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Analytical Study of Formulation for Electromagnetic Wave Scattering Behavior on a Cylindrically Shaped Dielectric Material

[1]E.I. Ugwu, [1]G.A. Agbo and [2]V.O.C. Eke
[1]Department of Industrial Physics
[2]Department of Computer Science, Ebonyi State University, Abakaliki, Nigeria

Abstract: An analytical formulation for electromagnetic (E.M) wave scattering from a dielectric cylinder is presented during which scattered E.M wave spectrum was associated with a vibrated object on which first order scattered field was developed. The incident wave impinging on the material is considered to cause a boundary deformation as a well as a dielectric inhomogeneity within the dielectric cylinder. A perturbation method is first developed to solve for E.M scattering from the deformed and inhomogeneous dielectric cylinder. From the solution both the surface displacement and the variation in dielectric constant fluctuation within the cylinder is found to contribute to the solution of the EM scattered field and that the shape variation of the dielectric fluctuations were found to contribute to the solution of both T.M and first order scattered wave.

Key words: Electromagnetic wave, dielectric material, perturbation, polarization, scattering

INTRODUCTION

The idea of EM scattering and its study has been evolved for long both classical and quantum mechanically (Morse and Feshach, 1953) due to its applications. For instance strong penetrating wave such as radar have been proposed for the detection and identification of buried object (Sarabandi and Lawrence, 1999). The acoustically induced Doppler spectra scattered has been used as a means for buried object detection. The idea of E.M scattering combined with acoustic excitation for detecting buried objects has been treated previously (Stewart, 1960; Smith, 1992; Kinsler *et al.*, 1982). Scatt and Martin (1998) reported experimental results, where E.M radar is used to measure surface displacement caused by a traveling surface acoustic wave where the approach was based on small changes in surface displacement when a buried object is introduced. In this technique both acoustic and E.M receivers are used to detect ground vibrations by incorporating what would facilitate isolating the characteristics of the object of interest such as the normal modes of free vibration.

In this paper, we associate the scattered E.M spectrum with an acoustically vibrated object and developed the first order-scattered fields that might propose a means of detections and identification scheme as shown in Fig. 1. Also to demonstrate the phenomena associated with the acoustic electromagnetic scattering behaviour of dielectric objects in two dimensions.

A vibrating dielectric cylinder exhibits two sources of scattering. The cylinder shape variation and the dielectric constant fluctuation within the cylinder are considered (Lawrence and Sarabandi, 2001).

As the incident wave vibrates the cylinder and its density profile vary with time thus making it possible to obtain the EM scattering solution for an arbitrary deformed, inhomogeneous dielectric cylinder either analytically or numerically (Wood, 1995). Consequently, analytically perturbation techniques were considered here to calculate the scattered field since the technique can provide analytical solutions to the defined problems when the surface irregularity and dielectric inhomogeneity are small and exact solutions exist for the unperturbed problems. Perturbations theory is an established analytical approach for scattering solutions and has been applied by many authors (Rayleigh, 1945; Maxwell, 1954) to certain scalar field problems. In this research, the scattering contribution from the boundary deformation is obtained through perturbation expansion of the exact eigenfunction solution for a homogeneous dielectric cylinder. Time varying perturbation co-efficient for an arbitrary deformed cylinder are needed and is derived using this similar technique (Bruschini and Gros, 1997). The contribution from the dielectric fluctuations is formulated and solved by expanding the interior fields in terms of a perturbation parameter related to dielectric inhomageneity and then using the induced volumetric current to calculate the scattered field.

MATERIALS AND METHODS

Analytical development: In the first case we consider shape perturbation following the methodology employed by Yeh (1963) in analytical solution of the EM scattering from a slightly deformed dielectric circular cylinder. Here, he derived and analyzed the solution using a perturbation method.

Corresponding Author: E.I. Ugwu, Department of Industrial Physics, Ebonyi State University, Abakaliki, Nigeria

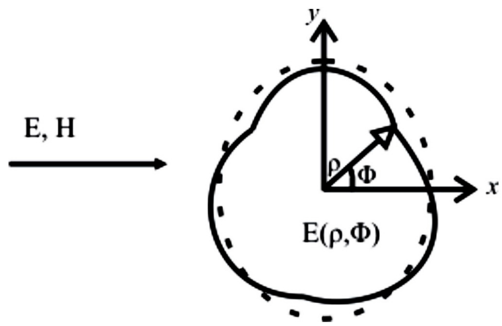

Fig. 1: The Geometry of Em scattering of a plane wave from a dielectric cylinder with perturbed shape and dielectric constant profile (Lawrence and Saraband, 2001)

If a plane wave is incident upon a perturbed dielectric cylinder as shown in fig1, the radius of the cylinder can be expressed in polar coordinates as

$$\rho = a + bf(\phi,t) \tag{1}$$

where a is unperturbed radius *f* (Ø, t) is periodic and smooth function of Ø; b is perturbation parameter assumed to be much smaller than the wavelength and the radius a. The standard method for calculating the scattered field from a dielectric cylinder is by way of eigenfunction expansion of the total field. From Harrington's work (Harrington and Hayt, 1961; Naqvi and Rizvi, 1998), incident E-M plane wave propagating along Z-direction with a plane polarization parallel to the cylinder's axis expressed as

$$E^{i} = e^{-jk_0\rho\cos\phi} = \sum_{n=\infty}^{\infty} j^{-n} J_n(K_0\rho)e^{jn\phi} \tag{2}$$

was considered. Where K_o is the wave number in free space. When there is no perturbation on the surface (i.e., b = 0), the scattered field external to the cylinder is given by

$$E_x^s = \sum_{n=-\infty}^{\infty} j^{-n} A_n^{TM} H_n^{(2)}(K_0\rho)e^{jn\phi} \tag{3}$$

with

$$A_n^{TM} = \frac{\frac{no}{nd}J_n(K_o a)J_n^{\prime}(K_d a) - J_n^{\prime}(K_o a)J_n(K_d a)}{J_n(K_d a)H_n^{(2)\prime}(K_o a) - \frac{n_o}{n_d}J_n^{\prime}(K_d a)H_n^{(2)}(K_o a)} \tag{4}$$

Where n_o and n_d are the wave impedances in free space and within the dielectric, respectively, and k_d is the wave number in the dielectric the scattered field internal to the cylinder is:

$$E_x^{sd} = \sum_{n=-\infty}^{\infty} j^{-n} B_n^{TM} J_n(K_d\rho)e^{jn\theta} \tag{5}$$

where,

$$B_n^{TM} = \frac{-2j/\pi k_o a}{J_n(K_d a)H_n^{(2)\prime}(K_o a) - \frac{n_o}{n_d}J_n^{\prime}(K_d\rho)e^{jn\theta}} \tag{6}$$

In a situation where a small perturbation is introduced on the boundary, the scattered field can be expanded in a perturbation series in $K_o b$ and $K_d b$, respectively. Considering the first order in $K_o b$, we may write the expression of the external field as:

$$E_X^S = \sum_{n=-\infty}^{\infty} jnA_n^{TM}(1 + C_n^{TM}K_{0b})H_n^{(2)}(K_0\rho)e^{Jn\phi} \tag{7}$$

where the internal field to the first order in $K_d b$ is:

$$E_x^{sd} = \sum_{n=-\infty}^{\infty} j^{-n} B_n^{TM}(I + D_n^{TM}K_d b)J_n(K_d b)e^{jn\phi} \tag{8}$$

C_n^{TM} and D_n^{TM} are unknown co-efficients to be determined from the boundary conditions at the surface of the perturbed cylinder. The boundary conditions explicitly made it possible for the tangential electric and magnetic fields to be continuous at the surface of the cylinder. In this case, the tangential magnetic field on the surface of the perturbed cylinder for the incident, externally scattered, and internally scattered field are, respectively written as:

$$H_I^i \frac{j}{n_o} = \sum_{n=-\infty}^{\infty} \alpha j^{-n} \left[\frac{Bjn}{k_o\rho^1} J_n(K_o\rho') - J_n'(K_o\rho j)\right]e^{jn\phi} \tag{9}$$

$$H_1^s = \frac{j}{n_d}\sum_{n=-\infty}^{\infty} \alpha j^{-n} A_n^{TM}(I + C_n^{TM}K_o b)\left[\frac{Bjn}{K_d\rho} H_n^{(2)}(K_o\rho - H_n^{(2)}(K_o\rho)\right]e^{jn\phi} \tag{10}$$

$$H_1^{sd} = \frac{j}{\eta_d} \sum_{n=-\infty}^{\infty} \alpha j^{-n} B_n^{TM} (I + D_n^{TM} (Kdb) \left[\frac{Bjn}{K_d \rho} J_n (Kd\rho) J_n (Ka\rho) \right] e^{jn\phi} \tag{11}$$

The eigenfunctions in the above expressions can be approximated to the first order in $K_o b$ and $K_d b$ using Taylor series expansion. Keeping terms up to the first order in K_b as considered, generate the two equations for the perturbation co-efficient C_n^{TM} and D_n^{TM} by applying the boundary conditions at the surface of the perturbed cylinder. These equations are solved by expanding $f(Ø^1,t)$ in terms of a fourier series with respect to $Ø^1$ with the Pth fourier series co-efficient denoted by f_P (t).

$$f(\phi t) = \sum_{\rho=-\infty}^{\infty} f\rho(t) e^{j\rho\phi'} \tag{12}$$

Where explicit form for C^{TM}_n is found to be:

$$C_n^{TM}(t) = \frac{j^n J_n(K_d a) \sum_{\rho=-\infty}^{\infty} fn-\rho(t) V\rho}{\eta_0 / \eta_d J_n(k_o a) J_n'(k_d a) - J_n'(k_o a) J_n(k_d a)} \tag{13}$$

$$V\rho = j^{-\rho} \begin{bmatrix} -J_\rho^H(k_o a) - A_\rho^{TM} H_\rho^{(2)''}(K_o a) \\ + \frac{K_d^2}{K_o^2} B_\rho^{TM} J_\rho''(K_d a) \end{bmatrix} \tag{14}$$

The expansion in Eq. (13) provides the Transverse Mode (TM) solution for the scattered field from a homogeneous dielectric cylinder with a perturbed cross-section.

RESULTS AND DISCUSSION

Dielectric Perturbation: A vibrating cylinder that is initially homogeneous will experience density fluctuations as the cylinder vibrates. As a result, the dielectric constant will fluctuate within the cylinder requiring a solution for the EM scattering from a slightly inhomogeneous dielectric cylinder. Since the dielectric fluctuations are small, we will once again develop a perturbation technique to solve for the scattered fields. The technique presented here is similar to that used for scattering from a low contrast dielectric cylinder shape as in Naqvi and Rizvi (1998) work. The dielectric constant of the cylinder can be written as:

$$\Sigma(\rho', \phi, t) = \Sigma d + \delta \overline{\Sigma}(\rho', \phi, t) \tag{15}$$

Where δ is a small-assumed perturbation parameter. (i.e., δ <<1).

In accordance to Gauss' Law in a source free region, we have that:

$$\nabla . [\Sigma(\rho, \phi', t) E] = \Sigma(\rho_I, \phi', t) \nabla . \Sigma + \nabla \varepsilon(\rho, \phi', t). E = 0 \tag{16}$$

which indicates that:

$$\nabla . E = \frac{1}{\varepsilon(\rho_I \phi', t)} \nabla \varepsilon(\rho \phi', t). E \tag{17}$$

Since $\epsilon(\rho, Ø, t)$ does not contain a x- component and the electric field has a x- component for TM case, we conclude that:

$$\nabla . E = 0 \tag{18}$$

From Maxwell's equation, wave equation can be written as:

$$\nabla^2 E_x + \omega^2 \mu \varepsilon_0 E_x = 0 \text{ External to cylinder} \tag{19}$$

$$\nabla^2 E_x + \omega^2 \mu \bar{\varepsilon}(\rho', \phi, t) E_x = 0 \text{ Internal to cylinder} \tag{20}$$

Substituting the perturbation expression from Eq. (15) into (20) gives:

$$\nabla^2 E_x + k_d^2 E_x = -\omega_m^2 \delta \bar{\varepsilon} (\rho, \phi, t) E_x \tag{21}$$

where. $K_d^2 = \omega^2 \mu \varepsilon_d$

Since δ is small, E_x can be expanded in a convergent perturbation series in δ, and is given by:

$$E_x = \sum_{n=0}^{\infty} E_x^{(n)} \delta^n \tag{22}$$

Where $E_x^{(n)}$ is the scattered field from the unperturbed cylinder? Substituting Eq. (22) into (21) and collecting equal power of δ, it can easily be shown that:

$$\nabla^2 E_x^{(n)} + k_d^2 E_x^{(n)} = -\omega_m^2 \delta \bar{\varepsilon} (\rho, \phi, t) E_x^{(n-1)} \tag{23}$$

Thus the (n-1)th order solution acts the source function for the nth order solution. Now the first-order scattered field external to the cylinder can be obtained by:

$$E_x^{(1)} = -w_m^2 \int_0^{2\pi}\int_0^{a} \bar{\varepsilon}(\rho',\phi,t) E_x^{(0)} G(\rho,\rho',\phi,\phi') \rho' d\rho' d\phi' \quad (24)$$

where, $G(\rho,\rho',\phi,\phi')$ is the Green's function for the source internal to the cylinder and the observation external cylinder. The Green's expression as shown in Eq. (24) can be solved to obtain:

$$G(\rho,\rho',\phi,\phi') = \sum_{M=-\infty}^{\infty} X_m J_m(K_d\rho') H_m^{(2)}(K_o\rho) e^{im(\phi-\phi')} \quad (25a)$$

$$X_M = \frac{1}{2\pi K_o a}\left[\frac{1}{J_M(k_d a) H_m^{(2)}(k_o a) - \frac{\eta_o}{\eta_d} J'_M(k_d a) H_m^{(2)}(k_o a)}\right]$$

Substituting Eq. (25) and (5) into (24) and simplifying gives:

$$E_x^{(1)} = 2\pi\omega^2\mu \sum_{M=-\infty}^{\infty} X_m H_m^{(2)}(K_o\rho) e^{im\phi} \sum_{m=-\infty}^{\infty} j^{-n} B^{TM} \int_0^a \bar{\varepsilon}_{n-m}(\rho',t) Jn(K_d\rho') J_m(K_d\rho')\rho' \quad (25b)$$

where, $\bar{\varepsilon}_{n-m}(\rho',t)$ is $(n-m)^{th}$ the Fourier series co-efficient of $\bar{\varepsilon}(\rho',t)$ when expanded in a Fourier series in Φ^1. This expression provides the TM solution for the first order-scattered fields from an inhomogeneous dielectric cylinder.

For TE, the EM scattering from a slightly deformed dielectric cylinder with a TE incident plane is derived using the perturbation method just like in the first case shape and dielectric perturbations are considered, respectively.

In the case of shape perturbation, the incident magnetic field can be expressed as:

$$H_x^i = e^{-jk_o\rho\cos\phi} = \sum_{n=-\infty}^{\infty} j^{-n} J_n(k_o\rho) e^{jn\phi} \quad (26)$$

the unperturbed scattered fields are given in Eq. (2) and (5) with E_x replaced by H_x and the co-efficient and are given in (5) and (7) with an interchange of η_o and η_d. When a small perturbation is introduced on the boundary, the perturbed fields can be expanded in a perturbation series in $k_0 b$ and $k_d b$, respectively.
The external field is by:

$$H_x^s = \sum_{n=-\infty}^{\infty} j^{-n} A_n^{TM}(1 + C_m^{TE} k_o b) H_n^{(2)}(k_o\rho) e^{jn\phi} \quad (27)$$

and the internal field:

$$H_x^{sd} = \sum_{n=-\infty}^{\infty} j^{-n} B_n^{TM}(1 + D_m^{TE} k_d b) J_n(k_d\rho) e^{jn\phi} \quad (28)$$

These unknown co-efficient C_n^{TE} and B_n^{TM} can be determined from the boundary conditions applied at the surface of the perturbed cylinder. The boundary conditions specify that the total tangential electric and magnetic fields be continuous at the cylinder boundary. The tangential electric fields on the surface of the perturbed cylinder for the incident, external scattered and internal scattered fields are given, respectively by:

$$H_\perp^i = \frac{j}{\eta_o} \sum_{n=-\infty}^{\infty} \alpha j^{-n} \left[\frac{Bjn}{K_o\rho'} J_n(k_o\rho') - J'_n(k_o\rho')\right] e^{jn\phi} \quad (29)$$

$$E_\perp^s = \frac{j}{\eta_o} \sum_{n=-\infty}^{\infty} \alpha j^{-n} A^{TE}(I + C_n^{TE} k_o b) \left[\frac{Bjn}{K_o\rho'} H_n^{(2)} k_o\rho' - H_n^{(2)}(k_o\rho')\right] e^{jn\phi} \quad 30)$$

$$E_\perp^{sd} = \frac{j}{\eta_o} \sum_{n=-\infty}^{\infty} \alpha j^{-n} B_n^{TE}(I + D_n^{TE} k_d b) \left[\frac{Bjn}{K_o\rho'} J_n(k_d\rho')\right] e^{jn\phi} \quad (31)$$

Using the Taylor series expansion for the Bessel and Hankel functions and applying the boundary conditions results in two equations for the unknown co-efficient C_n^{TE} and D_n^{TE}.

Expanding f(Ø!, t) in a fourier series as in Eq. (12), neglecting term containing $(Kb)^2$ and higher, and solving for C_n^{TE} boundary perturbation:

$$E_{\perp}^{sd} = \frac{j}{\eta_o} \sum_{n=-\infty}^{\infty} \alpha j^{-n} B_n^{TE} \left(I + D_n^{TE} k_d b\right) \left[\frac{Bjn}{K_o \rho} J_n(k_d \rho)\right] e^{jn\phi} \qquad (32)$$

where,

$$V_{\rho n} - \rho = j^{-\rho}\left[\frac{-\rho(n-\rho)}{(k_o a)^2} B_\rho^{TE} J\rho(k_d a)\left(I - \frac{\varepsilon_o}{\varepsilon_d}\right) - J_\varepsilon''(k_o a) - A_\rho^{TE} H_\rho^{(2)''}(k_o a) J_\rho''(k_d a)\right] \qquad (33)$$

and

$$U_\rho = j^{-\rho}\left[\frac{kd}{k_o} B_\rho^{TE} J'_\rho(k_d a) - A_\rho^{TE} H_\rho^{(2)'}(k_o a) - J'_\rho(k_o a)\right] \qquad (34)$$

In this case the dielectric fluctuation induced by vibration of the cylinder will contribute to the scattered field. The dielectric constant internal to the cylinder is expressed by Eq. (15). At this time, the cure of Ampere's law, extended to the time-varying case, in a source free region is given by:

$$\nabla_x \nabla_x H = j\omega \nabla_x \left[\varepsilon(\rho, \phi, t)\right] \qquad (35)$$

$$= j\omega \nabla \varepsilon(\rho, \phi, t) x E + j\omega \varepsilon(\rho, \phi, t) \nabla x E \qquad (36)$$

As it can easily be shown that from Maxwell's equations that the wave equation for H_x is:

$$\nabla^2 H_x + \omega^2 \mu \varepsilon(\rho, \phi, t) H_x = \left[-j\omega\rho\delta\bar{\varepsilon}(\rho, \phi, t) x E_x\right] \qquad (37)$$

$$\nabla^2 H_x + \omega^2 \mu \varepsilon(\rho, \phi, t) H_x = \left[-j\omega\rho\delta\bar{\varepsilon}(\rho, \phi; t) x E_x\right] \qquad (38)$$

Substituting Eq. (15) into (38), we obtain:

$$\nabla^2 H_x + K_d^2 H_x = \left[-j\omega\nabla\,\delta\bar{\varepsilon}(\rho; \phi, t) x E\right] - \omega_m^2 \delta\bar{\varepsilon}(\rho, \phi, t) H_x \qquad (39)$$

When δ is small, both H_x and E will be expended in a perturbation series in δ.

$$H_x = \sum_{n=0}^{\infty} H_x^{(n)} \delta^n \quad \text{and} \quad E = \sum_{n=0}^{\infty} E^n \delta^n \qquad (40)$$

Subsisting Eq. (40) into (39) and equating equal powers of δ; it can easily be shown that:

$$\nabla^2 H_x^{(n)} + K_d^2 H_x^{(n)} = \left[-j\omega\nabla\,\delta\bar{\varepsilon}(\rho, \phi; t) x E^{(n-1)}\right]_x - \omega^2 \mu \bar{\varepsilon}(\rho, \phi, t) H^{(n-1)} \qquad (41)$$

where (n-1)th order solution generate the source function for the nth solution as in the first case. The first order scattered field external to the cylinder can be obtained by the same solution as in Eq. (24) which will finally give:

$$H_x^{(1)} = 2\pi \sum_{n=-\infty}^{\infty} Y_m H_m^{(2)}(k_o \rho) e^{jm\phi} \sum_{n=-\infty}^{\infty} j^{-n} B_n^{TE} I_{m,n} \qquad (42)$$

where,

$$Y_m = \frac{1}{2\pi k_o a}\left[\frac{1}{J_m(k_d a) H_m^{(2)}(k_o a) - \frac{\eta_d}{\eta_o} J'_m(k_d a) H_m^{(2)}(k_o a)}\right]$$

and

$$I_{m,n} = \int_{\rho=1}^{a}\left\{\left[\frac{n(m-\rho)}{\rho^{12}} + K_d^2\right] J_n(k_a \rho)\,\bar{\varepsilon}_{m-n}(\rho, t) - K_d J'_n(k_a \rho) \frac{\delta}{\partial \rho} \bar{\varepsilon}_{m-n}(\rho; t)\right\} \frac{J_m(k_{d'} \rho')}{\varepsilon_d} \rho\, dp'$$

and $\bar{\varepsilon}(\rho, \phi, t)$ is the (m-n)th Fourier series co-efficient when $\bar{\varepsilon}_{m-n}(\rho, \phi, t)$ is expanded in a Fourier series in ϕ'.

Dielectric of perturbed Inhomogeneous material: The permittivity of a dielectric material under vibration of applied field becomes Inhomogeneous as the bulk density of the material becomes a function of position. The dielectric of a material is defined as (Harrington and Hayt, 1961).

$$\varepsilon = \frac{D}{E} = \frac{\varepsilon_o E + \rho}{E} \tag{43}$$

where P is the dipole moment per unit volume. P is related to the electric susceptibility (X) by:

$$\rho = \varepsilon_o \chi \varepsilon \text{ and } \varepsilon = \varepsilon_o(\chi + I) \tag{44}$$

The dipole moment per unit volume is related to the number of molecules per unit volume (N) multiplied by the dipole moment of each molecule (d):

$$P = Nd \tag{45}$$

Supposing the mass of each molecule is in so that the material bulk density ρ is given by:

$$P = N\mu \tag{46}$$

Now if we S(r) are the field impedance parameter that is related to the bulk density and the displacement vector U as given by (Kingler *et al.* 1982):

$$\text{Then } S(r) = \frac{P(r) - \rho_o}{e_o} = -\nabla . U(r) \tag{47}$$

Where ρ_o is the unperturbed bulk density of the material. Substituting Eq. (46) into (47), we obtain:

$$S(r) = \frac{N(r)m - N_o m}{\rho_o} = -\nabla . U(r) \tag{48}$$

$$S(r) = \frac{P(r) - \rho o}{\rho_o} = \frac{\varepsilon_o \chi(r) E - \varepsilon_o \chi_o E}{\varepsilon_o \chi_o E} \tag{49}$$

$$= \frac{\varepsilon_o(\chi(r) + I) - \varepsilon_o(\chi_o + I)}{\varepsilon_o(\chi_o + I) - \varepsilon_o} \tag{50}$$

$$= \frac{\varepsilon_o(r) - \varepsilon_d}{\varepsilon_d - \varepsilon_o} \tag{51}$$

where e_d is the dielectric constant for the unperturbed material. Hence, the fluctuating part of the dielectric constant is give by:

$$\delta \bar{\varepsilon}(r) = -(\varepsilon_d - \varepsilon_o)\nabla : U(r) \tag{52}$$

Analysis of result: From Eq. (3) a term ATM shows clearly that wave impedance contributes in the scattered wave for both the external and internal to the cylinder. With introduction of a small perturbation on the boundary, the scattered field for external and internal field made it possible to expand the Eq. (3) and (5) imperturbation specified in terms of $k_o b$ and $k_d b$. In the analysis, emphasis was placed more on the tangential electric and magnetic field to the cylinder due to the imposed boundary conditions for easy solution and analysis of the scatter wave impinging on the cylinder.

The solution to $\int(\phi^1, t)$ was obtained in terms of fourier series and from this solution TM scattered field from homogeneous dielectric cylinder with a perturbed cross-section were obtained in the problems. A perturbation technique was applied in solving for the scattered filed because of the density fluctuations resulting from the vibration of cylinder.

It was from this consideration that enable us to obtain the TM solution for the first order scattered fields from the inhomogeneous or perturbed dielectric cylinder as in Eq. (25) considering the dielectric constant to be a microscopic material parameter directly related to the dipole moment per unit volume, the relationship between the displacement interior to the cylinder and dielectric constant fluctuation as shown in Eq. (52) is thus:

$$\sigma(\rho, \phi, t) = -(\varepsilon_d - \varepsilon_o)\nabla u(\rho, \phi, t) \tag{53}$$

Where the divergence of the displacement vector is

$$\nabla U(\rho, \phi, t) = k_\perp^2 \sum_{n=0}^{\infty} a_n J_n(K_1 P^1) \cos n\phi \tag{54}$$

Finally, insertion of Eq. (54) into (53) gives the series co-efficient for the exponential fourier representation of $\bar{\varepsilon}(\rho, \phi, t)$ in ϕ' as:

$$\bar{\varepsilon}_n(\rho, t) = \frac{\varepsilon_n}{2\sigma}(\varepsilon_d - \varepsilon_o) k_\perp^2 \gamma_n(\rho) \cos \omega_o t + \gamma_n(\rho) \tag{55}$$

From the solution it is noted that both the shape variation and dielectric constant fluctuation of the cylinder were accounted for by the expansion of unperturbed scattered field in a perturbation expansion in terms of both b ands. The expansion in b and δ are treated independently since the included terms of bs are of second order and higher. Therefore, the total scattered fields for both shapes were obtained by adding coherently the results from both perturbation expansions described in this work. Where the time-varying scattered field that provided the desired scattered spectrum was based on taking the fourier transform.

CONCLUSION

This study has analytically considered the formulation of the electromagnetic wave scattering behaviour of a dielectric cylinder. Both the shape variation dielectric fluctuations have been found to contribute to the solution of both T.M and first order-scattered wave.

Also the spectrum of the scattered field is a strong function of the scattering angle Φ^1. Thus the phenomenological observation here can be applied in the detection and identification of buried objects especially when the frequency an E M line is within the range of acoustic

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