

Effects of Radiation, Heat Generation and Viscous Dissipation on MHD Free Convection Flow along a Stretching Sheet

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Abstract: The radiation and viscous dissipation effects on a steady two-dimensional magneto-hydrodynamics free convection flow along a stretching sheet with heat generation is analyzed. The non-linear partial differential equations, governing the flow field under consideration, have been transformed by a similarity transformation into a system of non-linear ordinary differential equations and then solved numerically by applying Nachtsheim-Swigert shooting iteration technique together with sixth order Runge-Kutta integration scheme. Resulting non-dimensional velocity, temperature and concentration profiles are then presented graphically for different values of the parameters of physical and engineering interest.

Key words: Boundary layer, Eckert number, heat transfer, magnetic field parameter, Prandtl number, rosseland approximation

INTRODUCTION

Boundary layer flow and heat transfer over a linearly stretched surface has received considerable attention in recent years. This is because of the various possible engineering and metallurgical applications such as hot rolling, wire drawing, metal and plastic extrusion, continuous casting, glass fiber production, crystal growing and paper production. Sakiadis (1961) was the first to study boundary layer flow over a stretched surface moving with a constant velocity. Erickson *et al.* (1966) extended the work of Sakiadis and later Chen and Char (1988), Elbashbeshy (1997) investigated the effects of variable surface temperature and heat flux on the heat transfer characteristics of a linearly stretched sheet subject to blowing or suction. The magneto-hydrodynamics (MHD) of an electrically conducting fluid is encountered in many problems in geophysics, astrophysics, engineering applications and other industrial areas. MHD free convection flow has a great significance for the applications in the fields of stellar and planetary magnetospheres, aeronautics. Kumar *et al.* (2002) studied MHD flow and heat transfer on a continuously moving vertical plate. The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and these concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips and semi conductor wafers. Vajravelu and Hadjinalaou (1993) studied the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or

absorption. Recently, Samad and Mohebujjaman (2009) investigated the case along a vertical stretching sheet in presence of magnetic field and heat generation. At high operating temperature, the effect of radiation on MHD flow can be quite significant. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Takhar *et al.* (1996) studied the radiation effects on MHD free-convection flow of a gas past a semi-infinite vertical plate. Ghaly (2002) considered radiation effect on a steady flow, whereas Raptis and Massalas (1998) and El-Aziz (2009) analyzed the unsteady cases. Shateyi *et al.* (2007) studied magneto-hydrodynamic flow past a vertical plate with radiative heat transfer. Recently, Mahmoud (2007) investigated variable viscosity effects on MHD flow in presence of radiation.

The problem of natural convection along a vertical isothermal or uniform flux plate is a classical problem. However, Gebhart (1962) was the first who studied the problem taking into account the viscous dissipation. Recently, Copiello and Fabbri (2008) studied the effect of viscous dissipation on the heat transfer in sinusoidal profile finned dissipators, Alam *et al.* (2007) considered the effect of viscous dissipation in natural convection over a sphere. Pantokratoras (2005) studied the effect of viscous dissipation in natural convection in a new way. However, in this work we have investigated the combined effect of thermal radiation, heat generation and viscous dissipation on steady free convection heat and mass transfer flow over a stretching sheet in the presence of magnetic field.

MATHEMATICAL FORMULATION

A steady two dimensional MHD free convection laminar boundary layer flow of a viscous incompressible and electrically conducting fluid along a vertical stretching sheet with heat generation under the influence of thermal radiation and viscous dissipation is considered (Fig. 1). This work is an extension of Samad and Mohebujjaman (2009) where they considered the MHD heat and mass transfer free convection flow with heat generation. Introducing the Cartesian coordinate system, the x -axis is taken along the stretching sheet in the vertically upward direction and the y -axis is taken as normal to the sheet. Two equal and opposite forces are introduced along the x -axis, so that the sheet is stretched keeping the origin fixed. The plate is maintained at a constant temperature T_w and the concentration is maintained at a constant value C_w . The ambient temperature of the flow is T_∞ and the concentration of uniform flow is C_∞ . The fluid is considered to be gray, absorbing-emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radioactive heat flux in the energy equation. The radioactive heat flux in the x -direction is considered negligible in comparison to the y -direction. The concentration is assumed to be non-reactive.

A strong magnetic field is applied in the y -direction. Here, we can neglect the effect of the induced magnetic field in comparison to the applied magnetic field. The electrical current flowing in the fluid gives rise to an induced magnetic field if the fluid were an electrical insulator, but here we have taken the fluid to be electrically conducting. Hence, only the applied magnetic field B_0 plays a role which gives rise to magnetic forces

$$F_x = \frac{\sigma B_0^2 u}{\rho}$$

in x -direction, where σ is the electrical conductivity, B_0 is the uniform magnetic field strength (magnetic induction) and ρ is the density of the fluid. We also bring into account the effect of temperature dependent volumetric heat generation (W/m^3), in the flow region that is given by Vajravelu and Hadjinicolaou (1993) as,

$$q''' = Q_0(T - T_\infty), \quad T \geq T_\infty \quad (1)$$

where Q_0 is the heat generation constant.

Governing equations: Under the above assumptions, the governing boundary layer equations are:

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

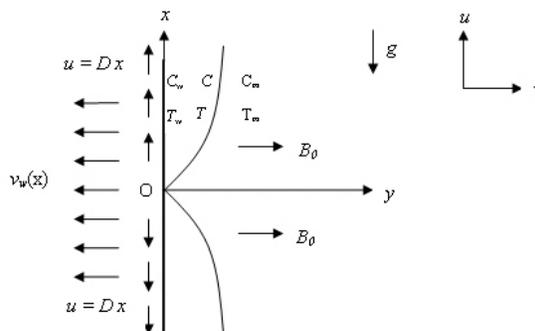


Fig.1: Sketch of the physical model and coordinate system

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2 u}{\rho} \quad (3)$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (4)$$

Concentration equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} \quad (5)$$

The appropriate boundary conditions are:

$$\left. \begin{aligned} u = D_x, v = v_w, T = T_w, C = C_w, \text{ at } y = 0 \\ u = 0, T = T_\infty, C = C_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (6)$$

where u and v are the velocity components in the x and y directions respectively, ν is the kinematic viscosity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, T is the fluid temperature in the boundary layer, C is the concentration of the fluid within the boundary layer, σ is the electric conductivity, B_0 is the uniform magnetic field strength, ρ is the density of the fluid, k is the thermal conductivity of the fluid, c_p is the specific heat at constant pressure, Q_0 is the heat generation constant, q_r is the radiative heat flux, D_m is the coefficient of mass diffusivity, D (>0) is the stretching constant and v_w is the velocity component at the

wall having positive value to indicate suction.

The radiative heat flux q_r is described by the Rosseland approximation such that:

$$q_r = \frac{4\sigma_1}{3k_1} \frac{\partial T^4}{\partial y} \tag{7}$$

where σ_1 is the Stefan-Boltzman constant and k_1 is the Rosseland mean absorption coefficient. Following Chamkha (1997), it is assumed that the temperature differences within the flow are sufficiently small such that T^4 can be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about the free stream temperature T_∞ and neglecting higher-order terms. This results in the following approximation:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

Using (7) and (8) in the last term of Eq. (4), we obtain:

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_1 T_\infty^3}{3k_1} \frac{\partial^2 T}{\partial y^2} \tag{9}$$

Introducing in Eq. (4), we obtain the following energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1} \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \tag{10}$$

Non-dimensionalisation: In order to obtain a similarity solution of the problem, we introduce a similarity parameter δ such that δ is a length scale. We now introduce the following dimensionless variables:

$$\left. \begin{aligned} \eta = \frac{y}{\delta} = y \sqrt{\frac{D}{\nu}} \quad \psi(x, y) = \sqrt{D\nu x} f(\eta) \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \right\} \tag{11}$$

where, ψ is the stream function, η is the dimensionless distance normal to the sheet, f is the dimensionless stream function, θ is the dimensionless fluid temperature and ϕ is the dimensionless concentration. Now:

$$\left. \begin{aligned} u = \frac{\partial \psi}{\partial y} = D_x f'(\eta) \\ v = \frac{\partial \psi}{\partial x} = -\sqrt{D\nu} f(\eta) \end{aligned} \right\} \tag{12}$$

Using the transformations from Eq. (11) and (12) in Eq. (3), (5) and (10), we get the following dimensionless equations:

$$f''' + ff'' - (f')^2 - Mf' + \lambda\theta = 0 \tag{13}$$

$$\theta'' + \frac{3N}{3N+4} \text{Pr} f\theta + \frac{3N}{3N+4} \text{Pr} Q\theta + \frac{3N}{3N+4} \text{Pr} \text{Ec} (f'')^2 = 0 \tag{14}$$

$$\phi'' + \text{Sc} f\phi' = 0 \tag{15}$$

where;

$$\lambda = \frac{Gr}{\text{Re}^2} = \frac{\beta g (T_w - T_\infty)}{D^2 x}$$

is the buoyancy parameter

$$M = \frac{\sigma B_0^2}{\rho D}$$

is the magnetic field parameter

$$\text{Pr} = \frac{\mu c_p}{k}$$

is the Prandtl number

$$N = \frac{kk_1}{4\sigma_1 T_\infty^3}$$

is the radiation parameter

$$Q = \frac{Q_0}{\rho c_p D}$$

is the heat source parameter

$$\text{Sc} = \frac{\nu}{D_m}$$

is the Schmidt number

$$\text{Ec} = \frac{D^2 x^2}{c_p (T_w - T_\infty)}$$

is the Eckert number

The transformed boundary conditions are:

$$\left. \begin{aligned} f' = 1, f = F_w, \theta = 1, \phi = 1 \text{ at } \eta = 0 \\ f' = 0, \theta = 0, \phi = 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \tag{16}$$

where, $F_w = -\frac{\nu_w}{\sqrt{D\nu}}$ is the suction parameter.

Numerical computation: The nonlinear differential Eq. (13)-(15) under the boundary conditions (16) have been

solved numerically by applying a standard initial value solver shooting method called Nachtsheim-Swigert (1965) iteration technique together with the sixth order Runge-Kutta-Butcher iteration scheme. We have chosen a step-size $\Delta \eta = 0.001$ to satisfy the convergence criterion of 10^{-6} in all cases. The value of η_{∞} was found to each iteration loop by $\eta_{\infty} = \eta_{\infty} + \Delta \eta$. The maximum value of η_{∞} to each group of parameters $\lambda, F_w, Pr, M, Sc, Q, N$ and Ec is determined when the value of the unknown boundary conditions at $\eta = 0$ change to successful loop with error less than 10^{-6} .

RESULTS AND DISCUSSION

In order to discuss the problem under consideration, the results of numerical calculations are presented in the form of non-dimensional velocity, temperature and concentration profiles. Numerical computations have been carried out for different values of buoyancy parameter λ ,

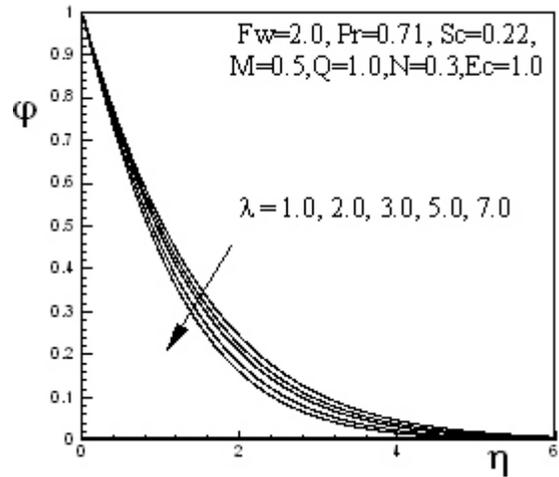


Fig. 2c: Buoyancy parameter (λ) effect on concentration profiles

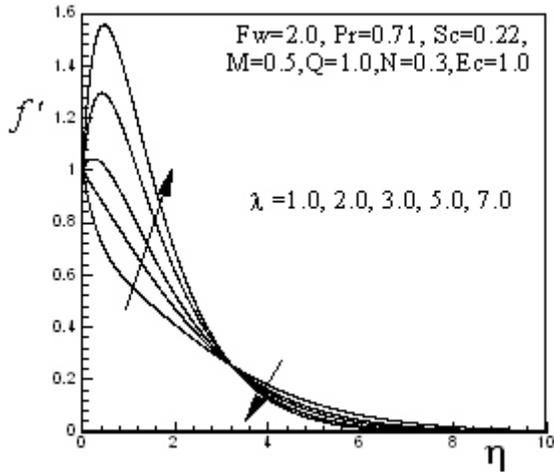


Fig. 2a: Buoyancy parameter (λ) effect on velocity profiles

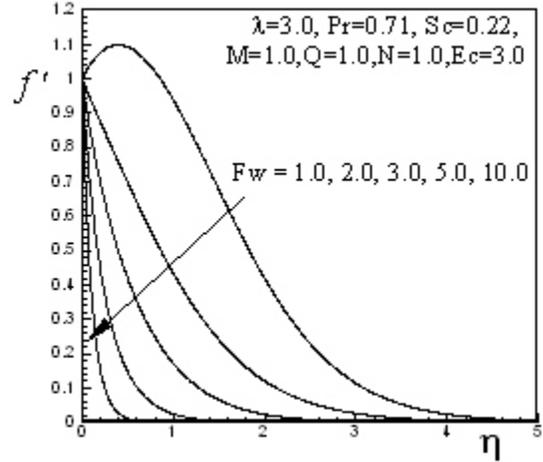


Fig. 3a: Suction parameter (F_w) effect on velocity profiles

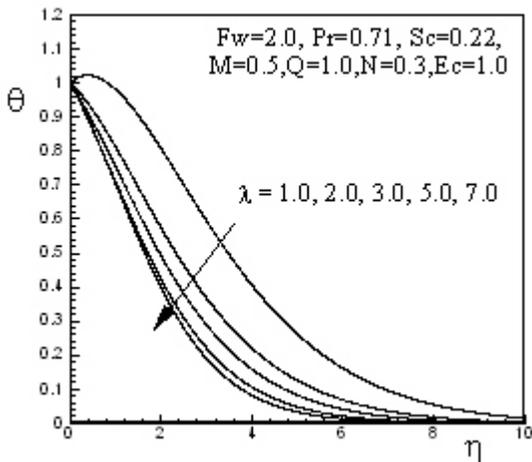


Fig. 2b: Buoyancy parameter (λ) effect on temperature profiles

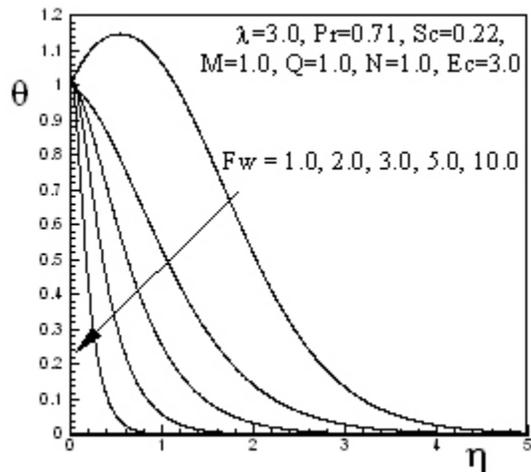


Fig. 3b: Suction parameter (F_w) effect on temperature profiles

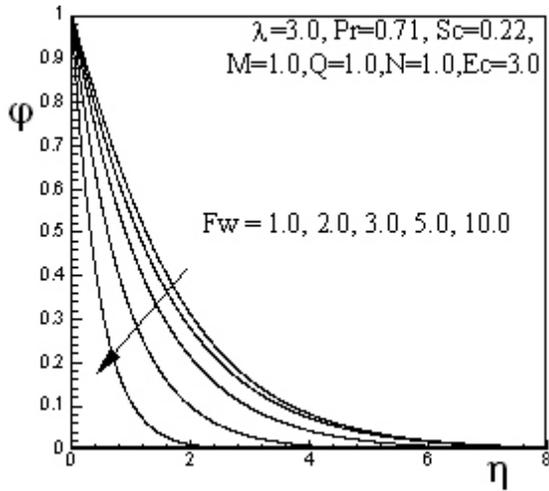


Fig. 3c: Suction parameter (F_w) effect on concentration profiles

suction parameter F_w , Prandtl number Pr , magnetic field parameter M , Schmidt number Sc , heat source parameter Q , radiation parameter N and Eckert number Ec .

Fig. 2a, b, c shows the variation of velocity, temperature and concentration profiles respectively with buoyancy parameter λ . Fig. 2a shows that the velocity rises steeply near the vertical stretching sheet as the buoyancy parameter λ is increased. Moving away from the wall, a cross flow in the velocity is induced as the velocity profiles turn to zero at slower rate for small buoyancy parameters λ . The thermal boundary layer and the concentration boundary layers reduce as the buoyancy parameter λ increases causing the fluid temperature to reduce at every point other than the wall. It is observed that the effect of buoyancy parameter λ is to reduce the concentration distribution as concentration species is dispersed away (Shateyi, 2008). This is clearly depicted in Fig. 2b and c. It should be mentioned here that for $\lambda = 1.0$, the flow represents the mixed convection.

In Fig. 3a, b and c, we have plotted the dimensionless velocity, temperature and concentration boundary layers respectively showing the effect of suction parameter F_w . We observe that the velocity, temperature and concentration boundary layers decrease with the increase of suction parameter F_w indicating the usual fact that suction stabilizes the boundary layer growth. For $F_w = 1.0$ we notice that the velocity and temperature profiles increase near the surface and then start to decrease. However, for other values of the suction parameter F_w , the velocity and the temperature profiles start to decrease monotonically from the very beginning. Thus sucking the decelerated fluid particles reduces the growth of the fluid boundary layer as well as thermal and concentration boundary layers.

The effect of Prandtl number Pr on velocity, temperature and concentration distributions is displayed

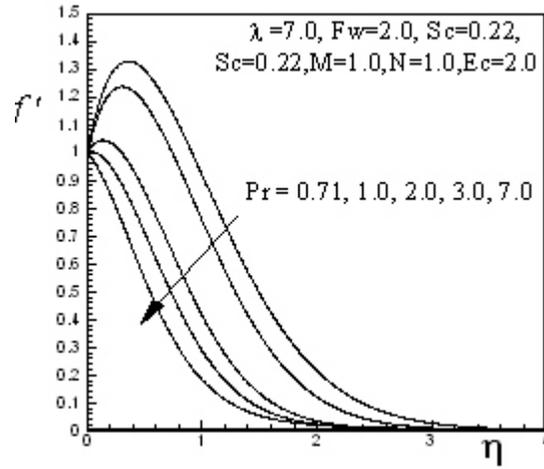


Fig. 4a: Prandtl number (Pr) effect on velocity profiles

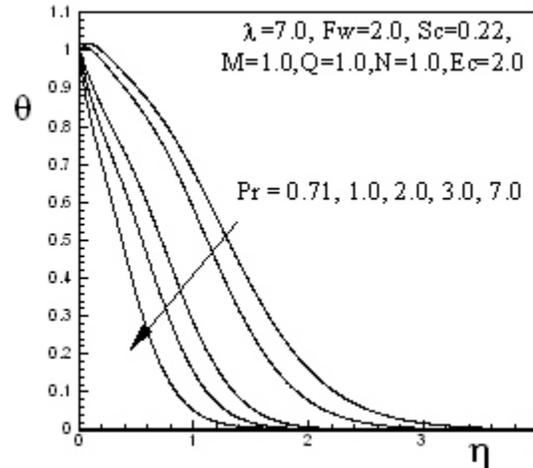


Fig. 4b: Prandtl number (Pr) effect on temperature profiles

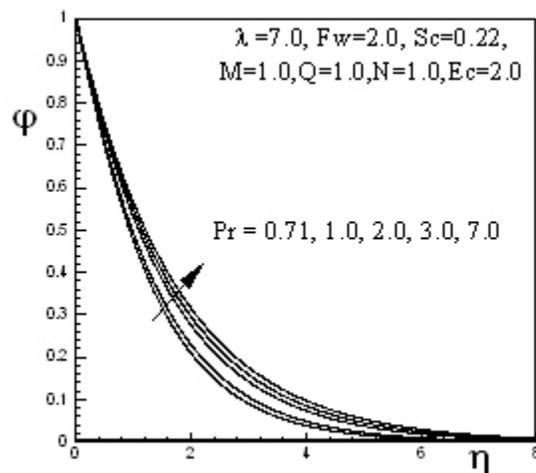


Fig. 4c: Prandtl number (Pr) effect on concentration profiles

in Fig. 4a, b and c, respectively. Figure 4a and b show that the velocity profiles and the temperature profiles decrease with the increase of Prandtl number Pr . However, Fig. 4c shows that the concentration profiles increase uniformly as Prandtl number Pr increases. From Fig. 4a, we observe that for $Pr = 0.71, 1.0$, there is a sharp rise, and for $Pr = 2.0$, there is a slight rise in velocity boundary layers near the stretching sheet. We can also see from Fig. 4b that for $Pr = 0.71, 1.0$, there is a slight rise in the thermal boundary layers near the surface.

The variation of velocity, temperature and concentration distributions with magnetic field parameter M are shown in Fig. 5a, b, c, respectively. The velocity curves in Fig. 5a show that the rate of transport is considerably reduced with the increase of magnetic field parameter M . It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is due to the fact that the variation of M leads to the variation of Lorentz force due to magnetic field and the Lorentz force produces more resistance to the transport phenomena (Ishak *et al.* 2008). From Fig. 5b we observe that temperature profiles also decrease with the increase of magnetic field parameter M . However, Fig. 5c shows that the concentration profiles increase with the increase of magnetic field parameter M .

Figure 6 exhibits the influence of Schmidt number Sc on the velocity, temperature and concentration profiles. From Fig. 6a, b and c, we see that the velocity, temperature and concentration profiles respectively decrease as Schmidt number Sc is increased. It can also be seen that for $Sc = 0.22, 0.4, 0.6, 1.0$, there is almost no variation in the corresponding velocity profiles and also in the temperature profiles. However, the concentration profiles show significant variation for different values of Schmidt number Sc .

In Fig. 7a, b, c, the effect of heat source parameter Q on the velocity, temperature and concentration boundary layers is displayed respectively. From Fig. 7a and b we see that the velocity and the temperature profiles increase as heat source parameter Q increases. On the other hand, the concentration profiles decrease with the increase of heat source parameter Q .

The variation of velocity, temperature and concentration distributions with radiation parameter N are shown in Fig. 8a, b, c respectively. From Fig. 8a and b, we observe that the velocity and the temperature profiles decrease with the increase of radiation parameter N whereas Fig. 8c shows that the concentration profiles increase as radiation parameter N increases. So, radiation can be used to control the velocity and the thermal boundary layers quite effectively. From Fig. 8a, we observe a rapid growth of velocity profiles near the stretching sheet for $N > 1.0$.

Figure 9 shows the Eckert number Ec effect on velocity, temperature and concentration profiles in the

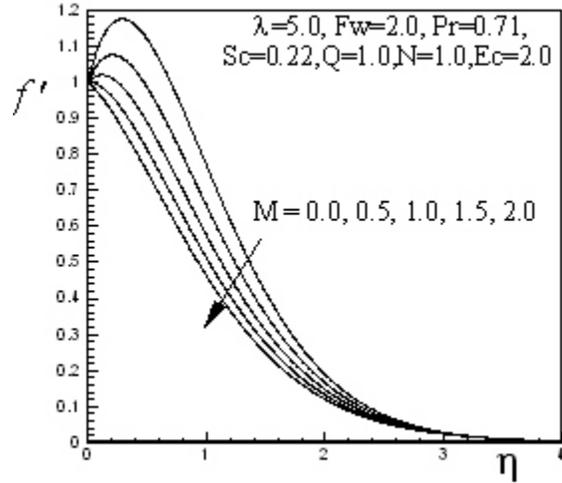


Fig. 5a: Magnetic field parameter (M) effect on velocity profiles

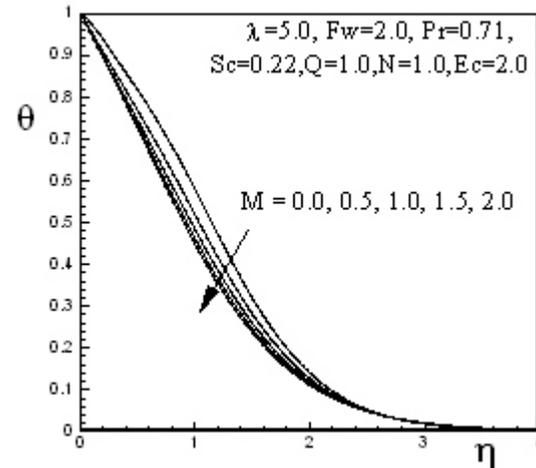


Fig. 5b: Magnetic field parameter (M) effect on temperature profiles

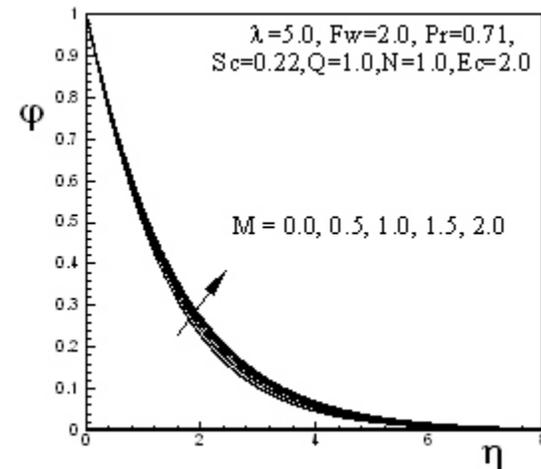


Fig. 5c: Magnetic field parameter (M) effect on concentration profiles

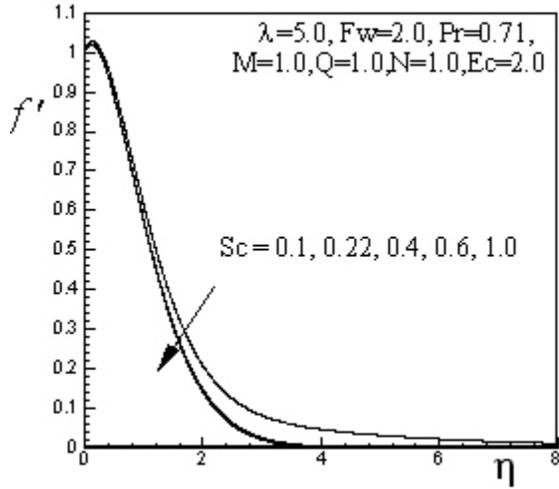


Fig. 6a: Schmidt number (Sc) effect on velocity profiles

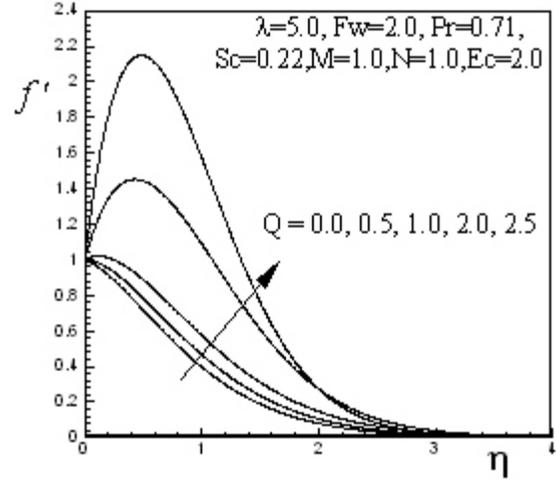


Fig. 7a: Heat source parameter (Q) effect on velocity profiles

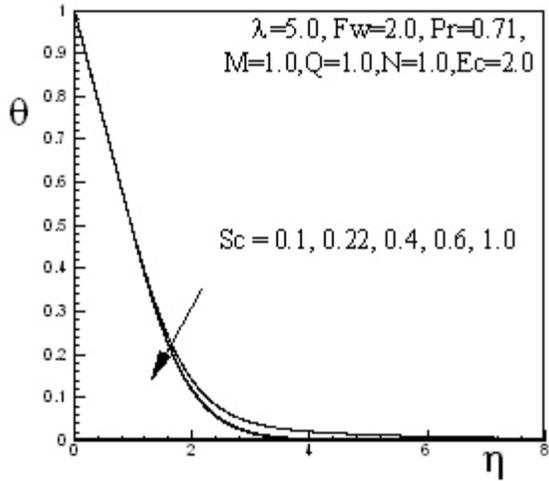


Fig. 6b: Schmidt number (Sc) effect on temperature profiles

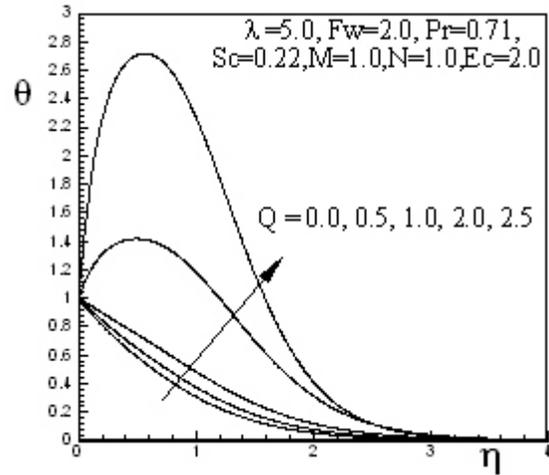


Fig. 7b: Heat source parameter (Q) effect on temperature profiles

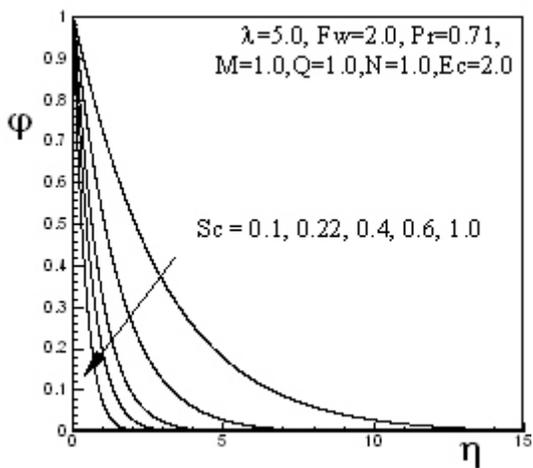


Fig. 6c: Schmidt number (Sc) effect on concentration profiles

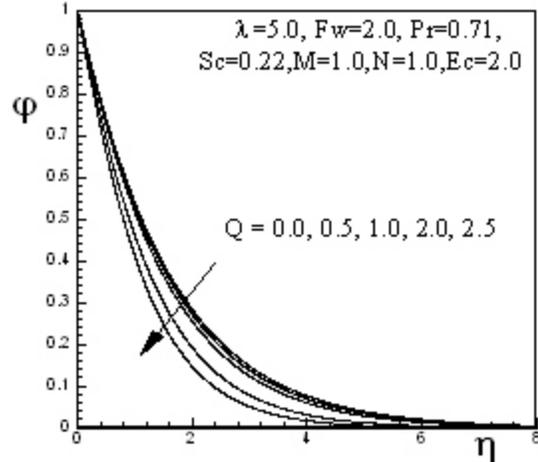


Fig. 7c: Heat source parameter (Q) effect on concentration profiles

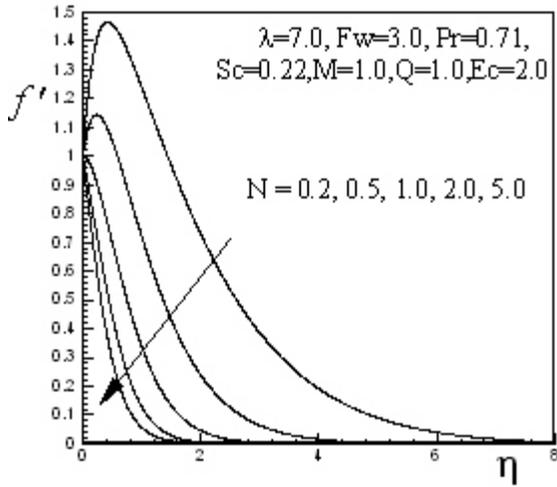


Fig. 8a: Radiation parameter (N) effect on velocity profiles

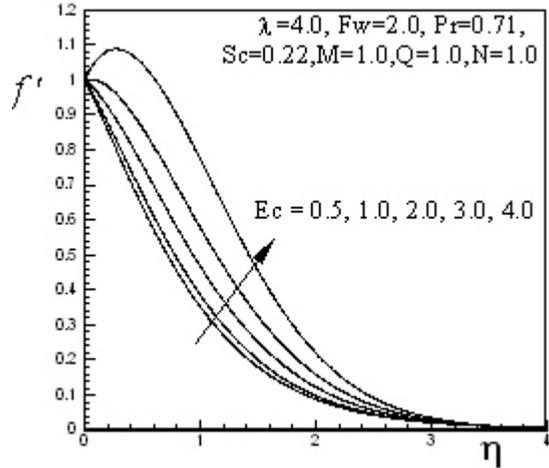


Fig. 9a: Eckert number (Ec) effect on velocity profiles

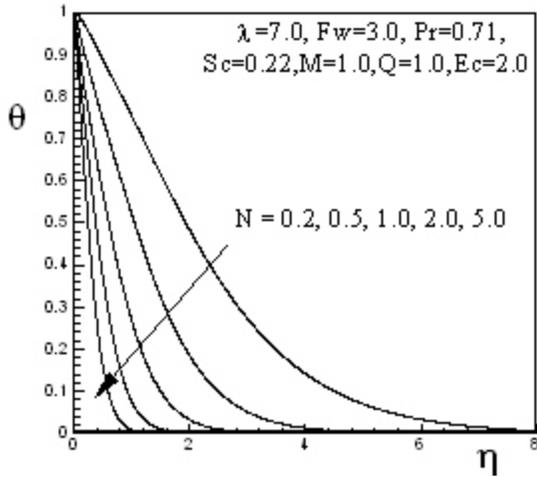


Fig. 8b: Radiation parameter (N) effect on temperature profiles

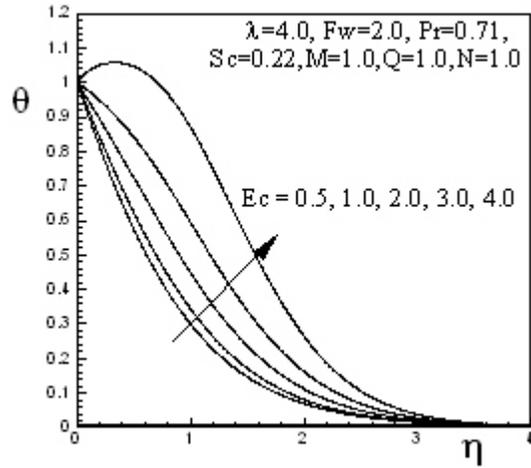


Fig. 9b: Eckert number (Ec) effect on temperature profiles

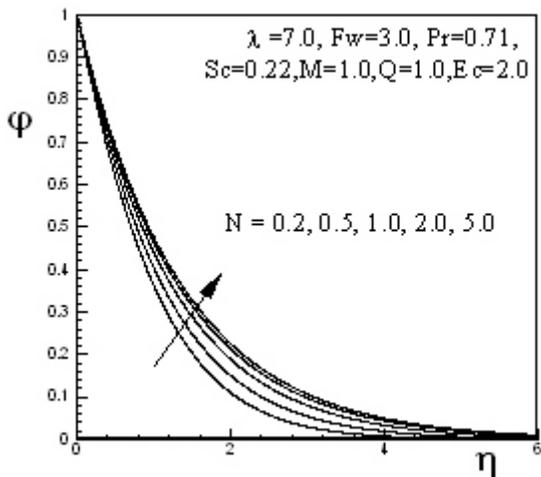


Fig. 8c: Radiation parameter (N) effect on concentration profiles

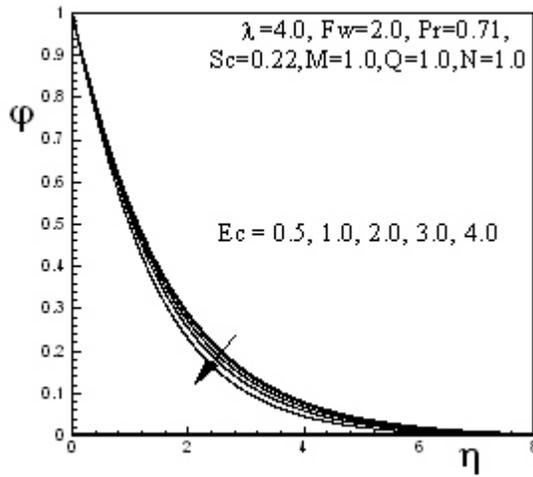


Fig. 9c: Eckert number (Ec) effect on concentration profiles

free convection flow. We see from Fig. 9a and b that the velocity and the temperature curves increase with the increase of Eckert number Ec whereas Fig. 9c shows that concentration profiles decrease with the increase of Eckert number Ec .

From the present results and the results obtained by Samad and Mohebujjaman (2009), we have observed that the flow field shows the same trend with the variation of buoyancy parameter (λ), Prandtl number (Pr), magnetic field parameter (M), suction parameter (F_w), heat source parameter (Q) and Schmidt number (Sc). However, the important part of this work in comparison with the previous work is that we have seen sharp rises in the momentum and thermal boundary layers near the stretching sheet when we have considered the effects of Prandtl number (Pr), Schmidt number (Sc), heat source parameter (Q) and the suction parameter (F_w). This can easily be explained as the usual effect of the high buoyancy force, radiation and viscous dissipation.

CONCLUSION

From the present study we can make the following conclusions:

- Larger values of buoyancy parameter λ can be used to control the temperature and concentration boundary layers
- Suction stabilizes the boundary layer growth.
- The boundary layers are highly influenced by Prandtl number Pr
- Using magnetic field we can control the flow characteristics and it has significant effect on heat and mass transfer
- The momentum boundary layer and the thermal boundary layer thicknesses reduce as a result of increasing radiation
- The presence of a heavier species (large Sc) in air decreases the fluid velocity, heat transfer and the concentration in the boundary layer
- Large values of heat source parameter Q have significant effect on the velocity and temperature distributions whereas it causes reduction in the concentration distribution in the boundary layer
- Eckert number Ec has significant effect on the boundary layer growth

NOTATIONS

Nomenclature:

C Species concentration in the flow field
 C_f Skin friction coefficient
 c_F Specific heat at constant pressure
 D Stretching constant
 D_m Coefficient of mass diffusivity

Ec Eckert number
 f Dimensionless stream function
 F_w Suction parameter
 Gr Grashof number
 k Thermal conductivity
 k_1 Mean absorption coefficient
 M Magnetic field parameter
 N Radiation parameter
 q_r Radiative heat flux, Rosseland Approximation
 Pr Prandtl number
 Q Heat source parameter
 Re Reynolds number
 Sc Schmidt number
 T Fluid temperature within the boundary layer
 u Velocity along x-axis
 v Velocity along y-axis
 v_w Suction velocity
 x Coordinate along the plate
 y Coordinate normal to the plate

Greek symbols:

β Coefficient of volume expansion
 δ Characteristic length scale
 η Similarity variable
 η Step-size
 λ Buoyancy parameter
 θ Dimensionless temperature
 ρ Density of the fluid
 σ Electric conductivity
 σ_1 Stefan-Boltzmann constant
 ψ Stream function
 ϕ Dimensionless concentration
 μ Coefficient of viscosity
 ν Kinematic viscosity

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