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Mathematical Modeling and Forecasting Population for Muslim of Rural Region in Bangladesh

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Abstract: In this research the population for Muslim of Rural region in Bangladesh is predicted by using the exponential growth rate method. For this link, the information of data for the Rural Muslim population for male and female of Bangladesh is obtained from 1991 and 2001 censuses. The predictions are computed in three phases. In the first phase, the predictions are computed using negative exponential growth model estimated by the Quasi-Newton method using STATISTICA for the years 1991 and 2001. Using the Cross Validation Predictive Power (CVPP) criterion and R^2, the shrinkage coefficient (λ) is constructed. The shrinkage coefficient determines the adequacy of the first phase prediction. In the second phase, these predicted values are used to estimate the growth rate, for different age groups, by using the exponential growth rate method. In the third phase, that is, finally considering the observed population for Muslim of Rural region in Bangladesh for the Census year 2001 as the base population and using the estimated exponential growth rate, at different age groups, of the second phase estimation, the predictions of the population of Muslim of Rural region are obtained for the years 2002 through to 2021 employing exponential growth rate method successively 20 times.

Key words: Cross-validity prediction power, exponential model, exponential growth rate method, F-test mathematical modeling, Muslim population of rural region in Bangladesh

INTRODUCTION

In this globe, Bangladesh is Muslim prevailing country and approximately 90% population of Bangladesh is Muslim. The Muslim population was increased 10.9% in 1911 over 1901. On the other hand, comparatively lower increasing trends have been observed of 6.8% and 9.2% in 1921 and in 1931 respectively due to causes of high mortality, floods, droughts and pandemic diseases. And, it was increased 26.9% in 1961 over 1951 and 49.3% in 1974 over 1961. It is the reason that the emigrated number of Hindus after independence of 1947 was relatively larger than the immigrated number of Muslims from India. And, it was increased more or less 18.3% in 2001 census (BBS, 2003).

Actually, the size, structure and distribution of population by religion, age or any other demographic and socio-economic characteristics in Bangladesh are obtained from national censuses usually carried out at ten year duration of time are the most important source of population data. The Government and Non-government organizations do require the up-to-the-minute information of population. Therefore, one can easily formulate reasonable and realistic strategy of development for the present and future time. The Government and demographers are not only the users of population estimates but also trade unions, social organizations, university and other social research institutes and centers, market research analysis, housing developers and business communities frequently do need these estimates for their own purposes. Forecasting or projections do help the planners by providing magnified picture of the consequences of current trends to show them for taking appropriate measures. The short and long term projections are required and important in reaching at important decisions in our deliberate economy at the preferred time frame. Population or population by religion (Muslim), age and sex serves as the denominator for the estimation of socio-demographic, health and development related indicators.

Moreover, future population is important and needed for sustainable development in the up to minute high technological, urbanization and industrialization era. In fact, population projections provide an indication to the planners as to how the economy is to be adjusted according to the population and also how the population should be adjusted according to the economy. Population projections can illustrate the possible population growth and structural consequences of such presumed development.

It was investigated that the age structure for male, female and both sexes population of Bangladesh followed either negative exponential model or modified negative exponential model (Islam, 2003, 2005). It was set up that age configuration for population of both sexes of Bangladesh follows negative exponential model (Islam *et al*., 2005). Islam *et al*. (2003) observed that age structure for male population of Bangladesh in 1991

census follows modified negative exponential model. It was showed that proportion of married women of Bangladesh in the reproductive length of life follow cubic polynomial model (Islam, 2004).

Therefore, the basic aims and objectives of this study are to build up mathematical models for the Rural Muslim population for male and female of Bangladesh during 1991-2001 censuses and to test the adequacy of the model and then, to forecast the Rural Muslim population for male and female of Bangladesh by employing exponential growth rate method during 2002-2021.

MATERIALS AND METHODS

Data and data source of this study: To fulfill the objectives mentioned above the data on Muslim population for male and female by age group of Rural area of Bangladesh is taken from the censuses of 1991 (BBS, 1994) and 2001 (BBS, 2003). These are utilized as raw materials of the current study and shown in Table 1-2.

Smoothing of age data: It is observed that there is some kind of unexpected distortions in the data aggregate when the Rural Muslim population of Bangladesh by age group is placed on graph paper. Therefore, an adjustment is important and needed to lessen these unpredicted distortions before going to fit the models to this data. As a consequence, a modification is made here using the Package Minitab Release 12.1 by the most up-to-date smoothing technique "4253H, twice" (Velleman, 1980). After that, the smoothed data are used to fit mathematical model for male and female Muslim population of Rural region in Bangladesh and these smoothed data is launched in Table 1 and 2 for the censuses of 1991 and 2001 respectively.

Model fitting: Using the scattered plot of smoothed age structure for Rural Muslim population of Bangladesh by ages in years, it appears that this population is negative exponentially dispersed in terms of different ages. Therefore, a negative exponential model is considered for the first phase prediction and the form of the model is:

$$y = e^{(-ax+b)} + u$$
$$or\ y = c + e^{(-ax+b)} + u \qquad (1)$$

where, x represents the middle value of the age group; y represents the Rural Muslim population for male and female; a, b and c are unknown parameters and u is the stochastic disturbance term of the model.

Model accuracy test: In this manuscript, the Cross Validity Prediction Power (CVPP), is applied to assess the accuracy and soundness of these models. The mathematical formula for CVPP is specified by

$$\rho_{cv}^2 = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}\left(1-R^2\right)$$

In which, n is the number of classes, k is the number of regressors in the fitted model and the cross-validated R is the correlation between observed and predicted values of the dependent variable (Stevens, 1996). The shrinkage and contraction of the model is the positive value of $\lambda = \left(\rho_{cv}^2 - R^2\right)$; where ρ_{cv}^2 is CVPP and R^2 is the coefficient of determination of the model. 1- λ is the stability of R^2 of the model. Closer the value of λ to zero, the better is the prediction. The estimated CVPP related to their R^2 and information on model fittings are presented in Table 3. Note that CVPP was also employed as model validation or accuracy test by Islam (2003 and 2005) and Islam *et al.* (2003, 2005).

F-test: To find out the measure of overall significance of the fitted models as well as the significance of R^2, the F-test is employed here (Gujarati, 1998).

Exponential growth rate method: The mathematical form of this method is given by

$$P_{t_2}^{a-a+5} = P_{t_1}^{a-a+5} \exp\left\{r^{a-a+5}\left(t_2 - t_1\right)\right\} \qquad (2)$$

(Shryock *et al.*, 1975)

Where,

$P_{t_1}^{a-a+5}$ is the predicted initial population at time t_1 in the age group a to a+5,

$P_{t_2}^{a-a+5}$ = the predicted terminal population at time t_2 in the age group a to a+5,

r^{a-a+5} is the inter-censual annual growth rate in the age group a to a+5 and

$\left(t_2 - t_1\right)$ = the time interval between inter-censual period.

For the second phase estimation, the r^{a-a+5} is computed for different age groups from (2) as follows.

$$r^{a-a+5} = \frac{1}{\left(t_2 - t_1\right)} \ln\left(\frac{P_{t_2}^{a-a+5}}{P_{t_1}^{a-a+5}}\right) \qquad (3)$$

Table 1: Observed, smoothed and predicted Muslim population by age group for male and female of rural area in Bangladesh in 1991 Census (in thousands)

	Male			Female		
Age group	Observed	Smoothed	Predicted	Observed	Smoothed	Predicted
0-4	6614	6614	6965	6502	6502	6551
5-9	6502	6352	6043	6334	5642	5467
10-14	6334	5626	5226	4198	4501	4562
15-19	4198	4494	4501	3018	3569	3808
20-24	3018	3569	3858	3271	3128	3178
25-29	3271	3128	3287	3311	2832	2652
30-34	3311	2832	2782	2266	2395	2213
35-39	2266	2395	2333	1928	1912	1847
40-44	1928	1912	1936	1537	1523	1542
45-49	1537	1523	1583	1182	1225	1287
50-54	1182	1227	1271	1078	9791	074
55-59	1078	997	993	625	780	896
60-64	625	804	747	759	672	748
65-69	759	592	529	327	647	624
70+	327	327	336	707	647	521
Total	42949	42392	42392	37043	36955	36969

Table 2: Observed, smoothed and predicted Muslim population by age group for male and female of rural area in Bangladesh in 2001 Census (in thousands).

	Male			Female		
Age group	Observed	Smoothed	Predicted	Observed	Smoothed	Predicted
0-4	6170	6193	6627	5719	5828	6179
5-9	6494	6031	5723	5854	5601	5526
10-14	5905	5394	4943	5167	5097	4923
15-19	4096	4353	4269	3694	4479	4366
20-24	2881	3433	3687	4026	4011	3851
25-29	3007	2953	3184	3971	3632	3375
30-34	2705	2750	2750	3013	3131	2936
35-39	2704	2541	2375	2592	2518	2529
40-44	2218	2200	2051	1936	1945	2154
45-49	1703	1783	1772	1388	1487	1808
50-54	1464	1382	1530	1310	1141	1487
55-59	890	1099	1321	748	920	1191
60-64	1085	1006	1141	946	850	918
65-69	573	1006	986	450	850	665
70+	1365	1006	851	1041	850	432
Total	43260	43129	43210	41855	42339	42339

Table 3: The results of CVPP and information on model fittings

Models	n	K	R^2	ρ_{cv}^2	Shrinkage Coefficient (λ)	Parameters	p-value	Cal. F	Tab. F (at 1% level)
1	15	1	0.99182	0.9898	0.00199			727.49	6.93
						a	0.00000		with
						b	0.0000		(2, 12) d.f.
						c	0.00		
2	15	1	0.9957	0.9947	0.00115			3010.26	9.07
						a	0.00000		with
						b	0.0000		(1, 13) d.f
3	15	1	0.98366	0.9797	0.00399			782.59	9.07
						a	0.00000		with
						b	0.0000		(1, 13) d.f
4	15	1	0.98203	0.9776	0.0044			327.89	6.93
						a	0.00000		with
						b	0.0000		(2, 12) d.f.
						c	0.00		

Years 1991 and 2001 are considered as the initial and the terminal populations respectively in estimating the age specific growth rates by using Eq. (3).

Finally, that is, for the third phase for forecasting purpose, 2001 census Rural Muslim observed population for male and female by age group is considered as base population and inter-censual annual growth rate by age groups during 1991-2001 obtained in the second phase is used in this study assuming fertility and mortality remaining unaltered during the forecasted period.

Actually, age wise forecasted population is obtained from Eq. (2) using age specific growth rates estimated of the second phase and observed initial population of 2001 census running Eq. (2) successively 20 times.

Application of Models and Forecasted Results: The negative exponential model is selected to fit the models for Rural Muslim population of Bangladesh for the censuses of 1991 and 2001 and the fitted models are described below:

$$y = -1178.18 + \exp(-0.02403x + 9.0651) \quad (1)$$

for male of 1991 census
t-stats (-4.62) (11.82607) (485.3952)

$$y = \exp(-0.03617x + 8.8777) \quad (2)$$

for female of 1991 census
t-stats (42.65868) (571.9266)

$$y = \exp(-0.02932x + 8.8722) \quad (3)$$

for male of 2001 census
t-stats (23.33166) (324.9421)

$$y = -2404.85 + \exp(-0.01582x + 9.0971) \quad (4)$$

for female of 2001 census
t-stats (-6.83) (11.18068) (280.0926)

The findings on model fittings and estimated CVPP, ρ_{cv}^2, similar to their R^2 of these models is shown in Table 3. From this table it appears that all the fitted models (1), (2), (3) and (4) are highly cross- validated and their shrinkages are 0.00199, 0.00115, 0.00399 and 0.0044 respectively. These imply that the fitted models (1), (2), (3) and (4) will be stable more than 98, 99, 97 and 97% respectively. Moreover, it is found that the parameters of the fitted (1), (2), (3) and (4) are highly statistically significant with large proportion of variance explained. The stability for R^2 of these models is more than 99%.

The enumerated values of F statistic of these models are shown in the 9th columns of Table 3 while the analogous tabulated values of these models are demonstrated in the last column of the same table at 1% level of significance. These results are showed that these models and their corresponding to R^2 are highly statistically significant and hence, these are well fitted to the data aggregate.

The graph of observed, smoothed and predicted Rural Muslim population for male and female by age group in years for 1991 and 2001 census years of Bangladesh are shown in Fig. 1-4 respectively.

It should be noted that the usual models, i.e., Gompertz, Makeham, logistic, linear, log linear and polynomial model were also tried to apply here but seems to be worse fitted with respect to their coefficient of determination and shrinkages. Therefore, the results of those models were not shown here.

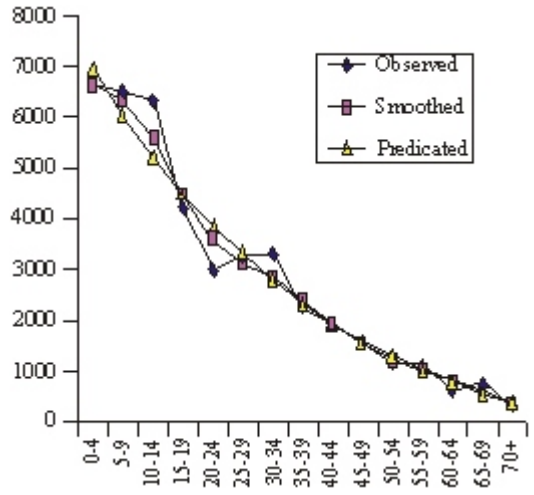

Fig. 1: Observed, smoothed and predicted Muslim population by age group for male of rural area in Bangladesh in 1991 census. X axis represents age group and Y axis represents male population.

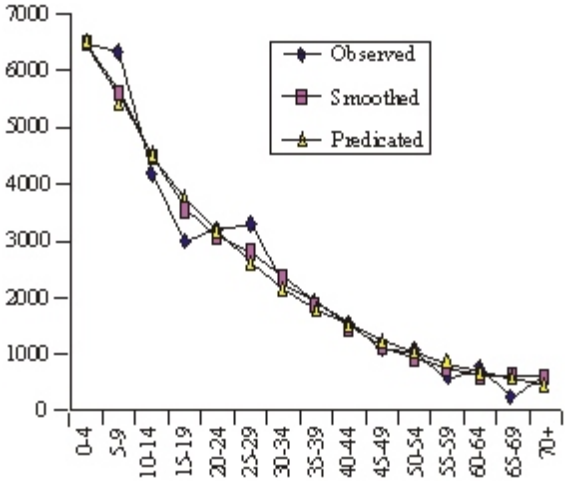

Fig. 2: Observed, smoothed and predicted muslim population by age group for female of rural area in Bangladesh in 1991 census. X-axis represents age and Y-axis represents female population.

For the second phase estimation, age specific annual growth rate during 1991-2001 are estimated by applying exponential growth rate method using the predicted values obtained by first phase procedure and these growth rates are demonstrated in Table 4. From the Table 4 it is found that growth rate for male population are negative in the age group 0-34, that is, population for male are decreasing in this age group. It is happened because total fertility rate (TFR) dramatically, significantly and substantially

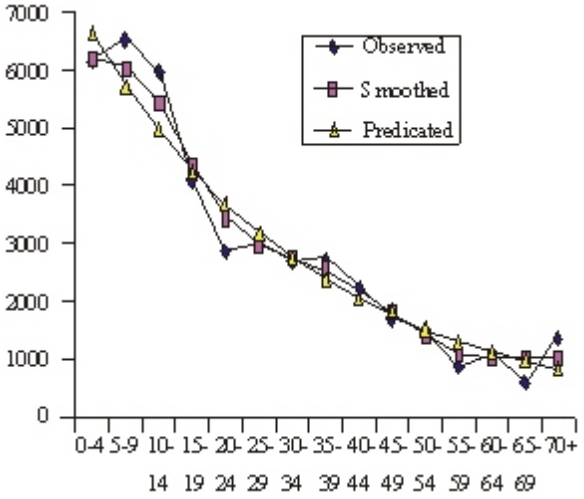

Fig. 3: Observed, smoothed and predicted Muslim population by age group for male of rural area in Bangladesh in 2001 census. X-axis represents age group and Y-axis represents male population.

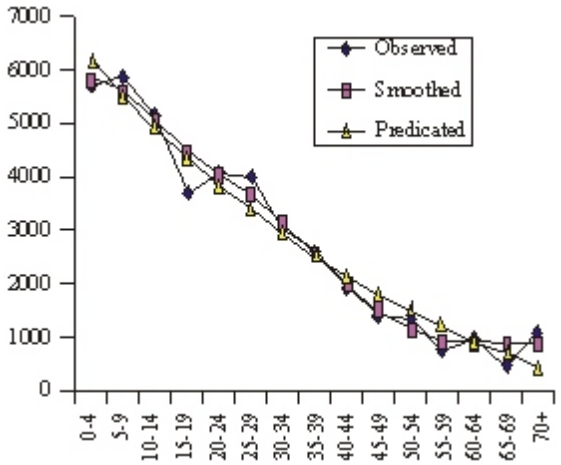

Fig. 4: Observed, smoothed and predicted muslim population by age group for female of rural area in Bangladesh in 2001 census. X axis represents age and Y axis represents female population.

reduced from early 198- 1991 period to late 1991-2001 period because of technological advancement, the development of socio-economic, demographic and health related characteristics of the common people, improvement in using contraceptive for both male and female, and advancement of social and cultural awareness about the curse of over increasing and accelerated population not only for a family but also for the country as a whole. In fact, it is also indicating that population is increasing slowly than the previous time. But beyond this age group population are increasing during 1991-2001

Table 4: The Estimated annual exponential growth rate during 1991-2001 for male and female rural Muslim population of Bangladesh at each age group

Age group	The Estimated Exponential Growth Rate for Male	The Estimated Exponential Growth Rate for Female
0-4	-0.004974539	-0.005846126
5-9	-0.005440743	0.001073421
10-14	-0.005567373	0.007615698
15-19	-0.005291999	0.013674314
20-24	-0.004533580	0.019208079
25-29	-0.003183703	0.024108125
30-34	-0.001156918	0.028269905
35-39	0.001784244	0.031426127
40-44	0.005770349	0.033424630
45-49	0.011278707	0.033990733
50-54	0.018546374	0.032537067
55-59	0.028541364	0.028460816
60-64	0.042359516	0.020479441
65-69	0.062266792	0.006363667
70 +	0.092930097	-0.018732445

period. On the other hand, growth rate for female population are positive, that is, population for female are increasing during 1991-2001 excepting the first age group 0-4.

Employing exponential growth rate method Rural Muslim population for male and female of Bangladesh by age wise are forecasted and presented in Table 5 and Table 6 respectively. It is found that the pattern of male and female Muslim population of Bangladesh is downward due to ages at each year. But the trend of total is increasing, i.e., upward with passing of time throughout the forecasting period 199-2021.

Age wise total Muslim population for both sexes of Rural area of Bangladesh is obtained by adding age wise male and female Muslim population of Rural area of Bangladesh. For that rationale, it is left out in the analysis of this dissertation.

CONCLUSION

It is observed that the age pattern of Muslim population for male and female of Rural region of Bangladesh of the censuses of 1991 and 2001 follow negative exponential model or modified negative exponential model.

Hope that one can use this 1st phase predicted population for Muslim of Rural area in Bangladesh for further highly advanced and sophisticated study as more smoothed data than observed. It is expected that these forecasted Rural Muslim population during the period 2002-2021 might be more used for higher developed research as well as it can be used in the planning of government and NGOs to boost the socio-economic, demographic, and health related characteristics and other social indicators as most of the cases the population is frequently used as denominator for the estimation of various socio-economic, demographic, health related, and

Table 5: Forecasted male Muslim population by age group of rural area in Bangladesh during 2002-2021 (in thousands)

Age Group	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
0-4	6594	6561	6529	6496	6464	6432	6400	6368	6337	6305
5-9	5692	5661	5630	5600	5569	5539	5509	5479	5450	5420
10-14	4916	4888	4861	4834	4807	4781	4754	4728	4701	4675
15-19	4246	4224	4202	4180	4158	4136	4114	4092	4070	4049
20-24	3670	3654	3637	3621	3604	3588	3572	3556	3540	3524
25-29	3174	3164	3154	3144	3134	3124	3114	3104	3094	3084
30-34	2747	2744	2740	2737	2734	2731	2728	2725	2722	2718
35-39	2379	2383	2388	2392	2396	2401	2405	2409	2413	2418
40-44	2063	2075	2087	2099	2111	2123	2136	2148	2160	2173
45-49	1792	1812	1833	1854	1875	1896	1918	1939	1961	1984
50-54	1559	1588	1618	1648	1679	1710	1742	1775	1808	1842
55-59	1359	1399	1439	1481	1524	1568	1613	1660	1708	1757
60-64	1190	1242	1296	1352	1410	1471	1535	1601	1671	1743
65-69	1049	1117	1189	1265	1346	1433	1525	1623	1727	1838
70+	934	1025	1125	1234	1354	1486	1631	1790	1964	2155
Total	43365	43537	43726	43936	44166	44418	44694	44996	45326	45685

Table 5: (Continued)

Age Group	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
0-4	6274	6243	6212	6181	6151	6120	6090	6059	6029	5999
5-9	5391	5361	5332	5303	5274	5246	5217	5189	5161	5133
10-14	4649	4624	4598	4572	4547	4522	4497	4472	4447	4422
15-19	4028	4006	3985	3964	3943	3922	3902	3881	3861	3840
20-24	3508	3492	3476	3460	3445	3429	3414	3398	3383	3367
25-29	3074	3065	3055	3045	3036	3026	3016	3007	2997	2988
30-34	2715	2712	2709	2706	2703	2700	2696	2693	2690	2687
35-39	2422	2426	2431	2435	2439	2444	2448	2453	2457	2461
40-44	2185	2198	2211	2224	2236	2249	2262	2275	2289	2302
45-49	2006	2029	2052	2075	2099	2122	2147	2171	2195	2220
50-54	1876	1911	1947	1984	2021	2059	2097	2136	2176	2217
55-59	1808	1861	1914	1970	2027	2086	2146	2208	2272	2338
60-64	1818	1897	1979	2065	2154	2247	2344	2446	2552	2662
65-69	1956	2082	2215	2358	2509	2670	2842	3024	3219	3425
70+	2365	2596	2848	3126	3430	3764	4131	4533	4975	5459
Total	46076	46502	46965	47467	48013	48606	49249	49946	50702	51522

Table 6: Forecasted female Muslim population by age group of rural area in Bangladesh during 2002-2021 (in thousands)

Age Group	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
0-4	6143	6107	6072	6036	6001	5966	5931	5897	5862	5828
5-9	5532	5538	5544	5550	5556	5562	5568	5574	5580	5586
10-14	4961	4999	5037	5075	5114	5153	5193	5232	5272	5313
15-19	4426	4487	4549	4611	4675	4739	4805	4871	4938	5006
20-24	3926	4002	4079	4159	4239	4321	4405	4491	4578	4667
25-29	3457	3542	3628	3717	3807	3900	3995	4093	4193	4295
30-34	3020	3107	3196	3288	3382	3479	3578	3681	3787	3895
35-39	2610	2693	2779	2868	2959	3054	3151	3252	3356	3463
40-44	2227	2303	2381	2462	2546	2632	2722	2814	2910	3009
45-49	1871	1935	2002	2071	2143	2217	2294	2373	2455	2540
50-54	1536	1587	1639	1694	1750	1808	1867	1929	1993	2059
55-59	1225	1261	1297	1335	1373	1413	1454	1496	1539	1583
60-64	937	956	976	996	1017	1038	1060	1081	1104	1127
65-69	669	674	678	682	686	691	695	700	704	709
70+	424	416	408	401	393	386	379	372	365	358
Total	42964	43606	44266	44944	45642	46359	47097	47855	48634	49436

development indicators because Bangladesh is Rural and Muslim dominant country. Moreover, population projections are needed and used for setting employment target, for manpower planning, for planning productivity increases, for opening schools, for teachers training, for planning expansion of hospitals, for supply of goods and services of various types, for providing inputs to agriculture and industry, for calculating food requirements, for planning distribution system, for looking after nutritional needs, for planning housing construction programed, for planning transportation needs, for planning energy supply needs, for developing mining, for planning information and communication technology to boost the knowledge in this sector to vie the competitive global market, for conducting and planning elections, for planning the organization of law and order machinery, etc.

Table 6: (Continued)

Age group	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
0-4	5794	5760	5727	5693	5660	5627	5594	5562	5529	5497
5-9	5592	5598	5604	5610	5616	5622	5628	5634	5640	5646
10-14	5353	5394	5435	5477	5519	5561	5603	5646	5689	5733
15-19	5075	5145	5215	5287	5360	5434	5509	5584	5661	5739
20-24	4757	4849	4943	5039	5137	5237	5338	5442	5547	5655
25-29	4400	4507	4617	4730	4845	4964	5085	5209	5336	5466
30-34	4007	4122	4240	4362	4487	4615	4748	4884	5024	5168
35-39	3573	3687	3805	3927	4052	4181	4315	4453	4595	4741
40-44	3111	3217	3326	3439	3556	3677	3802	3931	4065	4203
45-49	2628	2719	2813	2910	3010	3115	3222	3334	3449	3568
50-54	2127	2197	2270	2345	2423	2503	2585	2671	2759	2851
55-59	1629	1676	1724	1774	1825	1878	1932	1988	2045	2104
60-64	1150	1174	1198	1223	1248	1274	1300	1327	1355	1383
65-69	713	718	722	727	732	736	741	746	750	755
70+	352	345	339	332	326	320	314	308	303	297
Total	50260	51108	51979	52875	53796	54743	55717	56718	57748	58806

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