

A Simple and Direct Approach for Unbalanced Radial Distribution System three phase Load Flow Solution

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Abstract: The aim of the study is to develop a simple and direct approach for unbalanced radial distribution system three-phase load flow solution. The special topological characteristics of unbalanced radial distribution networks have been fully utilized to make the direct three phase load flow solution possible. Two developed matrices—the node-injection to line section-current matrix and the line section-current to node-voltage matrix—and a simple matrix multiplication are used to obtain the three phase load flow solution. Due to the distinctive solution techniques of the proposed method, the time-consuming Lower Upper decomposition and forward-backward substitution of the Jacobian matrix or admittance matrix required in the traditional load flow methods became not necessary. Therefore, the proposed method is observed as robust and time-efficient. Results obtained on 8 node and IEEE 13 node test systems demonstrate the validity of the proposed method. The proposed method has got good potential for the usage in distribution automation applications.

Key words: Distribution network, line section, node, radial, three phase load flow, unbalanced

INTRODUCTION

Many programs of real-time applications in the area of Distribution Automation (DA), such as network optimization, VAR planning, switching, state estimation and so forth, require a robust and efficient load flow method (William, 2002; Kersting, *et al.*, 1999; Zimmerman, 1995). Such a load flow method must be able to model the special features of distribution systems in sufficient detail. The well-known characteristics of an electric distribution system are radial or weakly meshed structure; multiphase and unbalanced operation; unbalanced distributed load; extremely large number of line sections and nodes; wide-ranging resistance and reactance values. Those features cause the traditional load flow methods used in transmission systems, such as the Gauss-Seidel and Newton-Raphson techniques, to fail to meet the requirements in both performance and robustness aspects in the distribution system applications. In particular, the assumptions necessary for the simplifications used in the standard fast-decoupled Newton-Raphson method (Chen *et al.*, 1991a) often are not valid in distribution systems. Therefore, a novel load flow algorithm for distribution systems is desired. To qualify for a good distribution load flow algorithm, the characteristics mentioned need to be considered. Several load flow algorithms specially designed for distribution systems have been proposed in the literature (Chen *et al.*, 1991b; Nanda, *et al.*, 2000; Jen-Hao, 2003).

Some of these methods were developed based on the general meshed topology like transmission systems (Chen *et al.*, 1991b; William, 2007). From those methods, the Gauss implicit Z-matrix method (Das, 1998) is one of the most commonly used methods; however, this method does not explicitly exploit the radial and weakly meshed network structure of distribution systems and, therefore, requires the solution of a set of equations whose size is proportional to the number of nodes. Recent research proposed some new ideas on how to deal with the special topological characteristics of distribution systems (Kersting, 2001), but these ideas require new data format or some data manipulations. A compensation-based technique is proposed (Kersting, 2001) to solve distribution load flow problems. Line Section (LS) power flows rather than line section currents were later used in the improved version and presented in (Distribution Test Feeders, 2001). Since the forward-backward sweep technique was adopted in the solution scheme of the compensation-based algorithm, new data format and search procedure is necessary. Extension of the method, which emphasized on modeling unbalanced loads and dispersed generators, was proposed in (Nanda *et al.*, 2000). One of the main disadvantages of the compensation-based methods is that new databases have to be built and maintained. In addition, no direct mathematical relationship between the system status and control variables can be found, which makes the applications of the compensation-based algorithm difficult.

The algorithm proposed in this paper is a “novel but classic” technique. The only input data for this algorithm is the conventional node-line section oriented data used by most power utilities. The objective of this paper is to develop a formulation, which takes advantages of the topological characteristics of Unbalanced Radial Distribution System (URDS) and solve the distribution load flow directly. It means that the time-consuming Lower Upper (LU) decomposition and forward-backward substitution of the Jacobian matrix or the admittance matrix, required in the traditional Newton Raphson and Gauss implicit matrix algorithms, are not necessary in the new algorithm proposed. Two developed matrices, the node-injection to line section-current matrix and the line section-current to node-voltage matrix and a simple matrix multiplication are utilized to obtain load flow solution. The proposed method is robust and very efficient compared to the conventional methods. Test results demonstrate the feasibility and validity of the proposed method.

MATERIALS AND METHODS

This study was conducted in 2010 in the Electrical Engineering Department of TRR Engineering College, Inole, Medak District, Hyderabad, AP, India

Modeling of unbalanced radial distribution system components: The distribution system components like voltage regulators; distribution transformers of various connections, capacitor banks, loads are modeled with three phase wye and delta configurations.

Proposed method for three phase load flow Algorithm of URDS: The algorithm development procedure is described in detail. For URDS, the equivalent-current-injection based model (Jen-Hao, 2003) is more accurate. The proposed method for three-phase load flow algorithm is developed using two derived matrices, the Node-Injection to Line Section-Current matrix (NILSC), the Line Section-Current to Node-Voltage matrix (LSCNV) and the equivalent current injections. For node i , the complex load W_i is expressed by:

$$W_i = (X_i + jK_i) \quad i = 1 \dots N \tag{1}$$

And the corresponding equivalent current injection at the k^{th} iteration of solution is

$$I_i^k = I_i^r (V_i^k) + jI_i^i (V_i^k) = \left(\frac{X_i + jK_i}{V_i^k} \right)^* \tag{2}$$

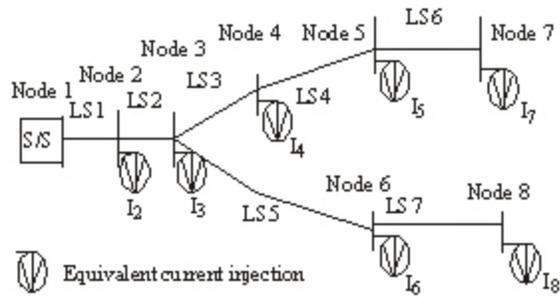


Fig. 1: A Sample 8 node URDS

where, V_i^k and I_i^k are the node voltage and equivalent current injection of node i at the k^{th} iteration, respectively. I_i^r and I_i^i are the real and imaginary parts of the equivalent current injection of node at the k^{th} iteration, respectively.

Development of relationship matrix: A simple 8 node URDS illustrated in Fig.1 is used as an example for the description of the proposed method. The power injections can be converted to the equivalent current injections by (2) and the relationship between the node current injections and line section currents can be obtained by applying Kirchhoff’s Current Law (KCL) to the URDS. The line section currents can then be formulated as functions of equivalent current injections. For example, the line section currents LS_1 , LS_3 , LS_5 and LS_7 can be expressed by their equivalent current injections as

$$\begin{aligned} LS_1 &= I_2 + I_3 + I_4 + I_5 + I_6 + I_7 + I_8 \\ LS_3 &= I_4 + I_5 + I_7 \\ LS_5 &= I_6 + I_8 \\ LS_7 &= I_8 \end{aligned} \tag{3}$$

Therefore the relationship between the node current injections and line section currents can be expressed as

$$\begin{bmatrix} LS_1 \\ LS_2 \\ LS_3 \\ LS_4 \\ LS_5 \\ LS_6 \\ LS_7 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} \tag{4a}$$

Eq. (4a) can be expressed in general form as

$$[LS] = [NILSC] [I] \tag{4b}$$

Where NILSC is the Node-Injection to Line section-Current (NILSC) matrix. The constant NILSC matrix is an upper triangular matrix and contains values of 0 and +1 only. The relationship between line section currents and node voltages can be obtained as follows. For the example given in Fig. 1, the voltages of node 2, 3, and 4 are:

$$V_2 = V_1 - LS_1 Z_{12} \quad (5a)$$

$$V_3 = V_2 - LS_2 Z_{23} \quad (5b)$$

$$V_4 = V_3 - LS_3 Z_{34} \quad (5c)$$

Where V_i is the voltage of node i , and Z_{ij} is the line impedance between node i and node j . Substituting (5a) and (5b) into (5c), (5c) can be rewritten as

$$V_4 = V_1 - LS_1 Z_{12} - LS_2 Z_{23} - LS_3 Z_{34} \quad (6)$$

From (6), it can be observed that the node voltage can be expressed as a function of line section currents, line parameters, and the substation voltage. By adopting similar procedure for other nodes, the relationship between line section currents and node voltages can be expressed as:

$$\begin{bmatrix} V1 \\ V1 \\ V1 \\ V1 \\ V1 \\ V1 \\ V1 \end{bmatrix} - \begin{bmatrix} V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{bmatrix} = \begin{bmatrix} Z12 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z12 & Z23 & 0 & 0 & 0 & 0 & 0 \\ Z12 & Z23 & Z34 & 0 & 0 & 0 & 0 \\ Z12 & Z23 & Z34 & Z45 & 0 & 0 & 0 \\ Z12 & Z23 & 0 & 0 & Z36 & 0 & 0 \\ Z12 & Z23 & Z34 & Z45 & 0 & Z57 & 0 \\ Z12 & Z23 & 0 & 0 & Z36 & 0 & Z68 \end{bmatrix} \begin{bmatrix} LS1 \\ LS2 \\ LS3 \\ LS4 \\ LS5 \\ LS6 \\ LS7 \end{bmatrix} \quad (7a)$$

Equation (7a) can be rewritten in general form as:

$$[\Delta V] = [LSCNV][LS] \quad (7b)$$

Where LSCNV is the Line Section-Current to Node-Voltage (LSCNV) matrix.

Algorithm development for radial system Matrices: From (4a and 4b), the algorithm developed for NILSC matrix is as follows:

- Step 1:** For an URDS with m -line sections and n -nodes, the dimension of the NILSC matrix is $m'(-1)$.
- Step 2:** If a line section (LS_k) is located between i^{th} node and j^{th} node, copy the column of the i^{th} node of the NILSC matrix to the column of the j^{th} node and fill a +1 to the position of the k^{th} row and the j^{th} node column.
- Step 3:** Repeat step 2 until all line sections are included in the NILSC matrix. From (7a and 7b), the algorithm developed for LSCNV matrix is as follows.
- Step 4:** For an URDS with m -line sections and n -nodes, the dimension of the LSCNV matrix is $(n-1)'m$.
- Step 5:** If a line section (LS_i) is located between node i and node j , copy the row of the i^{th} node of the LSCNV matrix to the row of the j^{th} node and fill the line impedance (Z_{ij}) to the position of the j^{th} node row and the k^{th} column.

Step 6: Repeat step 5 until all line sections are included in the LSCNV matrix.

The algorithm can easily be expanded to multiphase line sections or nodes. For example, if the line section between node i and node j is a three-phase line section, the corresponding line section current B_i will be a 3'1 vector and +1 in the NILSC matrix will be a 3'3 identity matrix. Similarly, if the line section between node i and node j is a three-phase line section, the (Z_{ij}) in the LSCNV matrix is a 3'3 impedance matrix.

It can also be observed that building algorithms for the NILSC and LSCNV matrices are similar. In fact, these two matrices were built in the same subroutine of the test program. In addition, the algorithms are developed based on the traditional node-line section oriented database, thus, the data preparation time can be reduced and the proposed method can be easily integrated with the existing Distribution Automation System.

Three phase load flow Solution Technique development: From 3.2 algorithm development, the NILSC and LSCNV matrices can be developed based on the topological structure of URDS. The NILSC matrix represents the relationship between node current injections and line section currents. The corresponding variations at line section currents, generated by the variations at node current injections, can be calculated directly by the NILSC matrix. The LSCNV matrix

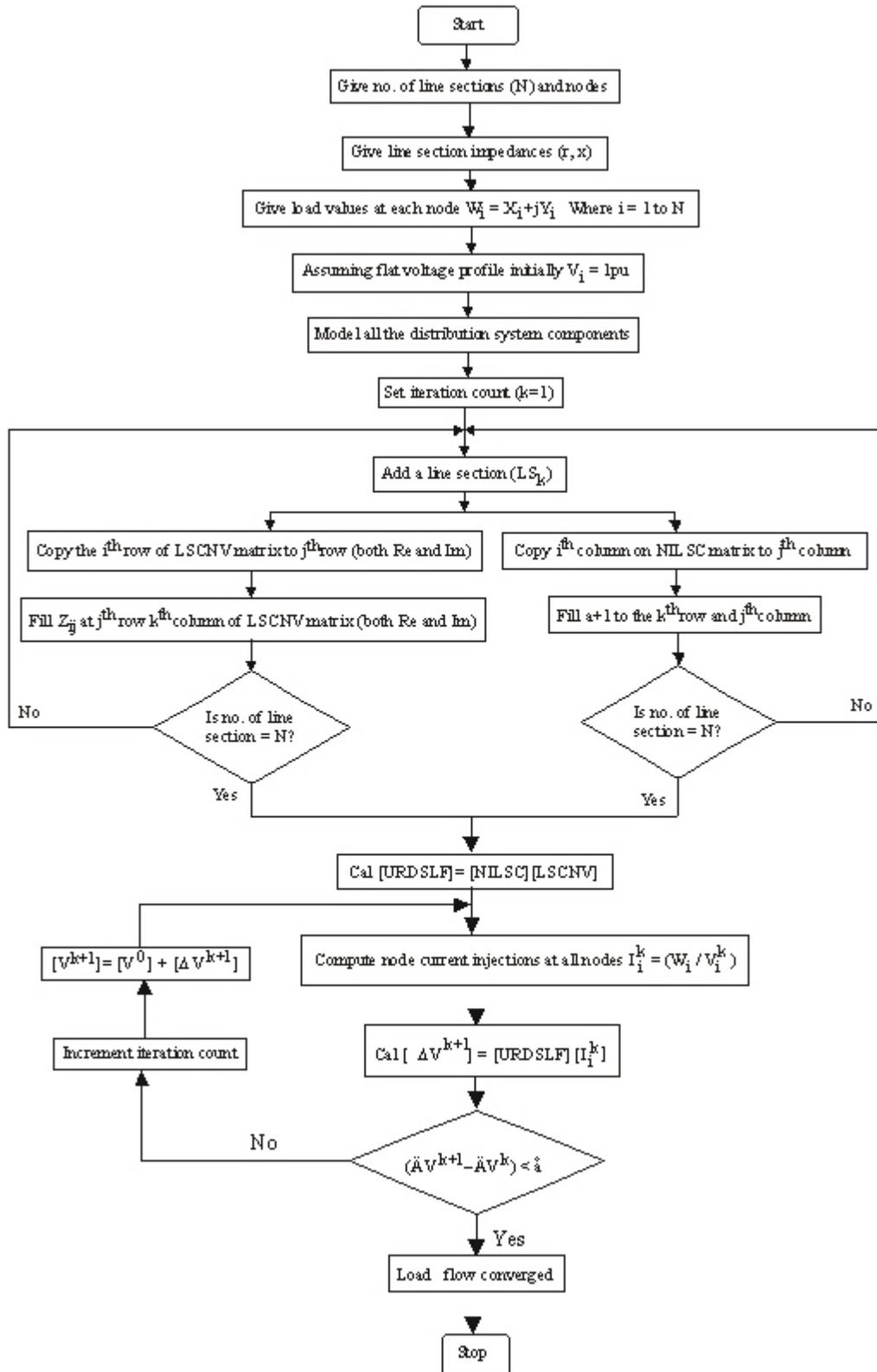


Fig. 2: Flowchart for URDS three phase load flow

represents the relationship between line section currents and node voltages. The corresponding variations at node voltages, generated by the variations at line section currents, can be calculated directly by the LSCNV matrix. From (4b) and (7b), the relationship between node current injections and node voltages can be expressed as

$$\begin{aligned} [\Delta V] &= [LSCNV][NILSC][I] \\ &= [URDSL F][I] \end{aligned} \quad (8)$$

and the solution for URDS load flow can be obtained by solving (2) iteratively

$$I_i^k = I_i^r(V_i^k) + jI_i^i(V_i^k) = \left(\frac{X_i + jK_i}{V_i^k} \right)^* \quad (9a)$$

$$[\Delta V^{k+1}] = [URDSL F][I^k] \quad (9b)$$

$$[V^{k+1}] = [V^0] + [\Delta V^{k+1}] \quad (9c)$$

As per the research, the arithmetic operation number of LU factorization is approximately proportional to N^3 (Autar, 1996), where N is the number of nodes. For a large value of N , the LU factorization occupies a large portion of the computational time. Therefore, if the LU factorization can be avoided, the load flow method can save tremendous computational resources. From the solution techniques described, the LU decomposition (Autar *et al.*, 1996) and forward-backward substitution of the Jacobian matrix or the admittance matrix is no longer necessary for the proposed method. Only the URDSL F matrix is necessary for solving load flow problem. Therefore, the proposed method can save considerable computation resources and this feature makes the proposed method suitable for online operations.

Convergence criterion: The voltage drop between different nodes is calculated using Eq. (9b). If the difference between the voltage drops in two successive iterations is less than the convergence tolerance 0.001, then the load flow is said to be converged.

The flowchart in Fig. 2 illustrates the three phase load flow algorithm in detail. The proposed three-phase load flow algorithm is implemented using C language on LINUX platform

RESULTS AND DISCUSSION

Case 1: The three phase load flow algorithm has been tested on an 8-node unbalanced test system (Jen-Hao, 2003) and compared with the results

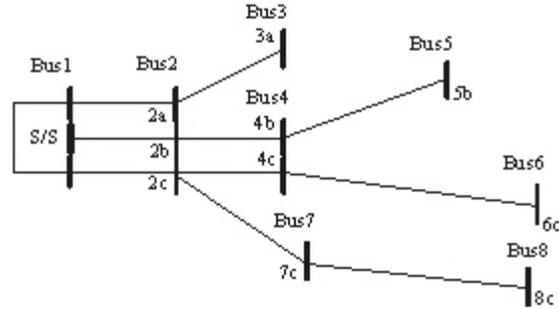


Fig. 3: 8-Node Unbalanced Radial Distribution System (URDS)

Table 1: Test feeder

Feeder No.	No. of nodes	Length (km)
1	45	1.5
2	90	2.5
3	135	3.2
4	180	4.0
5	270	7.4

Table 2: Final converged voltage solutions

Node No.	Implicit Z_{bus} Gauss method		Proposed method		Phase
	V (pu)	Angle (Rad.)	V (pu)	Angle (Rad.)	
1	1.0000	0.0000	1.0000	0.0000	A
1	1.0000	-2.0944	1.0000	-2.0944	B
1	1.0000	2.0944	1.0000	2.0944	C
2	0.9840	0.0032	0.9839	0.0032	A
2	0.9714	-2.0920	0.9711	-2.0920	B
2	0.9699	2.0939	0.9697	2.0939	C
3	0.9833	0.0031	0.9832	0.0031	A
4	0.9653	-2.0897	0.9652	-2.0897	B
4	0.9672	2.0932	0.9668	2.0932	C
5	0.9644	-2.0898	0.9640	-2.0898	B
6	0.9652	2.0930	0.9649	2.0930	C
7	0.9686	2.0937	0.9683	2.0938	C
8	0.9674	2.0936	0.9671	2.0936	C

obtained from Implicit Z matrix Gauss method (Jen-Hao, 2003). The 8-node unbalanced test system given in Fig. 3 is from the Taiwan Power Company (TPC). The single phase and double-phase laterals have been lumped to form the unbalanced loads for testing purpose. The main feeder trunk is with three phase nodes. This trunk is chopped into various sizes for tests as given in Table I. The substation is modeled as the slack node.

For any new method, it is important to make sure that the final solution of the new method is the same as that of the existing method. An 8-node unbalanced system, including the three-phase, double-phase, and single-phase line sections and nodes as given in Fig. 3 is used for comparison. The final converged voltage solutions of Implicit Z_{bus} Gauss Method and proposed method are given in Table 2. From Table 2, it is observed that the final converged voltage solutions of proposed method are very close to the solutions of Implicit Z_{bus} Gauss Method. It means that the accuracy of the proposed method is

Table 3: Number of iterations and normalized execution time

Feeder No.	Implicit Z_{bus} Gauss method		Proposed method	
	NET	IT	NET	IT
1	2.6229	3	1	2
2	14.426	3	2.162	2
3	52.131	4	4.366	3
4	131.15	4	8.923	3
5	432.79	4	12.033	4

NET = The normalized execution time, IT = Number of iterations, Performance 1.0 is set in proposed method for feeder 1

Table 4: IEEE 13 node test system line connectivity

IEEE node name	Assumed node as	From node	To node	Distance
650	1	1	2	10
RG0(regulator)	2	2	3	1990
632	3	3	4	500
633	4	4	5	0
634	5	3	6	500
645	6	6	7	300
646	7	3	8	1000
0(switch)	8	8	9	1000
671	9	9	10	0
692	10	10	11	500
675	11	9	12	300
684	12	12	13	300
611	13	9	14	1000
680	14	12	15	800
652	15	0	0	0

Table 5: IEEE 13 node test system loads defined

Sr. No.	Types of load	Defined as
1	Y-PQ	Constant power-y-load
2	D-PQ	Constant power-d-load
3	Y1	Star load
4	D1	Delta load
5	Y-Z	Constant impedance -y-load
6	D-Z	Constant impedance -d-load

Table 6: IEEE 13 node test system types of load presented

Node No.	Type of load presented
5	Y-PQ
6	Y-PQ
7	D-Z
8	Y-PQ
9	D-PQ
10	D1
11	Y-PQ
13	Y1
15	Y-Z

almost the same as the commonly used Implicit Z_{bus} Gauss Method.

Table 3 lists the number of iterations and the normalized execution time for both the methods. The convergence tolerance of 0.001PU has been considered. The power flow solution is converged in two iterations and the execution time is 0.0134 sec. It can be observed that proposed method is more efficient, especially when the network size increases, since the time-consuming processes such as LU factorization and forward-backward substitution of admittance matrix are not necessary for proposed method. It is also observed that for a 270-node

system; the proposed method is almost 24 times faster than Implicit Z_{bus} Gauss Method.

Case 2: The proposed 3 phase load flow algorithm has been tested on an IEEE 13 node unbalanced radial distribution test system given in Fig. 4, considering all the distribution system components modeling. The data for the IEEE 13 node test system is given in Table 4-6.

The results obtained with the proposed method for the IEEE 13 node test system are compared with the results obtained from the distribution analysis software Windmil. In these two methods the convergence tolerance of 0.001 pu has been considered.

The results obtained for all the three phases of the IEEE 13 node test system with the Windmil Software and with the proposed method are given in Table 7. For this test system the load flow solution is converged in three iterations and the execution time is 0.0156 sec.

CONCLUSION

A comprehensive, simple, direct and efficient three-phase load flow solution algorithm for the unbalanced radial distribution system has been presented in this paper. The algorithm takes into account the detailed modeling necessary for use in the distribution automation environment of the practical unbalanced radial distribution system. Based on the topological characteristics of the unbalanced radial distribution system, two matrices NILSC and LSCNV are developed for solving the three-phase load flow problem. The NILSC matrix represents the relationship between node current injections and line section currents and the LSCNV matrix represents the relationship between line section currents and node voltages. These two matrices are combined to form a direct approach for solving the unbalanced load flow problem. Due to this direct approach the time consuming procedures, such as the LU factorization and forward-backward substitution of the Jacobian matrix or the admittance matrix are not necessary in the proposed method.

Hence the proposed method is observed as robust, effective and time efficient. The distribution system components like voltage regulators; distribution transformers of various connections, capacitor banks, loads are modeled with three phase wye and delta configurations. This modeling has also been considered on an IEEE 13 node unbalanced radial distribution test system in the proposed approach. The proposed method has been tested on an 8-node unbalanced test system and the results obtained are closure to the results obtained by Implicit Z_{bus} Gauss Method. The proposed method has also been tested on an IEEE 13 node unbalanced radial distribution test system and the results obtained are in agreement with the results obtained from the distribution

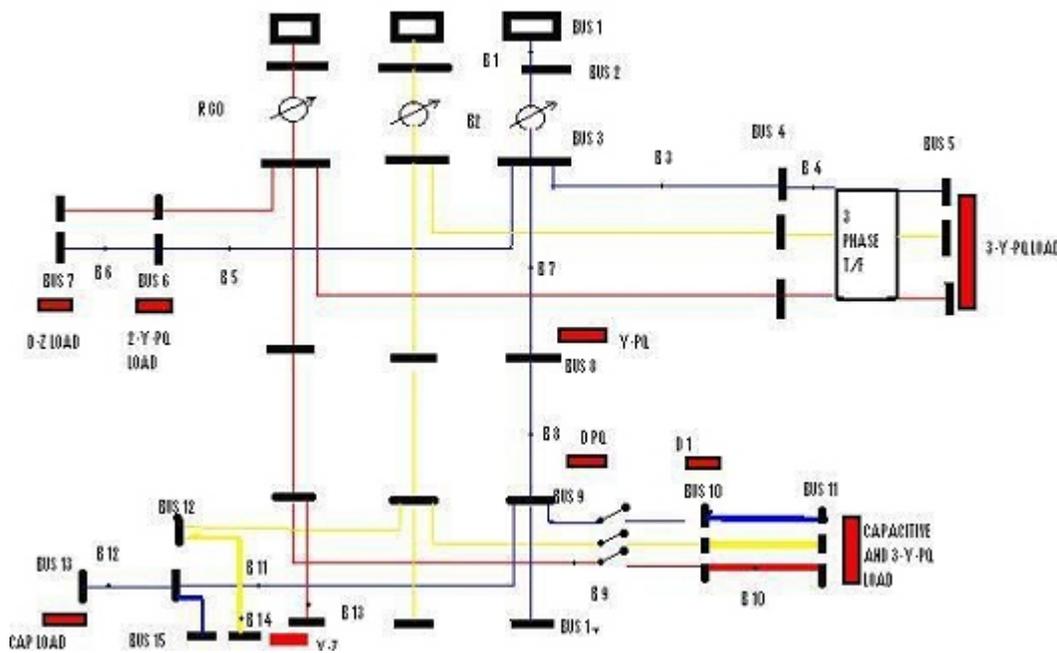


Fig. 4: IEEE 13 node test system

Table 7: Results obtained for all the three phases for the IEEE 13 node test system by proposed method and Windmil software

Node	Windmil software Phase A		Proposed method Phase A		Windmil software Phase B		Proposed method Phase B		Windmil software Phase C		Proposed method Phase C	
	V (pu)	ANG. (deg)	V (pu)	ANG. (deg)	V (pu)	ANG. (deg)	V (pu)	ANG. (deg)	V (pu)	ANG. (deg)	V (pu)	ANG. (deg)
650	1.0000	0.00	1.0000	0.00	1.0000	-120.00	1.0000	-120.00	1.0000	120.00	1.0000	120.00
RG60	1.0625	0.00	1.0646	0.00	1.0500	-120.00	1.0521	-120.00	1.0687	120.00	1.0755	120.00
632	1.0210	-2.49	1.0219	-2.51	1.0420	-121.72	1.0423	-122.02	1.0174	117.83	1.0191	117.73
633	1.0180	-2.56	1.0189	-2.60	1.0401	-121.77	1.0404	-122.06	1.0148	117.82	1.0165	117.72
634	0.9440	-3.23	0.9449	-3.31	1.0218	-122.22	1.0220	-122.52	0.9960	117.34	0.9977	117.25
645	-	-	-	-	1.0329	-121.90	1.0312	-122.28	1.0155	117.86	1.0148	117.81
646	-	-	-	-	1.0311	-121.98	1.0283	-122.41	1.0134	117.90	1.0114	117.88
671	0.9900	-5.30	0.9900	-5.36	1.0529	-122.34	1.0527	-122.60	0.9778	116.02	0.9789	116.01
680	0.9900	-5.30	0.9900	-5.37	1.0529	-122.34	1.0527	-122.60	0.9778	116.02	0.9789	116.01
684	0.9881	-5.32	0.9882	-5.39	-	-	-	-	0.9758	115.92	0.9770	115.91
611	-	-	-	-	-	-	-	-	0.9738	115.78	0.9751	115.78
652	0.9825	-5.25	0.9830	-5.32	-	-	-	-	-	-	-	-
692	0.9900	-5.31	0.9900	-5.36	1.0529	-122.34	1.0527	-122.60	0.9777	116.02	0.9789	116.01
675	0.9835	-5.56	0.9834	-5.61	1.0553	-122.52	1.0550	-122.77	0.9758	116.03	0.9769	116.02

analysis software Windmil. Test results conclude that the proposed method is suitable for analyzing large-scale unbalanced radial distribution systems.

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