most ovens are rectangular in shape using round pans is not efficient with respect to using the space in an oven. To the best of the author's knowledge, there are little works concerning shape design problem of the pan in a baking oven. In this study, we will develop a model to show the distribution of heat across the outer edge of a pan for pans of different shapes - rectangular to circular and other shapes in between.

### MODEL OF HEAT DISTRIBUTION IN THE BAKING OVEN

**Heat distribution model:** In this section, we will develop a model to show the distribution of heat across the outer edge of a pan for pans of different shapes and thus providing an explanation to why a rectangular pan tends to be overcooked at the corners and edges while in a round pan heat is evenly distributed over the entire outer edge and the product is not overcooked at all. The shape of the oven can be seen in Fig. 1.

Through reasonably assuming that the problem is under an ideal condition that the oven is homothermal, the matter of heat exchange between the oven and the pan can be simplified. We know that in order to bake brownies the oven must be preheated to a certain temperature and therefore, we suppose that, once the pan is inside the oven, the boundary conditions for every surface except the top surface of the pan are exactly the same after some time  $T_0$  (meaning that

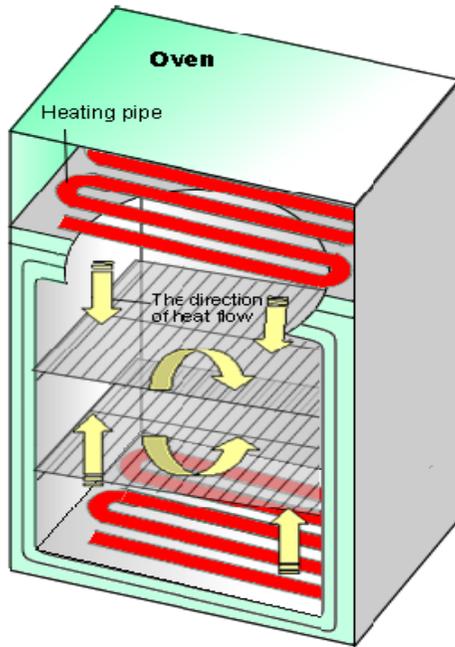


Fig. 1: The shape of baking oven

boundary temperatures have all reached the preheated temperature except the top surface).

**Heat transfer model:** The problem can then be interpreted as a problem of heat exchange among objects with equal initial temperatures under the first boundary condition. According to Fourier's Law in reference (Broyart and Trystram, 2002), a mathematical model for this problem can be described as follows:

$$\alpha \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) = \frac{\partial t}{\partial \tau}, \alpha = \frac{\lambda}{\rho C_p} \quad (1)$$

where,

- $\alpha$  = The thermal diffusivity
- $\lambda$  = Heat transfer coefficient
- $\rho, C_p$  = The density and the specific heat capacity
- $t$  = The temperature

The above three-dimensional heat conduction model is based on the conduction of elementary area in space. But in this study, we assume every surface except the top surface can be heated uniformly. Then the conduction of each surface can be treated as one-dimensional heat conduction model. The model can be simplified as:

$$\begin{cases} \alpha \frac{\partial^2 t}{\partial x^2} = \frac{\partial t}{\partial \tau} \\ t(x, \tau)|_{x=0} = f(\tau) \\ \lim_{x \rightarrow 0} t(x, \tau) = t_0 \\ t(x, \tau)|_{\tau=0} = t_0 \end{cases} \quad (2)$$

Let us define  $T(x, \tau) = t(x, \tau) - t_0$ , then (2) can be rewritten as:

$$\begin{cases} \alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial \tau} \\ T(x, \tau)|_{x=0} = f(\tau) \\ \lim_{x \rightarrow 0} T(x, \tau) = 0 \\ T(x, \tau)|_{\tau=0} = 0 \end{cases} \quad (3)$$

By using the Laplace transformation (Broyart and Trystram, 2002) with respect to time  $\tau$ , we have:

$$\begin{cases} \frac{d^2 \bar{T}(x, p)}{dx^2} - \frac{p}{\alpha} \bar{T}(x, p) = 0 \\ \bar{T}(x, p)|_{x=0} = \bar{f}_0(p) \\ \lim_{x \rightarrow 0} \bar{T}(x, p) = 0 \end{cases} \quad (4)$$

And Eq. (4) can be solved as:

$$\bar{T}(x, p) = C_1 e^{x\sqrt{\frac{p}{\alpha}}} + C_2 e^{-x\sqrt{\frac{p}{\alpha}}} \quad (5)$$

From Eq. (4) we know that  $C_2 = 0$  and  $C_1 = \bar{f}_0(p)$ . Substituting  $C_2 = 0$  into Eq. (5), we can obtain:

$$\bar{T}(x, p) = C_1 e^{-x\sqrt{\frac{p}{\alpha}}} \quad (6)$$

Since  $C_1 = \bar{f}_0(p)$ , we have:

$$\bar{T}(x, p) = \bar{f}_0(p) e^{-x\sqrt{\frac{p}{\alpha}}} \quad (7)$$

From above discussion, we can get the heat transfer model as:

$$T(x, \tau) = \frac{x}{\sqrt{4\pi\alpha}} \int_0^\tau \frac{f_0(\tau')}{(\tau - \tau')^{3/2}} e^{-\frac{x^2}{4\alpha(\tau - \tau')}} d\tau' \quad (8)$$

where,  $f_0(\tau) = T_s = t_s - t_0$ , with  $t_s$  is boundary temperature.

**Model of Instantaneous heat flux density:** According to Fourier's law, we can know that the instantaneous heat flux density pass through the tangent plane with the distant  $d$  apart for the surface:

$$q_d = -\lambda \frac{\partial T}{\partial d} = \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{d^2}{4\alpha\tau}} \quad (9)$$

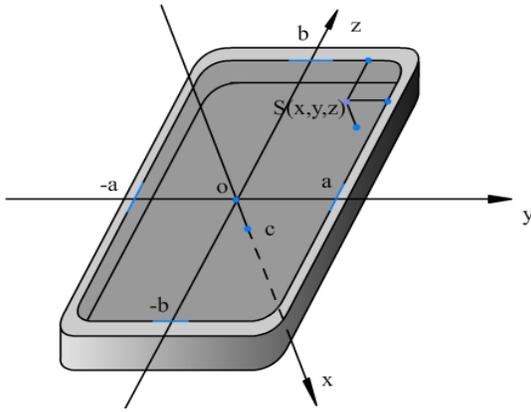


Fig. 2: Heat conduction on the pan in rectangular shape

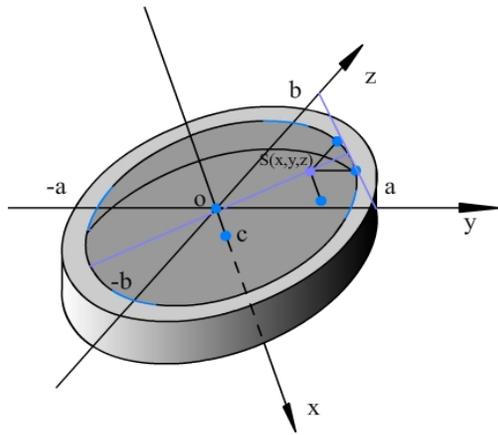


Fig. 3: Heat conduction on the pan in circle shape

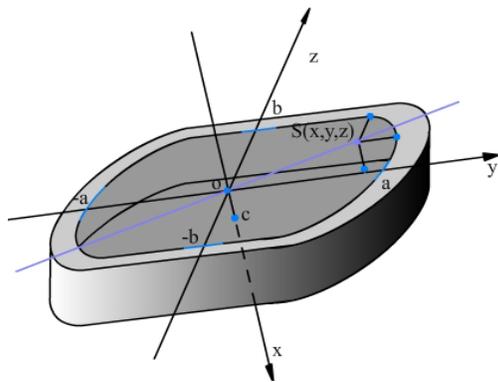


Fig. 4: Heat conduction on the pan in shape between circle and rectangle

Then we will further discuss the distribution of heat on the brownie pan by the three example, namely rectangular, circle and rectangular to circular and other shapes in between.

The instantaneous heat distribution on a rectangular pan can be seen in Fig. 2.

From Eq. (9) and Fig. 1, we can obtain the following equation:

$$\begin{aligned}
 q_s &= q_x + q_y + q_z \\
 &= \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{c^2}{4\alpha\tau}} + \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{(a-y)^2}{4\alpha\tau}} \\
 &+ \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{(y+a)^2}{4\alpha\tau}} + \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{(b-z)^2}{4\alpha\tau}} \\
 &+ \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{(z+b)^2}{4\alpha\tau}}
 \end{aligned}
 \tag{10}$$

The instantaneous heat distribution on a circle pan is shown in Fig. 3.

Similarly available, we have the following equation:

$$\begin{aligned}
 q_s &= q_x + q_y + q_z \\
 &= \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{c^2}{4\alpha\tau}} + \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{m^2}{4\alpha\tau}} \\
 &+ \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{(2r-m)^2}{4\alpha\tau}}
 \end{aligned}
 \tag{11}$$

There are many probable shapes between rectangular and circular shape. In this study, we construct the following shape as described in Fig. 4. We use two arcs to take the place of two parallel lines in the rectangle. From above discussions, we can obtain the following equation:

$$\begin{aligned}
 q_s &= q_x + q_y + q_z \\
 &= \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{c^2}{4\alpha\tau}} + \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{(d-z)^2}{4\alpha\tau}} \\
 &+ \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{(z+d)^2}{4\alpha\tau}} + \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{n^2}{4\alpha\tau}} \\
 &+ \lambda \frac{T_s}{\sqrt{\pi\alpha\tau}} e^{-\frac{(2r-n)^2}{4\alpha\tau}}
 \end{aligned}
 \tag{12}$$

### SIMULATION STUDIES

When a pan full of products with room temperature is put in the oven, in fact the pan's surface can no be heated to the temperature of the oven immediately. In this study we can think that after a small constant  $T_0$ , the surface except the top surface will be heated to the environmental temperature inside the oven. In the simulation studies, we begin compute at time  $T_0$  and the other parts of the pan are assumed to have the room temperature. The environmental temperature and temperature in the oven are assumed to be  $20^\circ$  and  $190^\circ$ , respectively.

Firstly we study the case that the pan has a rectangular shape. As the statement in Savoye *et al.* (1992), Sablani *et al.* (1998) and Sakin *et al.* (2009),

Table 1: Parameter values in the computation of the model

Variable	Value	Unit
A	0.02	m <sup>2</sup>
a	0.1	m
b	0.05	m
c	0.006	m
$\alpha$	$1.47 \times 10^{-5}$	m <sup>2</sup> /s
$\lambda$	54	w/(m[] °C)

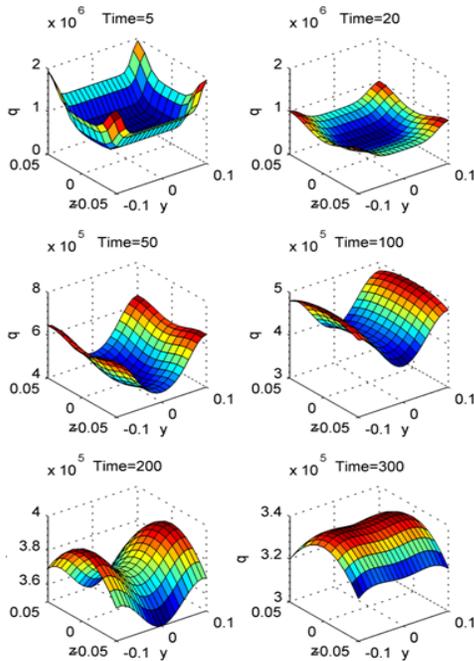


Fig. 5: Instantaneous heat flux density of rectangular pan at different time

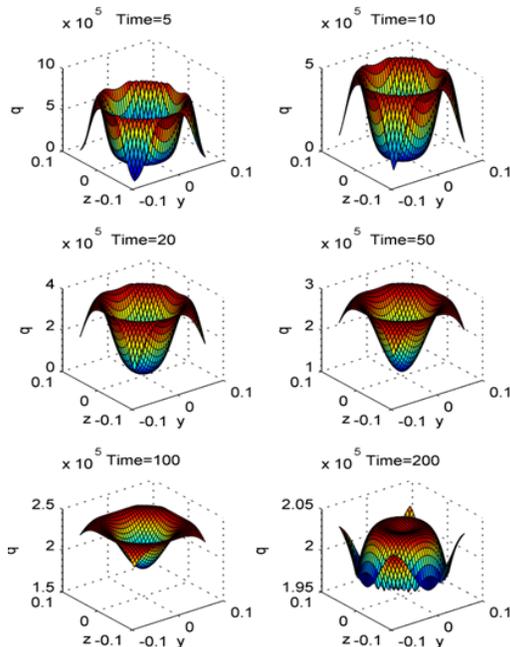


Fig. 6: Instantaneous heat flux density of circular pan at different time

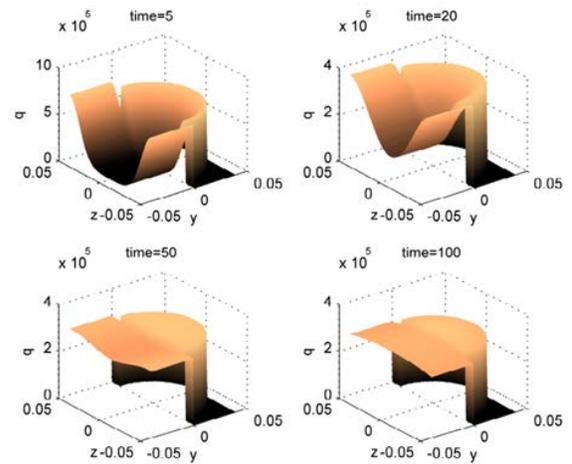


Fig. 7: Instantaneous heat flux density of the pan in different shape

in this study, the parameters used in the simulation are chosen as Table 1.

The simulation results are shown in Fig. 5. The q axis represents the instantaneous heat flux density.

From the results we can see that the heat in the corners of the rectangular brownie pan rise quickly and to a lesser extent at other places.

Secondly, with respect to the circular brownie pans, the computation results are presented in Fig. 6. Then we can easily get the radius  $r = \sqrt{A/\pi} = 0.0798m$ . From the Fig we can see that when the products are baked on the circular pan, the heat is distributed evenly over the entire outer edge. And 100 seconds later, the whole pan approximately achieves the same instantaneous heat flux density.

Then, we utilize the pan in shape between circular and triangle to test the model. To simplify the computation, we let the arcs be semi-circles. The length and width of the rectangle are chosen as 0.015 m, 0.01m, respectively. Then we can conclude that the radius of the semi-circles is  $\sqrt{0.005/\pi}$ . The computation results are shown in Fig. 7. Since the pan is symmetrical, we only draw the part of  $y \geq 0$ . From the results we can see that the shape of the brownie pan we constructed between circle and rectangle have good performance in the baking. And the heat is distributed evenly over the entire outer edge of the pan.

### CONCLUSION

Normally, we use rectangular pans in which heat is concentrated in the four corners and distributed unevenly around the outer edges and thus the brownie gets overcooked at the corners and loses the desired taste. However, with a round pan this can be avoided, though then the number of pans fit in the oven won't be maximized. With these problems in mind and through computer simulation, we analyzed diagrams of heat

distribution around the pan's outer edges for shapes from rectangular to circular and ones in between, to find a solution to bake a maximum quantity of perfect homemade brownies in the shortest amount of time. It is clear to see that the circular pan possesses a wonderfully even distribution of heat around the outer edge and this is confirmed through the fact that we have ran simulations to prove that the rapidity of temperature rise is directly related to the sharpness of the corners on the pan, which determines whether the brownie is overcooked or not. In order to avoid overcooking the brownie, we changed the pointy corners on the rectangular pan to the circular edges on a round pan, creating what we call the "track-shaped" design. This design comprises the advantages of both rectangular and round pans, meaning that not only does it distribute heat evenly around the outer edge, but it can also maximize the number of pans inside the oven.

#### ACKNOWLEDGMENT

Project is supported by the Natural Science Foundation of the Anhui Higher Education Institutions of China (Grant No. KJ2011Z359) and the Natural Science Foundation of the Anhui Higher Education Excellent Young Talents of China (Grant No. 2012SQRL179).

#### REFERENCES

- Broyart, B. and G. Trystram, 2002. Modelling heat and mass transfer during the continuous baking of biscuits. *J. Food Eng.*, 51: 47-57.
- Lostie, M., R. Peczalski, J. Andrieu and M. Laurent, 2002. Study of sponge cake batter baking process. II. Modeling and parameter estimation. *J. Food Eng.*, 55(4): 349-357.
- Ozilgen, M. and J.R. Heil, 1994. Mathematical modelling of transient heat and mass transport in a baking process. *J. Food Process. Preserv.*, 18: 133-148.
- Ploteau, J.P., V. Nicolas and P. Glouannec, 2012. Numerical and experimental characterization of a batch bread baking oven. *Appl. Thermal Eng.*, 48: 289-295.
- Sablani, S.S., M. Marcotte, O.D. Baik and F. Castaigne, 1998. Modeling of simultaneous heat and water transport in the baking process. *Food Sci. Technol.*, 31: 201-209. (In German)
- Sakin, M., F. Kaymak-Ertekin and C. Ilicali, 2007a. Modeling the moisture transfer during baking of white cake. *J. Food Eng.*, 80: 822-831.
- Sakin, M., F. Kaymak-Ertekin and C. Ilicali, 2007b. Simultaneous heat and mass transfer simulation applied to convective oven cupcake baking. *J. Food Eng.*, 83: 463-474.
- Sakin, M., F. Kaymak-Ertekin and C. Ilicali, 2009. Convection and radiation combined surface heat transfer coefficient in baking ovens. *J. Food Eng.*, 94: 344-349.
- Savoie, I., G. Trystram, A. Duquenoy, P. Brunet and F. Marchin, 1992. Heat and mass transfer dynamic modeling of an indirect biscuit baking tunnel-oven. Part I: Modeling principles. *J. Food Eng.*, 16: 173-196.
- Tong, C.H. and D.B. Lund, 1990. Effective moisture diffusivity in porous materials as a function of temperature and moisture content. *Biotechnol. Progr.*, 6: 67-75.