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Study on the Complexity of Closed-loop Supply Chain Based on Price Difference between New and Remanufactured Products

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Abstract: This study studied a more realistic closed-loop supply chain model, which is based on price difference between new and remanufactured products and it contains manufacturers, two recyclers and customers. In this model, assumed that new and remanufactured products have price difference, first we build a decision-making dynamic system model of the manufacturers, two recyclers and customers and then analyze the possibility of the existence of the system equilibrium points and their stability. Through numerical simulation, we use bifurcation diagram, Maximum Lyapunov index variation diagram and chaos attractor to estimate the complexity and chaos of the system comprehensively, observe the profit trends of the manufacturers and recyclers when system change from stability to chaos and analyze the system initial value sensitivity. The conclusion of the numerical simulation has a lot of guidance and reference value to the decision-makers in a closed-loop supply chain.

Keywords: Complexity, closed-loop supply chain, price difference, remanufactured products

INTRODUCTION

The application of game theory and complexity theory in supply chain, using it to guide the decision-makers, is intensively concerned by scholars home and abroad. The equilibrium selection problem in a nonlinear duopoly game with adaptive expectations is studied, proved that the iteration results was sensitive to the initial value (Bischi and Kopel, 2001). An output game model based on various decision rules and the stability of its equilibrium were discussed (Yali, 2012). A supply chain model which is constrained by forbidden returning and limited supply capacity was build and its bifurcation phenomenon was discussed (Jing and Xun, 2012). A duopoly game model with bounded rationality and time delayandits complexity were analyzed (Elsadany, 2010). Aduopoly advertising model and its chaos control based on heterogeneous expectations were studied (Juan *et al*., 2012). The price game model for four oligarchs with different decision rules and its chaos control were analyzed (Junling and Junhai, 2012). A supply chain output game model and its chaos and initial value sensitivity were discussed (Guanhui *et al*., 2011).

Meanwhile, scholars apply game and complexity theory to various economic environment successfully, such like pollution abatement (Dragone *et al*., 2010) and so on. In the field of closed-loop supply chain, a closed-loop supply chain model with uncertain demand was build and studied (Huiling, 2012); a duopoly manufacturers recycling game model was build and its chaos and initial value sensitivity when recovery price and adjustment rates vary were studied (Yuehong *et al*., 2011).

Based on the former work, to be more realistic, this study introduces the assumption that new and remanufactured products have price difference into the closed-loop supply chain model which is consist of manufacturers, recyclers and customers. After building the decision-making dynamic system model of the manufacturers and recyclers, this study analyzes the possibility of the existence of the system equilibrium points and their stability. Then through numerical simulation, we use bifurcation diagram, Maximum Lyapunov index variation diagram and chaos attractor to estimate the complexity and chaos of the system comprehensively, observe the profit trends of the manufacturers and recyclers when system change from stability to chaos and discuss the system initial value sensitivity. At last, we summarize and put forward the direction of further research.

MODEL ESTABLISHMENT

Model description: At present, the research about closed-loop supply chain is mostly focused on the analysis of Nash equilibrium point in price and output model and mostly only concerned the recycling market; seldom used complexity theory to study closed-loop supply chain and rarely combined the new product market and recycling market together. To be more realistic to the product market, this study concerns such a situation: new products and remanufactured products are the same in performance, but different in customer

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approval degree, then builds a three-tier closed-loop supply chain model, which contains manufacturers, recyclers and customers. In this model, the only manufacturer produces remanufactures and sales the products, two recyclers are in a duopoly state and recovery products.
For this model, assumed that:

- The manufacturer M and the two recyclers R1, R2 are independent decentralized decision-makers, at the discrete time periods t = 0, 1, 2, …, they all maximize their own profit (Fig. 1)
- The recovery quantity is only relevant to the recovery price, manufacturer must recovery them all from the recyclers and it can recycle all of them, no waste
- New and remanufactured products have price difference
- Price reduces when output rises
- The amount of remanufactured products is less than demand, so it can sell out

The parameters in this model are denoted as follows:

- **Manufacturer M:** market price of the new products p_n, market price of the remanufactured products p_r, manufacturer recovery price p_0, new product cost per unit c_n, recycling cost per unit c_r
- **Recycler R1:** Recovery price $p_1 = r_1 p_0$, r_1 is the price transfer ratio of R1
- **Recycler R2:** Recovery price $p_2 = r_2 p_0$, r_2 is the price transfer ratio of R2

Model and description: For manufacturer, the price function of new and remanufactured products is:

$$\begin{cases} p_n = a_1 - b_1 Q_1 - d_1 Q_2 \\ p_r = a_2 - b_2 Q_2 - d_2 Q_1 \end{cases} \quad (1)$$

$a_i > 0$, $b_i, d_i > 0 (i = 1,2)$ are the replacement ratio between new and remanufactured products, Q_1 is the amount of new products, Q_2 is the amount of remanufactured products, $Q_2 = q_1 + q_2$.
For recyclers, the recovery amount function is:

$$\begin{cases} q_1 = k_1 + e_1 p_1 - f_1 p_2 \\ q_2 = k_2 + e_2 p_2 - f_2 p_1 \end{cases} \quad (2)$$

$k_i \ge 0 (i = 1,2)$ is the environmental protection index, is the recovery quantity when recovery price is zero, $e_i > 0 (i = 1,2)$ is the customers' recovery price sensitive coefficient, $f_i > 0 (i = 1,2)$ is the recyclers' competition coefficient, $e_i > f_i (i = 1,2)$. Since recyclers can't decide the environmental protection index, so here assumes $k_i = 0$, then the function between q_i and r_i, p_0 is:

$$\begin{cases} q_1 = (e_1 r_1 - f_1 r_2) p_0 \\ q_2 = (e_2 r_2 - f_2 r_1) p_0 \end{cases} \quad (3)$$

So, the profit function of manufacturer M and recyclers R1, R2 at period t is:

$$\begin{cases} \pi_m(t) = (p_n(t) - c_n) Q_1(t) + \\ \qquad (p_r(t) - p_0(t) - c_r) Q_2(t) \\ \pi_{r1}(t) = (p_0(t) - p_1(t)) q_1(t) \\ \pi_{r2}(t) = (p_0(t) - p_1(t)) q_2(t) \end{cases} \quad (4)$$

The decision variables of manufacturer M are recovery price p_0 and new products amount Q_1, the decision variables of recyclers R1, R2 are $r_i (i = 1,2)$, then we got the marginal profit function:

$$\begin{cases} \frac{\partial \pi_m}{\partial Q_1} = a_1 - 2b_1 Q_1(t) - (d_1 + d_2)((e_1 - f_2) r_1(t) \\ \qquad + (e_2 - f_1) r_2(t)) p_0(t) - c_n \\ \frac{\partial \pi_m}{\partial p_0} = (a_2 - (d_1 + d_2) Q_1(t) - 2b_2((e_1 - f_2) r_1(t) \\ \qquad + (e_2 - f_1) r_2(t)) p_0(t) - 2 p_0(t) - c_r) \\ \qquad ((e_1 - f_2) r_1(t) + (e_2 - f_1) r_2(t)) \\ \frac{\partial \pi_{r1}}{\partial r_1} = p_0^2 (e_1 + f_1 r_2 - 2e_1 r_1) \\ \frac{\partial \pi_{r2}}{\partial r_2} = p_0^2 (e_2 + f_2 r_1 - 2e_2 r_2) \end{cases} \quad (5)$$

As in real economic world, no participants can have complete information, so based on the assumption that decision-makers are limited rational, their decision way at period $t+1$ is:

$$\begin{cases} Q_1(t+1) = Q_1(t) + v_1 Q_1(t) \frac{\partial \pi_m}{\partial Q_1} \\ p_0(t+1) = p_0(t) + v_2 p_0(t) \frac{\partial \pi_m}{\partial p_0} \\ r_1(t+1) = r_1(t) + v_3 r_1(t) \frac{\partial \pi_{r1}}{\partial r_1} \\ r_2(t+1) = r_2(t) + v_4 r_2(t) \frac{\partial \pi_{r2}}{\partial r_2} \end{cases} \quad (6)$$

$v_i > 0 (i = 1,2,3,4)$ are the adjustment speed of Q_1, p_0, r_1, r_2 respectively.

Synthesize Eq. (1)-(6), we got the discrete dynamic system model:

$$\begin{cases} Q_1(t+1) = Q_1(t) + v_1 Q_1(t)(a_1 - 2b_1 Q_1(t) - (d_1 + d_2) \\ \qquad ((e_1 - f_2) r_1(t) + (e_2 - f_1) r_2(t)) p_0(t) - c_n) \\ p_0(t+1) = p_0(t) + v_2 p_0(t)((a_2 - (d_1 + d_2) Q_1(t) - \\ \qquad 2b_2((e_1 - f_2) r_1(t) + (e_2 - f_1) r_2(t)) p_0(t) - \\ \qquad 2 p_0(t) - c_r)((e_1 - f_2) r_1(t) + (e_2 - f_1) r_2(t))) \\ r_1(t+1) = r_1(t) + v_3 r_1(t)(p_0^2 (e_1 + f_1 r_2 - 2e_1 r_1)) \\ r_2(t+1) = r_2(t) + v_4 r_2(t)(p_0^2 (e_2 + f_2 r_1 - 2e_2 r_2)) \end{cases} \quad (7)$$

Model analysis: Through the above analysis, we have built the discrete dynamic system model (7) of the manufacturer M and recyclers R1, R2, now we will solve the equilibrium of the system and analyze the stability of the equilibrium points.

Solve the discrete dynamic system model (7) and the equilibrium points are:

$$E_1=(0,0,0,0), E_2=(\frac{a_1-c_n}{2b_1},0,0,0),$$
$$E_3=(0,R,0,0), E_4=(0,0,R,0)$$

$$E_5=(0,0,0,R), E_6=(\frac{a_1-c_n}{2b_1},R,0,0),$$
$$E_7=(\frac{a_1-c_n}{2b_1},0,R,0), E_8=(\frac{a_1-c_n}{2b_1},0,0,R)$$

$$E_9=(0,\frac{a_2-c_r}{2+b_2(e_1-f_2)},\frac{1}{2},0),$$
$$E_{10}=(0,\frac{a_2-c_r}{2+b_2(e_1-f_2)},0,\frac{1}{2}), E_{11}=(0,0,R,R)$$

$$E_{12}=(\frac{2A_2(a_1-c_n)-A_1(d_1+d_2)(e_1-f_2)}{4b_1A_2},\frac{A_1}{A_2},\frac{1}{2},0)$$

$$E_{13}=(\frac{2A_3(a_1-c_n)-A_1(d_1+d_2)(e_2-f_1)}{4b_1A_3},\frac{A_1}{A_3},0,\frac{1}{2}),$$
$$E_{14}=(\frac{a_1-c_n}{2b_1},0,R,R)$$

$$E_{15}=(0,\frac{a_2-c_r}{2(b_2A_4+1)},\frac{e_2f_1+2e_1e_2}{4e_1e_2-f_1f_2},\frac{e_1f_2+2e_1e_2}{4e_1e_2-f_1f_2})$$

$$E_{16}=(\frac{A_5(a_1-c_n)-A_1A_4(d_1+d_2)}{2b_1A_5},\frac{A_1}{A_5},\frac{e_2f_1+2e_1e_2}{4e_1e_2-f_1f_2},\frac{e_1f_2+2e_1e_2}{4e_1e_2-f_1f_2})$$

$$A_1=(a_1-c_n)(d_1+d_2)-2b_1(a_2-c_r)$$

$$A_2=\frac{1}{2}(d_1+d_2)^2(e_1-f_2)-2b_1(b_2(e_1-f_2)-2)$$

$$A_3=\frac{1}{2}(d_1+d_2)^2(e_2-f_1)-2b_1(b_2(e_2-f_1)-2)$$

$$A_4=\frac{(e_1-f_2)(e_2f_1+2e_1e_2)+(e_2-f_1)(e_1f_2+2e_1e_2)}{4e_1e_2-f_1f}$$

$$A_5=A_4(d_1+d_2)^2-4b_1(b_2A_4+1)$$

$E_1 \sim E_{15}$ are boundary equilibrium points, E_{16} is the Nash equilibrium point.

The stability of the equilibrium point depends on the eigenvalues of the Jacobian matrix of system (7). The Jacobian matrix of system (7) is:

$$J(Q_1,p_0,r_1,r_2)=$$
$$\begin{bmatrix} 1+v_1(a_1-4b_1Q_1-(d_1+d_2)((e_1-f_2)r_1+(e_2-f_1)r_2)p_0-c_n) & -v_1(d_1+d_2)((e_1-f_2)r_1+(e_2-f_1)r_2)Q_1 & -v_1(d_1+d_2)(e_1-f_2)Q_1p_0 & -v_1(d_1+d_2)(e_2-f_1)Q_1p_0 \\ -v_2(d_1+d_2)((e_1-f_2)r_1+(e_2-f_1)r_2)p_0 & 1+v_2((a_2-(d_1+d_2)Q_1-4b_2((e_1-f_2)r_1+(e_2-f_1)r_2)p_0-4p_0-c_r)((e_1-f_2)r_1+(e_2-f_1)r_2)) & v_2(e_1-f_2)(a_2-(d_1+d_2)Q_1-4b_2((e_1-f_2)r_1+(e_2-f_1)r_2)p_0-2p_0-c_r)p_0 & v_2(e_2-f_1)(a_2-(d_1+d_2)Q_1-4b_2((e_1-f_2)r_1+(e_2-f_1)r_2)p_0-2p_0-c_r)p_0 \\ 0 & 2v_3r_1p_0(e_1+f_1r_2-2e_1r_1) & 1+v_3p_0^2(e_1+f_1r_2-4e_1r_1) & v_3f_1r_1p_0^2 \\ 0 & 2v_4r_2p_0(e_2+f_2r_1-2e_2r_2) & v_4f_2r_2p_0^2 & 1+v_4p_0^2(e_2+f_2r_1-4e_2r_2) \end{bmatrix}$$

Calculate the Jacobian matrices of the 16 equilibrium points separately; judge their stability according to the value of the eigenvalues of their Jacobian matrix. Take equilibrium point E_1 as an example, its Jacobian matrix is:

$$J_1=\begin{bmatrix} 1+v_1(a_1-c_n) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve the matrix and the Eigen values are: $\lambda_1=1+v_1(a_1-c_n)$, $\lambda_2=\lambda_3=\lambda_4=1$, according to the assumption before, $v_1>0$, $a_1-c_n>0$, so $\lambda_1>1$, then E_1 is unstable. As the same, $E_2,E_3,\cdots,E_{15}$ are all unstable equilibrium points, E_{16} is a local stable Nash equilibrium point.

NUMERICAL SIMULATIONS

In order to study the properties and characteristics of this system better, we will use certain value data to simulate the dynamic system. Assume the parameters values as follows:

$a_1=9$, $a_2=6$, $b_1=1.3$, $b_2=0.8$, $d_1=0.5$, $d_2=0.6$, $c_n=5$, $c_r=1$, $e_1=2.3$, $e_2=1.9$, $f_1=1.4$, $f_2=1.2$.

In the study, we'll use bifurcation diagram, Maximum Lyapunov index variation diagram and chaos attractor to study the dynamic properties of the system, observe the influence on all the participators' profits when the adjustment speed of decision variables change and analyze the initial value sensitivity of the decision variables.

Bifurcation and chaos numerical simulation: In this part, as the situation of v_2 is like v_1 and v_4 is similar to v_3, so we'll only research v_1 and v_3. First, we study the bifurcation and chaos phenomenon of the system: when v_2 = 0.3, v_3 = 0.45, v_4 = 0.5, the trends of Q_1, p_0, r_1, r_2 when v_1 changes are showed in Fig. 2 and

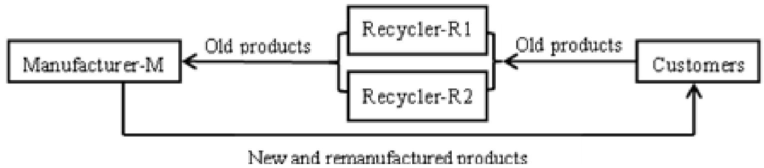

Fig. 1: The CLSC structure

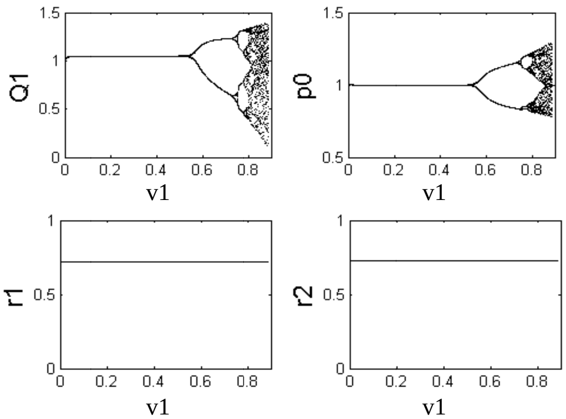

Fig. 2: Bifurcation when v_1 increases

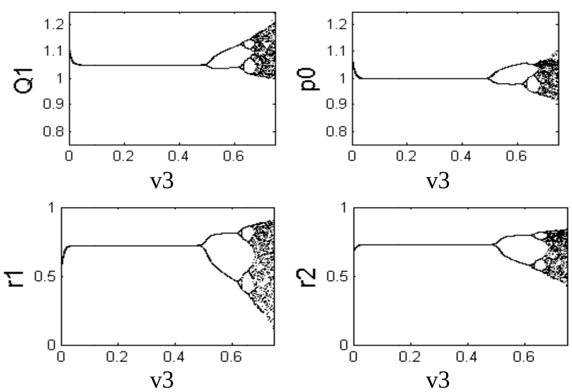

Fig. 3: Bifurcation when v_3 increases

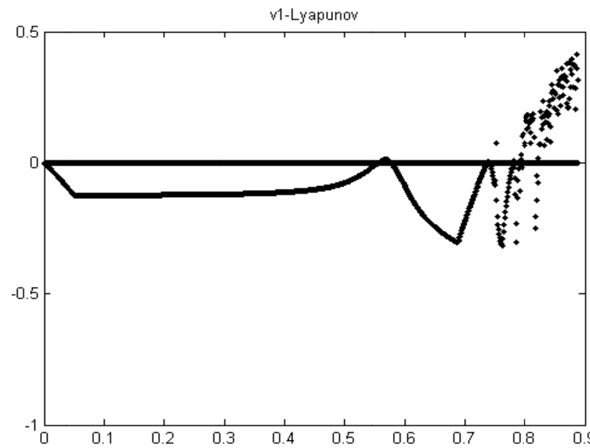

Fig. 4: v_1-maximum Lyapunov index

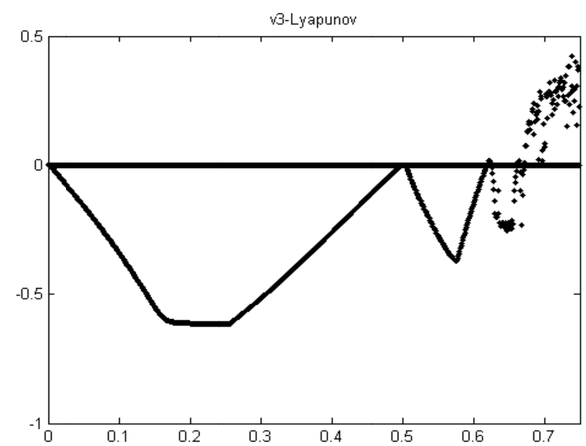

Fig. 5: v_3-maximum Lyapunov index

3 shows the trends of Q_1, p_0, r_1, r_2 when v_3 changes and v_1 = 0.25, v_2 = 0.18, v_4 = 0.5.

From Fig. 2 and 3, we can see that when v_1, v_3 increase, the decision variables of the system ($v_1: Q_1, p_0$; $v_3: Q_1, p_0, r_1, r_2$) will change from stability to the first Bifurcation and then to period-doubling, at last into chaos. If adjust decision variables too fast, the market will be chaotic and the decision will be very complicated.

Besides, in this model, without considering customers' environmental protection consciousness, the adjustment speed of new products amount Q_1 and recovery price p_0 by manufacturer will only cause the chaos of itself, it has no influence to the stability of the recyclers' decision; however, the adjustments peed of price transferratio r_i by there cyclers will in fect both of them.

Next, we'll measure the property of the system through the Maximum Lyapunov index variation diagram, the Maximum Lyapunov index is showed in Fig. 4 and 5 when v_1 and v_3 change.

We can see that the variation status of the Maximum Lyapunov index in Fig. 4 and 5 fits well the situation of bifurcation and chaos in Fig. 2 and 3, when v_1 and v_3 vary. When v_i is small, the Maximum

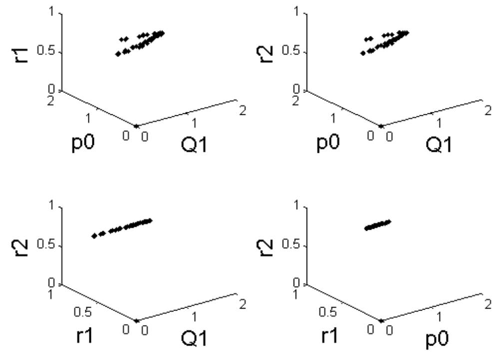

Fig. 6: Attractor when system is in chaos caused by the increasing of v_3

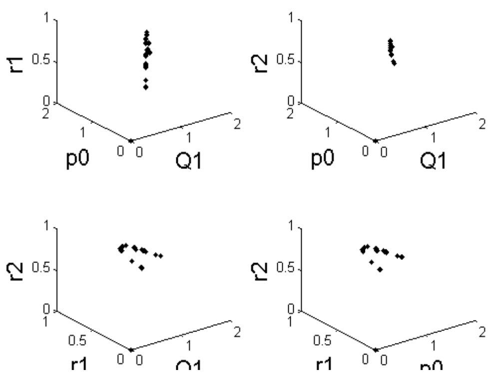

Fig. 7: Attractor when system is in chaos caused by the increasing of v_3

Lyapunov index is less than zero; with the increase of v_i, the system appears bifurcation phenomenon, the Maximum Lyapunov index equals zero; when v_i is too big and the system becomes chaotic, the Maximum Lyapunov index is greater than zero.
Value that:

- $v_1 = 0.85,\ v_2 = 0.3,\ v_3 = 0.45,\ v_4 = 0.5$
- $v_1 = 0.25,\ v_2 = 0.18,\ v_3 = 0.7,\ v_4 = 0.5$

in both values, the Maximum Lyapunov index of the system is greater than zero and the system is chaotic, as the chaos attractor confirmed in Fig. 6 and 7.

The situation in Fig. 6 and 7 is accordant to the one in Fig. 2 to 5.

Influence of DVAS on profits: Profit S is the key factor of an enterprise, how would profits vary when Decision Variables Adjustment Speed (DVAS) v_i increases? The trends of the profits are showed in Fig. 8 when v_1 increases, in Fig. 9 when v_3 increases. As before, the trend of v_2 is similar to v_1 and v_4 is nearly the same as v_3. In the picture, π_m, π_{r1}, π_{r2}, π_{total} stand for the profits of the manufacturer, recycler R1, recycler R2 and the sum of them, respectively.

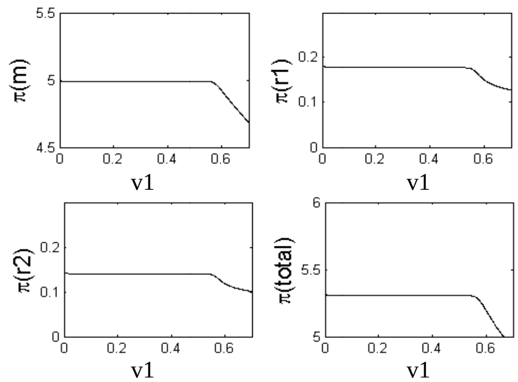

Fig. 8: Profits when v_1 increases

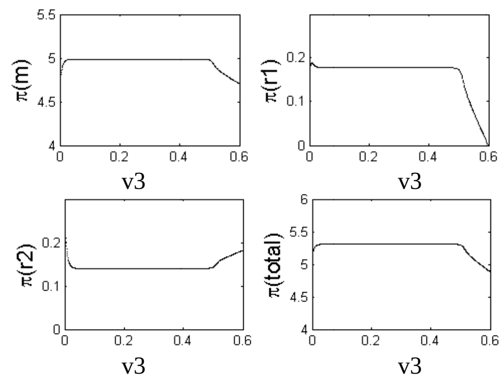

Fig. 9: Profits when v_3 increases

In Fig. 8, when v_1 increases and reach a critical point, the profits of the manufacturer, recycler R1, recycler R2 and the total profit all begin to fall. The critical point value is also the bifurcation point in Fig. 2 and the point when the Maximum Lyapunov index equals zero in Fig. 4. This means that, if the manufacturer adjusts its decision variable too fast, the system becomes chaos, the profits of all the ones will decrease and it's bad for the whole system.

In Fig. 9, when v_3 increases and reach a critical point, the profits of the manufacturer, recycler R1 and the total profit all fall, but the profit of recycler R2 increases. The critical point value is also the one where system changes from stability to chaos. This means recycler R2 can profit from market disorder, but the others' behalf will be harmed. This phenomenon claims the necessity of contract coordination and it is important.

Initial value sensitivity: Figure 10 to 13 showed the initial value sensitivity of the system variables, in the iteration process, $v_1 = 0.25, v_2 = 0.18, v_3 = 0.75$ and $v_4 =$ 0.5.

From Fig. 10 to 13 we can see that, the system variables are highly sensitive to the initial value. Set up two groups of initial values, each variable only differs

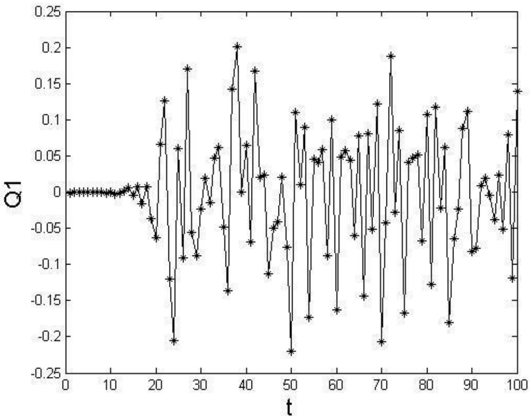

Fig. 10: Q_1-initial value sensitivity

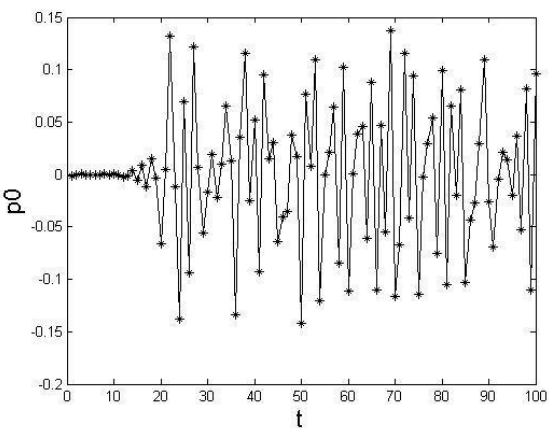

Fig. 11: p_0-initial value sensitivity

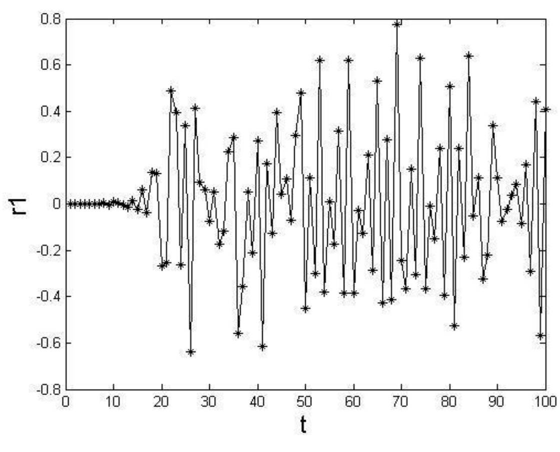

Fig. 12: r_1-initial value sensitivity

0.001. After about twenty times iterations, all the values of the variables had significant differences. After about fifty times iterative operation, the value of Q_1 differs0.2198, it means a difference of 219.8 times; the value of p_0 differs 0.1416, it means a difference of 141.6 times; the value of r_1 differs 0.4775, it means a

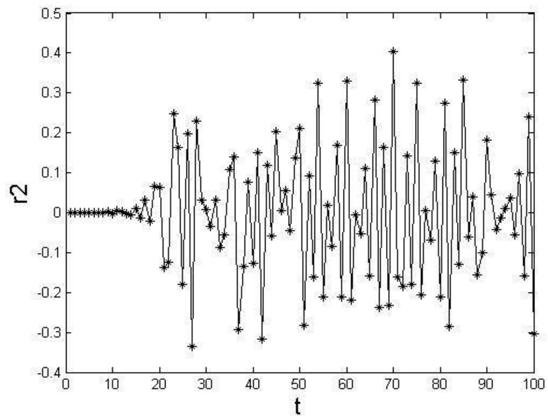

Fig. 13: r_2-initial value sensitivity

difference of 477.5 times; the value of r_2 differs 0.2824, it means a difference of 282.4 times. This phenomenon is the characteristics of chaos movement, as time goes on; the adjacent orbital has a great deviation. This gives the enterprises a useful advice that chose your initial value carefully, or it may cause huge losses.

CONCLUSION

This study assumed that new and remanufactured products have price difference, discussed the market situation of a three-tier supply chain, which is consist of the manufacturer, two recyclers and customers. Through numerical simulation, the conclusions are:

- When the decision-makers adjust their decision variables too fast, the market will become disordered; however, without considering the customers' environmental protection consciousness, the manufacturer and the recyclers have different influence
- If the manufacturer adjusts its two decision variables too fast, all the members' profit of the supply chain will decrease; while any of the recyclers adjusts its decision variable too fast, for one of the recyclers, its profit increase, the profit of the rest in this supply chain would fall. This claims the necessity of contract coordination
- All decision variables are sensitive to initial values

The model and methods in this study would help the manufacturers and recyclers make more reasonable decisions, improve the stability of their profits and the market and have theoretical guidance and practical reference value. The price or output models of multiple manufacturers and products, with one period delay or even more, the retailers are involved and so on, are the direction of further research in future.

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