Research Journal of Applied Sciences, Engineering and Technology 2(3): 233-238, 2010
ISSN: 2040-7467

Submitted Date: March 08, 2010 Accepted Date: March 24, 2010 Published Date: May 10, 2010

Statistical Study of Complex Eigenvalues in Stochastic Systems

[1,3]Seifedine Kadry, [1,2]Hussam Kassem, [2]Mohamad Smaili and [2]Abdo Abou Jaoude
[1]ISAE-CNAM,
[2] Faculty of Sciences, Lebanese University, Lebanon
[3]Arts, Sciences and Technology University, Lebanon

Abstract: In this research we analyze the complex modes arising in multiple degree-of-freedom non-proportionally damped discrete linear stochastic systems. The complex eigenvalues intervene when unstable states like resonances, happened. Linear dynamic systems must generally be expected to exhibit non-proportional damping. Non-proportionally damped linear systems do not possess classical normal modes but possess complex modes. The proposed method is based on the transformation of random variables. The advantage of this method which give us the probability density function of real and imaginary part of the complex eigenvalue for stochastic mechanical system, i.e. a system with random output (Young's modulus, load). The proposed method is illustrated by considering numerical example based on a linear array of damped spring-mass oscillators. It is shown that the approach can predict the probability density function with good accuracy when compared with independent Monte-Carlo simulations.

Key words: Complex modes, discrete linear stochastic systems, non-proportionately damped, probabilistic transformation method

INTRODUCTION

Dynamics of linear systems with statistical parameter uncertainties is currently an active area of research. The primary concern here lies in the probabilistic modeling of the uncertainties in specifying elastic, mass and damping properties of the structure. Problems of undamped linear deterministic structural dynamics have been extensively treated in the existing literature using classical modal analysis. Now, classical modal analysis has also been generalized to deal with undamped stochastic systems. Over the years many efficient methods have emerged to solve what is called random eigenvalue problems that arise in the dynamics of undamped stochastic systems. Several review papers have appeared in this field, which discuss the current as well as the earlier works (Benaroya, 1992; Ibrahim, 1987; Manohar and Ibrahim, 1999).

It is well known that if the damping is ‘proportional’ then classical modal analysis also holds for damped systems. Conditions for existence of classical normal modes were derived by Caughey and O’Kelly (1965).

It must be noted that proportional damping or ‘classical damping’ is purely a mathematical abstraction. There is no physical reason or mathematical basis for why a general system should behave like this. If the damping is not proportional then linear systems possess complex modes instead of real normal modes.

Apart from the mathematical consistency, practical experience in modal testing also shows that most real life structures possess complex modes. As Sestieri and Ibrahim (1994) have put it: it is ironic that the real modes are in fact not real at all, so that in practice they do not exist, while complex modes are those practically identifiable from experimental tests. This implies that real modes are pure abstraction, in contrast with complex modes that are, therefore, the only reality! However, consideration of complex modes in stochastic structural dynamics has not been considered till now. Central theme of this paper is the analysis of the complex eigenvalues in the dynamics of non-proportionally damped discrete linear stochastic systems.

Due to the non-proportional nature of the damping, eigenvalues become complex random variables and eigenvectors become complex random processes. This fact makes the analysis quite different from the traditional random eigenvalue problems arising in the analysis of stochastic undamped systems where the random eigensolutions remain real.

The complex eigenvalues analysis, however, may be performed by extending the currently available techniques of random eigenvalue problems in conjunction with the first-order formulation of the equations of motion. The first-order or state-space formalism, although straightforward, requires significant numerical effort to obtain statistics of the eigensolutions as the size of the problem doubles. Moreover, this approach also lacks some of the intuitive simplicity of the traditional ‘N-space’ based modal analysis method of random structural systems.

Corresponding Author: Seifedine Kadry, LIU, Khyara, West Bekaa, Faculty of Engineering, P.O. Box 5, Jeb Janeen, Lebanon

Due to manufacture error and measurement error of the structure, the respective materials and geometric characteristics have certain randomness. It is important to investigate the uncertain eigenvalue problem. There are references in this respect (Collins and Thomson, 1969; Shinozuka and Astill, 1972; Nakagiri *et al.* 1987; Ganesan *et al.*, 1993; Ganesan, 1996). The probabilistic finite element method for structural dynamics was formulated. Excellent survey papers (Vanmarcke *et al.*, 1986; Benaroya and Rebak, 1988) are available for reviews of the structural dynamics with parameter uncertainties.

Recently Adhikari (2000), propose an approach to obtain the first and second moment, not the probability density function, of complex eigenvalues of non-proportionally damped linear stochastic systems. Also, he assumed that the randomness is small so that the first-order perturbation method can be applied.

For these reasons, in this paper an effective method is proposed to obtain the "exact" probability density function (pdf), the most important statistical characteristics, of complex eigenvalues.

MATERIALS AND METHODS

The theory of Transformation method by Collins and Thomson (1969) is based on the following theorem:

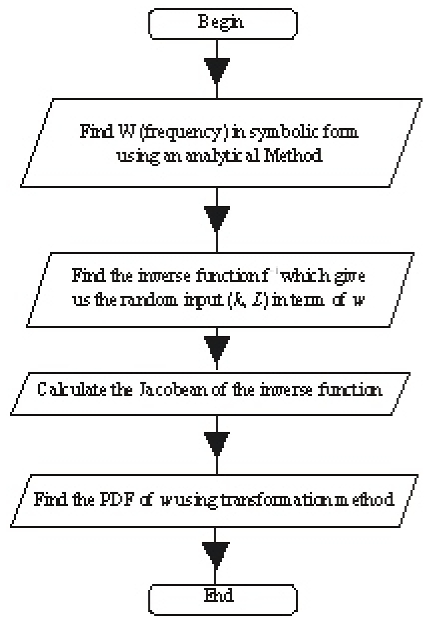
Theorem: Suppose that X is a random variable with PDF (probability density function) $f_x(x)$ and $A \subset \Re$ is the one–dimensional space where $f(x)>0$ Consider the random variable (function of x) $Y=u(X)$, where $y=u(x)$ defines a one-to-one transformation that maps the set A onto a set $B \subset \Re$ so that the equation $y=u(x)$ can be uniquely solved for x in terms of y, say $x=w(y)$ Then, the PDF of Y is: $gy(y) = fx[w(y)]|J|, y \in B$, where;

$$J = \frac{dx}{dy} = \frac{dw(y)}{dy} \tag{1}$$

is the Jacobian of the transformation.

The proposed method is a combination of the deterministic method to calculate the complex eigenvalues and the random variable transformation technique. In this technique, the solution of the eigenvalues system is solved firstly then this solution is used to obtain the probability density function of the eigenvalues using the random variable transformation between the input random variables and the output variable.

Algorithm: The following flowchart describes our algorithm



RESULTS AND DISCUSSION

Let us consider the system shown in the Fig. 1 with 3 degree-of-freedom system, with a strong contrast of amortization case, corresponding for an example of a weakly dissipative structure except in the neighborhood of its junction.

After simplification, the third complex eigenvalue of the previous system is:

$$w_3 = 0.5889\sqrt{\frac{k}{m}} + 0.1121\sqrt{\frac{k}{m}}i$$

i.e.,

$$R(\omega_3) = 0.5889\sqrt{\frac{k}{m}}, \operatorname{Img}(\omega_3) = 0.1121\sqrt{\frac{k}{m}}$$

If we apply our proposed method to the above equation, we obtain the "exact" probability density function of the real and imaginary part. As opposed to numerical method, because, unfortunately we don't have in the literature an analytical method to evaluate the "exact" pdf of eigenvalue of stochastic system, no series expansion is involved in this expression.

Case 1: $k \rightarrow U(12,15)$, k is random variable uniformly distributed in the range of Ibrahim (1987) and Papoulis (2002). The application of our method gives:
Step 1:

$$R(\omega_3) = 0.5889\sqrt{\frac{k}{m}}, \operatorname{Im}g(\omega_3) = 0.1121\sqrt{\frac{k}{m}}$$

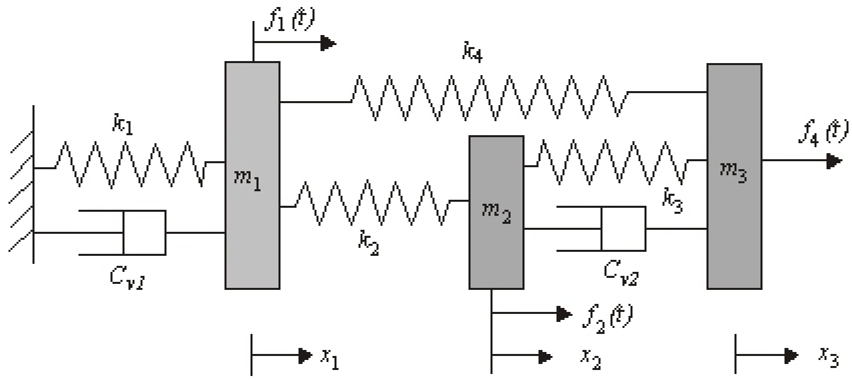


Fig. 1: Springs-mass system

Step 2:

For the real part $k = \dfrac{m(R(\omega_3))^2}{0.5889^2}$

For the imaginary part $k = \dfrac{m(\mathrm{Img}(\omega_3))^2}{0.1121^2}$

Step 3:

For the real part $|J| = \dfrac{2mR(\omega_3)}{0.5889^2}$

For the imaginary part $|J| = \dfrac{2m\,\mathrm{Img}(\omega_3)}{0.1121^2}$

Step 4:

$$pdf(R(\omega_3)) = \frac{2m\omega_3}{(0.5889)^2}.pdf(k)$$

and

$$pdf(\mathrm{Img}(\omega_3)) = \frac{2m\omega_3}{(0.1121)^2}.pdf(k)$$

Numerical value: *m=1kg*. i.e.,

$$pdf(R(\omega_3)) = \begin{cases} \dfrac{2\omega_3}{1.04} & if\ 2.04 < \omega_3 < 2.2 \\ 0 & if\ not \end{cases}$$

as illustrated in Fig. 2, and

$$pdf(\mathrm{Img}(\omega_3)) = \begin{cases} \dfrac{2\omega_3}{0.04} & if\ 0.38 < \omega_3 < 0.43 \\ 0 & if\ not \end{cases}$$

as shown in Fig. 3.

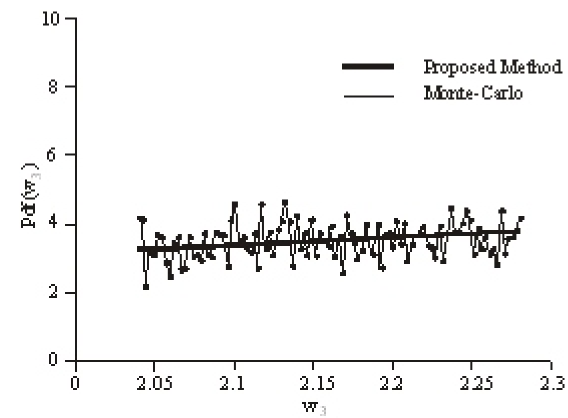


Fig. 2: Pdf of the real part of w_3

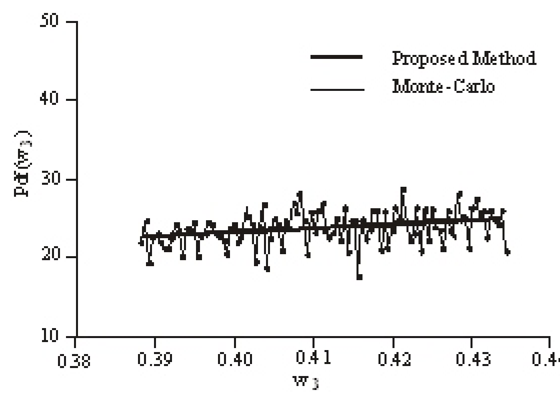


Fig. 3: pdf of the imaginary part of w_3

Case 2: $k \rightarrow \log(0,1)$, k is random variable follows a lognormal distribution log[0, 1]. The application of our method gives (Fig. 4):

$$pdf(R(\omega_3)) = \frac{2}{\omega_3\sqrt{2\pi}}\exp\left[-\frac{\left(\log\left(\omega_3^2/5889^2\right)\right)^2}{2}\right]$$

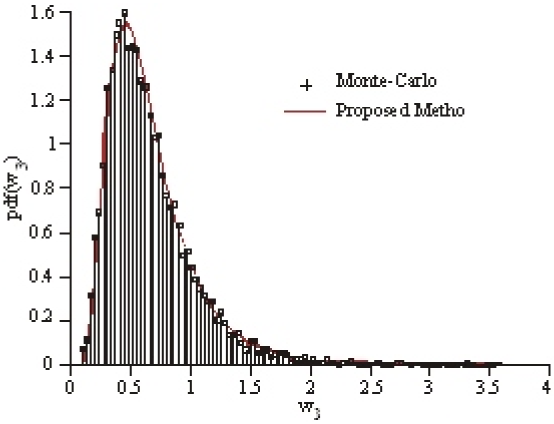


Fig. 4: pdf of the real part of w_3

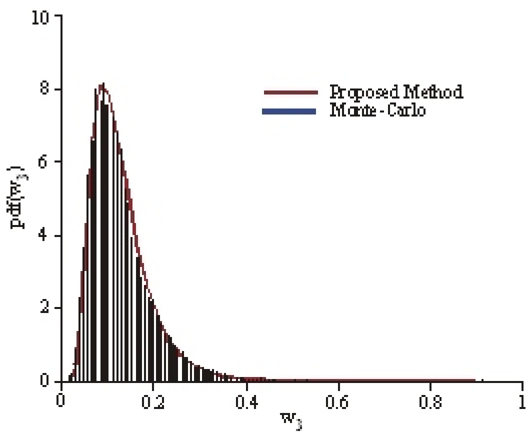


Fig. 5: pdf of the imaginary part of w_3

and (Fig. 5)

$$pdf(R(\omega_3)) = \frac{2}{\omega_3\sqrt{2\pi}}\exp\left[-\frac{\left(\log\left(\omega_3^2/5889^2\right)\right)^2}{2}\right]$$

Case 3: $m \rightarrow U(12,15)$, m is random variable uniformly distributed in the range of Ibrahim (1987) and Papoulis (2002). The application of our method gives:

Step 1:

$$R(\omega_3) = 0.5889\sqrt{\frac{k}{m}}, \mathrm{Im}\,g(\omega_3) = 0.1121\sqrt{\frac{k}{m}}$$

Step 2:

For the real part $m = \dfrac{0.5889^2 k}{(R(\omega_3))^2}$

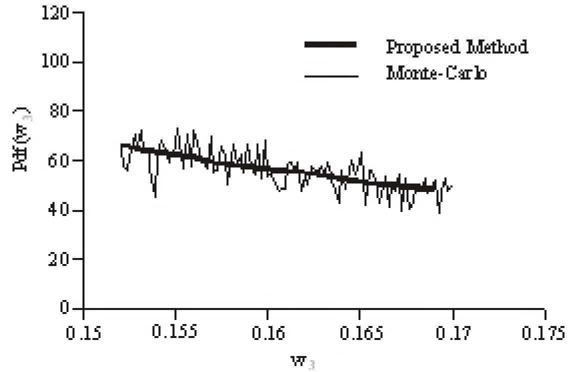


Fig. 6: Pdf of the real part of w_3

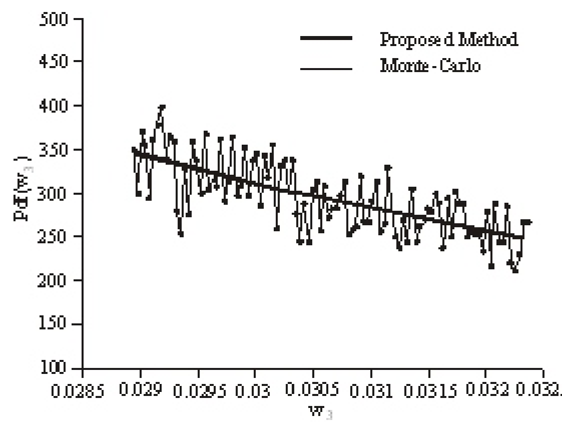


Fig. 7: Pdf of the imaginary part of w_3

For the imaginary part $m = \dfrac{0.1121^2 k}{(R(\omega_3))^2}$

Step 3:

For the real part $|J| = \dfrac{2(0.5889)^2}{R(\omega_3)^3}$

For the imaginary part $|J| = \dfrac{2(0.1121)^2}{R(\omega_3)^3}$

Step 4:

$$pdf(R(\omega_3)) = \frac{2(0.5889)^2}{R(\omega_3)^3}.pdf(m)$$

and

$$pdf(\mathrm{Im}\,g(\omega_3)) = \frac{2(0.1121)^2}{R(\omega_3)^3}.pdf(m)$$

Numerical value: $k=1M/m$ i.e.,

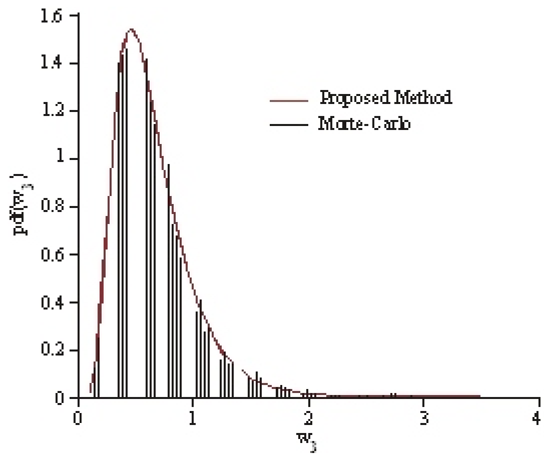


Fig. 8: Pdf of the real part of w_3

$$pdf(R(\omega_3)) = \begin{cases} \dfrac{2(0.5889)^2}{R(\omega_3)^3} & if\ 0.15 < \omega_3 < 0.17 \\ 0 & if\ not \end{cases}$$

as illustrated in Fig. 6, and

$$pdf(\mathrm{Im}\ g(\omega_3)) = \begin{cases} \dfrac{2(0.1121)^2}{R(\omega_3)^3} & if\ 0.028 < \omega_3 < 0.032 \\ 0 & if\ not \end{cases}$$

as shown in Fig. 7.

Case 4: $m \rightarrow \log(0,1)$, m is random variable follows a lognormal distribution log[0, 1]. The application of our method gives (Fig. 8):

$$pdf(R(\omega_3)) = \frac{2}{\omega_3\sqrt{2\pi}} \exp\left[-\frac{\left(\log\left(5889^2/\omega_3^2\right)\right)^2}{2}\right]$$

and (Fig. 9)

$$pdf(R(\omega_3)) = \frac{2}{\omega_3\sqrt{2\pi}} \exp\left[-\frac{\left(\log\left(1121^2/\omega_3^2\right)\right)^2}{2}\right]$$

CONCLUSION

In this study, we analyzed statistically the complex modes arising in multiple degrees of freedom with stochastic parameters. In this context, we developed a new method to calculate analytically the probability density function of the output knowing the probability density function of the input. The efficiency of our method is very closed to the result of the Monte-Carlo simulations.

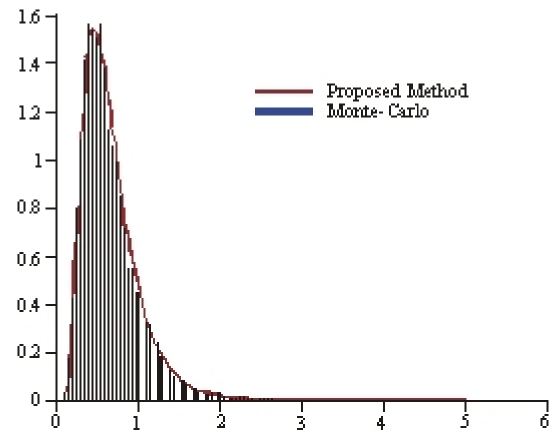


Fig. 9: Pdf of the imaginary part of w_3

REFERENCES

Adhikari, S., 2000. Complex modes in linear stochastic systems. Proceedings of VETOMAC-I, Bangalore, India, Oct., pp: 25-27.

Benaroya, H., 1992. Random eigenvalues, algebraic methods and structural dynamic models. Appl. Math. Comput., 52: 37-66.

Benaroya, H. and M. Rebak, 1988. Finite element methods in probabilistic structural analysis: A selective review. Appl. Mech. Rev., 41: 201-213.

Caughey, T.K. and M.E.J. O'Kelly, 1965. Classical normal modes in damped linear dynamic systems. Trans. ASME, J. Appl. Mech., 32: 583-588.

Collins, J.D. and W.T. Thomson, 1969. The eigenvalue problem for strctural systems with statistical properties. AIAA J., 7(4): 642-648.

Ganesan, R., 1996. Probabilistic analysis of non-self-adjoint mechanical systems with uncertain parameters. Int. J. Solids Struct., 33(5): 675-688.

Ganesan, R., T.S. Sankar and S.A. Ramu, 1993. Non-conservatively loaded stochastic columns. Int. J. Solids Struct., 30(17): 2407-2424.

Ibrahim, R.A., 1987. Structural dynamics with parameter uncertainties. Appl. Mech. Rev., ASME, 40(3): 309-328.

Manohar, C.S. and R.A. Ibrahim, 1999. Progress in structural dynamics with stochastic parameter variations: 1987 to 1998. Appl. Mech. Rev., ASME, 52(5): 177-197.

Nakagiri, S., H. Takabake and S. Tani, 1987. Uncertain eigenvalue analysis of composite laminated plates by the stochastic finite element method. J. Eng. Ind., 109(2): 9-12.

Papoulis, A., 2002. Probability, Random Variables and Stochastic Processes. 4th Edn., McGraw-Hill, Boston, USA.

Sestieri, A. and R. Ibrahim, 1994. Analysis of errors and approximations in the use of modal coordinates. J. Sound Vib., 177(2): 145-157.

Shinozuka, M. and C.J. Astill, 1972. Random eigenvalue problems in structural analysis. AIAA J., 10(4): 456-462.

Vanmarcke, E., M. Shinozuka, S. Nakagiri, G. Schueller and M. Grigoriu, 1986. Random fields and stochastic finite element methods. Struct. Saf., 3: 143-166.