

Modulation Technique Using Boundary-Pulse-Width for a Single-phase Power Inverter

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Abstract: The pulse-width-modulation technique plays a very important role in the inverter gain control. New varied-pulse width techniques for power inverters of two schemes have been introduced in this study: Small Boundary-Pulse-Width Technique (SBPW), and Large Boundary-Pulse-Width technique (LBPW). These techniques are developed to improve the inverter operation, based on minimum harmonic contents in the output voltage. The original pulse-width of an inverter operation is divided into multiple pulses of variable widths per half cycle. For the SBPW method a middle pulse and several boundary pulses per half cycle are deduced. The middle pulse is given the largest width and the boundary pulses are taken small widths. For LBPW method the opposite is considered. A software-model was developed by the authors to determine the conduction angles and their gating angles required for the inverter drive. The advantage of these techniques allow a modifying in the number of pulses per half cycle and their widths, leading to an inverter operation with less generation of harmonics. The SBPW technique makes the inverter output voltage closer to a sinusoidal form. Therefore, the optimal inverter performance is obtained with this technique.

Key word: Converters, multiple-pulse width modulation control, power inverters, pulse-width modulation control

INTRODUCTION

Inverters are widely used in industrial applications to have a variable a.c output from d.c. input (Ohnishi and Okitsu, 1983; Dewan *et al.*, 1984, Taniguchi and Irie, 1986). The gain of the inverter can be changed using various techniques such as a Pulse-Width Modulation technique (PWM) (Rashid, 2004; Kawabata *et al.*, 1990; Dujic *et al.*, 2007). This variation of the inverter gain provides an efficient control drive for a.c applications (Thorbrog and Nystorm, 1988; Salsbury, 2002; Sean, 1989.). The pulse- width modulation control also provides a variable inverter frequency. The output of an ideal inverter is a sinusoidal waveform but this is invalid for the practical inverter in which its output is discontinuous and contains harmonics (Hamman and Merwe, 1988). This discontinuous operation is undesirable for practical applications due to its problems such as extra power losses.

The single pulse-width-modulation control (Rashid, 2004; Jahmeerbacus and Soyjaudah, 2000), implies two signals: reference signal and carrier signal to generate a single pulse per half cycle. The width of this pulse varies and it is used to control the gain of the inverter. Even harmonics are not generated by this technique due to the symmetry of the output voltage, but the low orders of odd

harmonics are present and dominant. The multiple (uniform)-pulse-width modulation control (Rashid, 2004; Ahmed, 2005.) comprises several pulses of equal widths per half cycle instead of a single pulse of output voltage. The amplitudes of low order harmonics are smaller compared to that of single pulse-width-modulation technique. In a sinusoidal pulse-width-modulation control (Rashid, 2004; Fukuda *et al.*, 1990; Iqbal *et al.*, 2006) a sinusoidal reference signal is produced and it compares with a triangular carrier signal to generate several pulses per half cycle, in which the width of each pulse is varied depending on its corresponding amplitude of the sinusoidal reference signal. The inverter output voltage along with this technique has little harmonic amplitudes compared to that of single pulse-width-modulation and uniform pulse-width-modulation techniques especially for low order harmonics. Although the most used is the sinusoidal pulse-width-modulation technique but its fundamental output voltage is low. Such techniques like trapezoidal modulation and delta modulation are proposed trying to improve the fundamental output inverter voltage (Schiop and Trip, 2007; Chaudhari and Fernandes, 1999).

For the multiple pulse-width-modulation and sinusoidal pulse-width-modulation techniques several hardware circuits are required for generating the multiple pulses per half cycle, and moreover it is difficult to

modify the number of pulses per half cycle due to the complexity of the hardware circuits. In this study, the authors developed software-models for new pulse-width techniques to split the original pulse of an inverter operation into multiple and variable-width pulses per half cycle. The software-model determines the conduction angles for the variable-width pulses and applied directly to the transistor drive of the inverter. The proposed technique; Small Boundary-Pulse-Width technique (SBPW) gives an improved inverter operation due to its ability for making the inverter output voltage closer to the sinusoidal form and then produces less generation of harmonics. The results show that the harmonics can be modified and significantly decreased compared to that of the conventional pulse-width techniques, due to software ability of changing the widths of the pulses. Moreover, it is easy to modify the number of pulses per half cycle and their widths; thereafter an optimum inverter operation is reached and found.

The research has introduced new pulse-width techniques to control the inverter output voltage.

MATERIALS AND METHODS

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Conventional pulse-width-modulation techniques: Many PWM techniques were developed to control the power inverter gain, and tried to improve the inverter operation, based on minimum harmonic contents in the output voltage.

Single pulse-width-modulation control (SPWM): In the single pulse-width modulation technique (Rashid, 2004; Jahmeerbacus and Soyjaudah, 2000), the triangular carrier signal is compared with the rectangular reference signal to generate the positive and negative pulses required for inverter output voltage, as shown in Fig. 1. The reference signal determines the frequency of the inverter output voltage. The intersected points of the reference signal and the carrier signal represent the gating angles (a), or the starting and ending instants of conduction angles (d) for the transistors in the inverter drive.

Due to discontinuous operation of the inverter, the harmonics will be generated in the output voltage. The cosine-sine Fourier series for inverter output voltage $v(t)$ is represented as (Iqbal *et al.*, 2006):

$$v(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(\omega t) + b_n \sin n(\omega t)) \quad (1)$$

n is the harmonic order

Due to the symmetry of the inverter output voltage, a_0 and a_n are zeros and even harmonics are absent. The Fourier coefficient b_n is given by:

$$b_n = \frac{2}{2\pi} \int_0^{2\pi} v(t) \sin n(\omega t) d\omega t \quad (2)$$

With intervals as shown in Fig. 1, the following equation is produced:

$$b_n = \frac{2}{2\pi} \left[\int_{\frac{\pi}{2} - \frac{\delta}{2}}^{\frac{\pi}{2} + \frac{\delta}{2}} V \sin n(\omega t) d\omega t - \int_{\frac{3\pi}{2} - \frac{\delta}{2}}^{\frac{3\pi}{2} + \frac{\delta}{2}} V \sin n(\omega t) d\omega t \right] \quad (3)$$

Equation (3) is derived by (Rashid, 2004) and substituted it in Eq. (1), leading to the inverter output voltage:

$$v(t) = \sum_{n=1}^{\infty} \left[\left(\frac{4V}{n\pi} \sin \frac{n\delta}{2} \right) \sin n(\omega t) \right] \quad (4)$$

V is the dc supply voltage.

The conduction angle (d) in Eq. (4) is a function of gating angle (a) and then a function of the modulation index (m). The modulation index (m) is determined as in equations below (Rashid, 2004):

$$m = \frac{A_r}{A_c} \quad (5)$$

It can be shown from Fig. 1 that:

$$\sin(\theta) = \frac{A_r}{\frac{\pi}{2} - \alpha} = \frac{A_c}{\frac{\pi}{2}}, \text{ Then;}$$

$$\frac{A_r}{A_c} = \frac{\frac{\pi}{2} - \alpha}{\frac{\pi}{2}}$$

$$\frac{\pi}{2} m = \frac{\pi}{2} - \alpha$$

and

$$\alpha = \frac{\pi}{2}(1 - m) \quad (6)$$

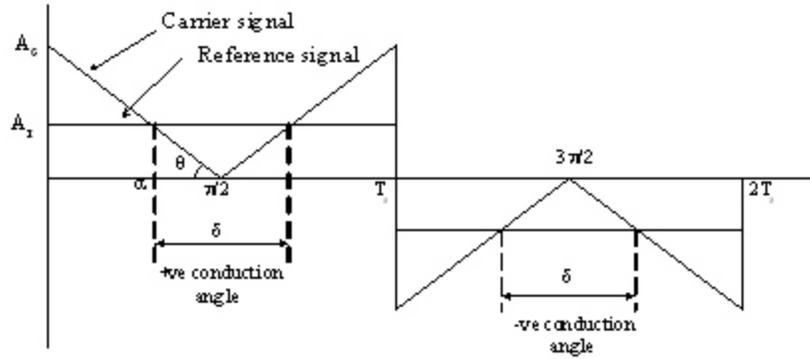


Fig. 1: Control of Single pulse-width modulation (SPWM)

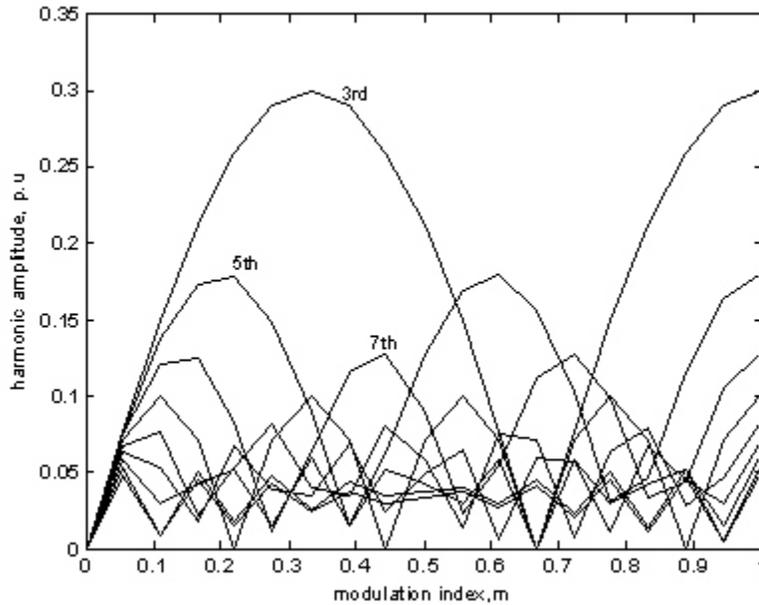


Fig. 2: Harmonic profile of an inverter gain using single pulse-width-modulation control (SPWM):

The gating signals (α) and ($\alpha + \pi$) are applied to the transistor driver of the inverter to produce the output voltage for positive and negative half cycles. Figure 2 shows the harmonic profile of inverter gain for SPWM. The dominant harmonic component is the third one.

Uniform pulse-width-modulation technique (UPWM):

Instead of generating a single pulse per half cycle of inverter output voltage, multiple pulses of uniform widths were produced as shown in Fig. 3. This technique is reduced the amount of harmonics injected into output voltage (Rashid, 2004; Ahmed, 2005) in this technique, two signals are generated to produce p-pulses per half cycle. The rectangular reference signal determines the frequency of the inverter output voltage (f_o). The

frequency of the triangular carrier signal (f_c) determines the number of pulses per single output cycle. Assuming the number of pulses per half cycle is (p), we get Eq. (7):

$$2P = \frac{f_c}{f_o}$$

or:

$$P = \frac{1}{2} \frac{f_c}{f_o} \tag{7}$$

For a single pulse per half cycle, the center of the pulse is ($p/2$) rad. If there are two boundary pulses added around the middle point of the half cycle of ($p/2$ rad), the center of the most-left pulse (MLP) becomes:

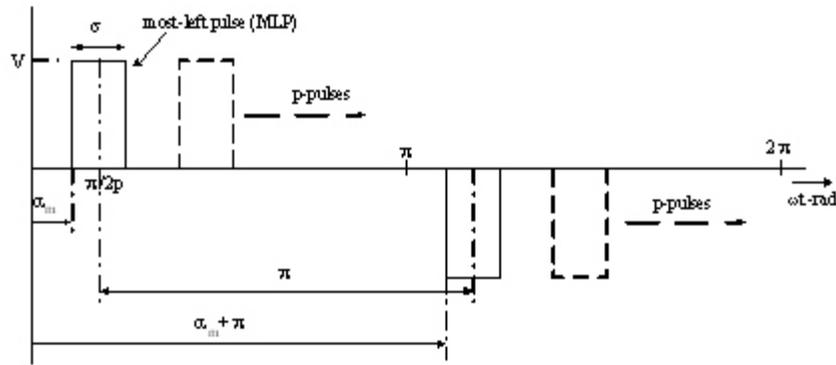


Fig. 3: Uniform pulse-width-modulation technique (UPWM) spectrum

$$\frac{1}{2} \times \frac{\pi}{2} \quad \text{rad}$$

If there are four boundary pulses added around the middle point of the half cycle of (p/2 rad), the center of MLP becomes:

$$\frac{1}{4} \times \frac{\pi}{2} \text{ rad}$$

Accordingly, if p-boundary pulses are added around the middle point of the half cycle of (p/2 rad), the center of the MLP becomes:

$$\frac{1}{p} \times \frac{\pi}{2} \quad \text{rad}$$

Assuming the width of the MLP is (σ rad) and its angle is (α_m rad), as shown in Fig. 3, then the effective out put voltage of the inverter is given in Eq. (8) (Rashid, 2004):

$$V_{eff} = V \left[\frac{p \cdot \sigma}{\pi} \right]^{\frac{1}{2}} \quad (8)$$

The harmonic profile calculated by this technique (Rashid, 2004; Ahmed, 2005) gives a lower distortion factor compared to that of single-pulse width modulation control.

Sinusoidal pulse-width-modulation technique (SSPWM): The width of each pulse can be varied by generating a sinusoidal reference signal instead of a rectangular reference signal (Ohnishi and Okitsu, 1983; Fukuda *et al.*, 1990; Iqbal *et al.*, 2006). This SSPWM technique produces variable pulse widths, which give a harmonic profile of lower distortion factor compared to that of uniform pulse-width-modulation technique and

single pulse-width-modulation technique. In the UPWM technique, the pulse width is uniform and the effective output voltage of the inverter that is determined in Eq. (8) depends on number of pulses per half cycle (p). The widths of the pulses in the SSPWM technique is varied, and the effective value of the inverter output voltage is determined by modifying Eq. (8) to get the sum of the pulses for variable widths as in equation Eq. (9):

$$V_{eff} = V \left[\sum_{k=1}^{2p} \frac{\sigma_k}{\pi} \right]^{\frac{1}{2}} \quad (9)$$

Boundary-pulse-width techniques: In the multiple-pulse width modulation and sinusoidal-pulse width modulation control-model the hardware circuits for producing reference signal and carrier signals make the modification of the number of pulses per half cycle and then an improved harmonic profile is difficult. For practical applications, it is reasonable to develop a control technique leading to minimum harmonic contents in the inverter output voltage towards to an optimum operation. In this work, two techniques are developed to produce several and varied-pulse widths per half cycle, without the need of hardware circuits. Depending on the algorithm proposed by each technique, different inverter performances are obtained and the simulated results will show that a high inverter performance is possible using Small Boundary-Pulse-Width technique (SBPW). The software-models of these techniques make the modification of number of pulses per half cycle accessible and easier.

Small boundary-pulse-width technique (SBPW): In this method, several uniform pulses will be given equal widths except the middle-pulse. The widths of all boundary-pulses shown in Fig. 3 are reduced to the half of the uniform pulse-width. The subtracted widths from the boundary-pulses are added to the middle-pulse. This is done somehow to ensure that the sum of the boundary-

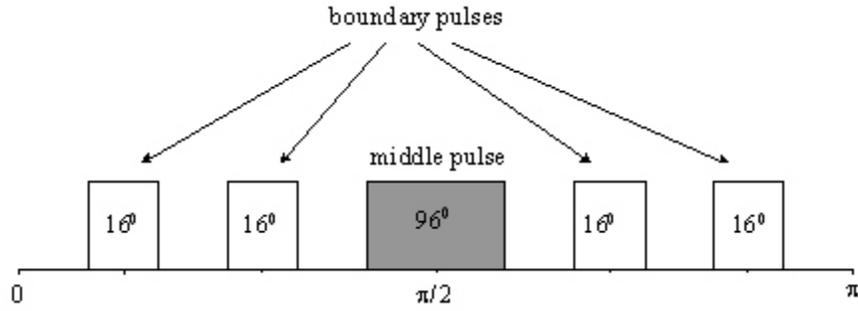


Fig. 4: Middle and boundary pulses per half cycle, (a) Small boundary-pulse-width technique (SBPW) for original single pulse $d = 160^\circ$ and $p = 5$, (b) Large boundary-pulse-width technique (LBPW): for original single pulse $d = 140^\circ$ and $p = 7$

pulse widths and the middle-pulse width equal to the width of the original pulse width (d) to maintain the same condition for the inverter operation. This method makes the inverter output voltage closer to the sinusoidal form by lengthening the center pulse and shortening the boundary pulses. Consequently, this technique will modify the harmonic contents of the inverter output and gives a high inverter performance. The uniform pulse-width is given as in Eq. (10):

$$\sigma = \frac{\delta}{p} \tag{10}$$

The width of each boundary-pulse is reduced to half of the uniform pulse-width, leading to Eq. (11):

$$\sigma_b = \frac{\sigma}{2} \tag{11}$$

The width of the middle pulse (s_m) is increased by (s_{mA}) and becomes:

$$\sigma_m = \sigma + \sigma_{mA} \tag{12}$$

The addition width (s_{mA}) is given as:

$$\sigma_{mA} = \sigma \left(\frac{p-1}{2} \right) \tag{13}$$

The sum of widths for the boundary-pulses and the middle-pulse must equal the width of the original single-pulse, and Eq. (14) must be verified:

$$\delta = \sigma_m + p \cdot \sigma_b \tag{14}$$

Testing example: To verify that the algorithm proposed above can keep the original pulse width after producing the variable pulse widths for middle and boundary pulses;

an inverter operation is assumed at the original single pulse of: $\delta = 160^\circ$. The number of pulses per half cycle is assumed to be (5):

$$\sigma = \frac{\delta}{p} = \frac{160^\circ}{5} = 32^\circ$$

Boundary pulses are reduced by:

$$\frac{\sigma}{2} = \frac{32^\circ}{2} = 16^\circ$$

and then become:

$$\begin{aligned} \sigma_b &= \sigma - \frac{\sigma}{2} = 16^\circ \\ \sigma_{mA} &= 32^\circ \times \frac{p-1}{2} = 64^\circ \end{aligned}$$

The middle Pulse width becomes:

$$\sigma_m = 32^\circ + 64^\circ = 96^\circ$$

The distribution of middle and boundary pulses per half cycle is shown in Fig. 4. The sum of ($\sigma_m + 5\sigma_p = 96^\circ + 5 \times 16^\circ = 160^\circ$) which verifies Eq. (14) and equals the same width of the original single pulse of: $\delta = 160^\circ$.

Large boundary-pulse-width technique (LBPW): In this method, the width of the middle-pulse is reduced to the half of the uniform pulse-width. The width of each boundary-pulse is increased somehow to ensure that the sum of the boundary-pulse widths and the middle-pulse width equal to the width of the original pulse width (δ) to maintain the same condition for the inverter operation.

The width of the middle-pulse becomes:

$$\sigma_m = \frac{\sigma}{2} \tag{15}$$

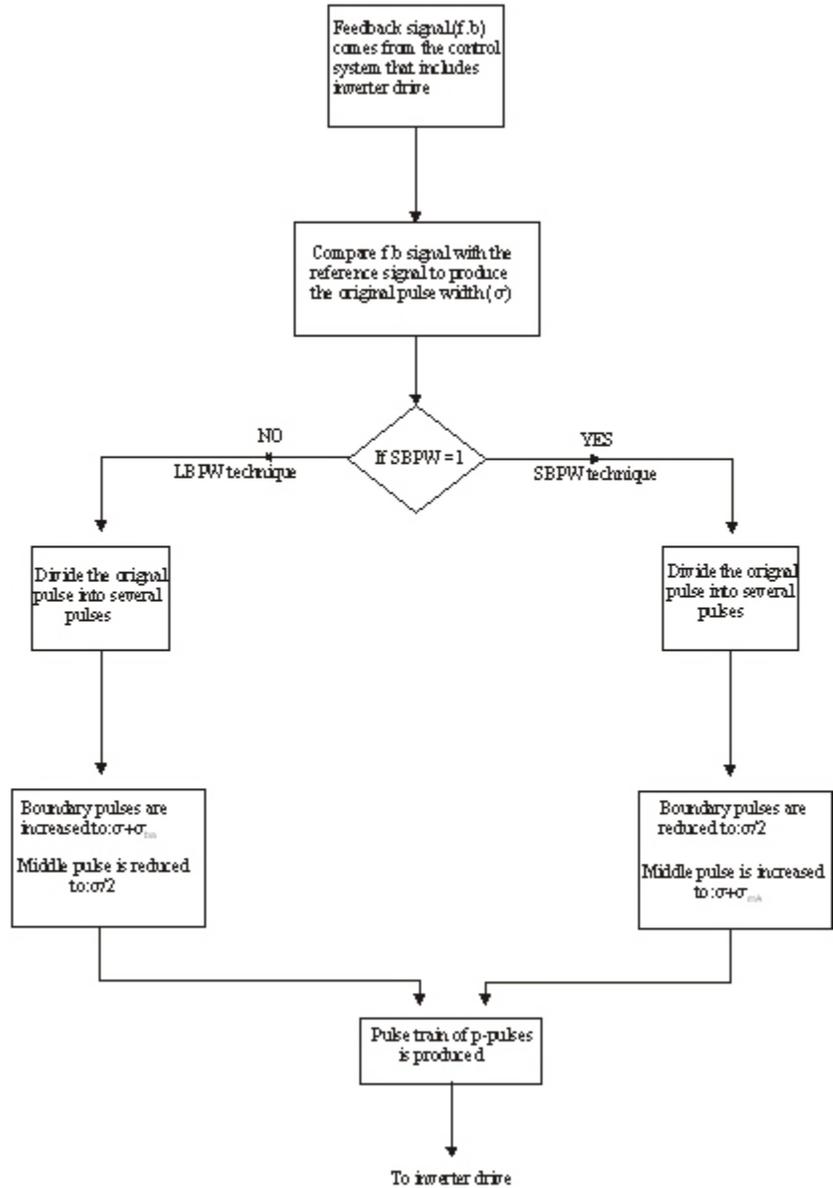


Fig. 5: State diagram for p-pulse calculations

The subtracted width from the middle-pulse is added to each pulse in the boundaries, so that the width of the boundary pulse (σ_b) is increased by (σ_{bA}) and becomes:

$$\sigma_b = \sigma + \sigma_{bA} \quad (16)$$

The addition width (σ_{bA}) is given as:

$$\sigma_{bA} = \sigma \left(\frac{1}{2(p-1)} \right) \quad (17)$$

The sum of widths for the boundary-pulses and the middle-pulse must equal the width of original single-pulse, and also the Eq. (14) must be verified:

The state diagram in Fig. 5 summarizes the SBPW and LBPW techniques for generation of p-pulses.

Testing example: To verify that the algorithm proposed above can keep the original pulse width after producing the variable pulse widths for middle and boundary pulses, an inverter operation is assumed at the original single pulse of: $\delta = 140^\circ$. The number of pulses per half cycle is assumed to be (7):

$$\sigma = \frac{\delta}{p} = \frac{140^\circ}{7} = 20^\circ$$

The middle Pulse width is decreased to:

$$\sigma_m = \frac{20^\circ}{2} = 10^\circ$$

$$\sigma_{bA} = 20 \times \left[\frac{1}{2(7-1)} \right] = [1.666666667]^\circ$$

Boundary pulse width becomes:

$$\sigma_b = 20^\circ + 1.666666667^\circ = (21.666666667)^\circ$$

The distribution of middle and boundary pulses per half cycle is shown in Fig. 4. The sum of $(\sigma_m + 6\sigma_p = 10^\circ + 6 \times 21.666666667^\circ = 140^\circ)$ which verifies Eq. (14) and equals the same width of the original single pulse of: $\delta = 140^\circ$.

Determination of the pulse center: For SBPW and LBPW techniques, it is necessary to determine the pulse center of each pulse. This pulse center is employed to calculate the gating angle of the pulse. The gating signals together with their corresponding pulse widths are composed to generate the pulse train required for inverter drive. Starting from the middle pulse, the widths of pulses per group are assumed: $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \dots, \sigma_R$, and their corresponding centers are: $C_1, C_2, C_3, C_4, \dots, C_R$, respectively. The center of the middle pulse is $p/2$ rad. If there are p - pulses per half cycle, the center of the last pulse (MLP) is given as in Eq (18):

$$C_R = \frac{\pi}{2p}, \text{ rad}$$

The double center value (D_{cen}) represents the difference between the centers of two consecutive pulses and it is given by Eq. (19):

$$D_{cen} = 2 \times C_R, \text{ rad} \quad (19)$$

Starting from the most left pulse (MLP), the center of the next pulse (C_{next}) can be determined from the following equation:

$$C_{next} = C_{rior} + D_{cen} \quad (20)$$

Discontinuous operation of the inverter: The discontinuous operation of the inverter is undesired for

practical applications due to the generation of harmonics. The practical purpose of the new techniques is to incorporate modifications in the inverter drive for higher control performance. The modifications were made for the conduction angles and their gating angles of inverter drive in order to reduce harmonic contents, and achieving the optimum operation for the inverter. The general Fourier series of the inverter output voltage is given by Eq. (21) (Hayt *et al.*, 2002.):

$$v(t) = \alpha_0 + \sum_{n=1}^{\infty} (\alpha_n \cos n(\omega t) + b_n \sin n(\omega t)) \quad (21)$$

Due to symmetry of the inverter output voltage, the Fourier coefficients a_0 and a_n are zeros. In Fig. 10, the Fourier coefficient b_n of a pulse of width σ_m and gating angle σ_m including positive and negative parts is given by Eq. (22):

$$b_n = \frac{2V}{\pi} \left[\begin{array}{c} \alpha_m + \sigma_m \\ \int \sin n(\omega t) d\omega t - \\ \alpha_m + \frac{\sigma_m}{2} \\ \int \sin n(\omega t) d\omega t \\ \alpha_m + \pi \\ \alpha_m + \pi \end{array} \right] \quad (22)$$

Where, V is the d.c supply voltage of inverter. The coefficient b_n is derived in appendix (A) as shown in Eq. (23):

$$b_n = \sum_{k=1}^{2p} \frac{2V}{\pi n} [\cos n \alpha_m - \cos n(\alpha_m + \sigma_m)] \quad (23)$$

RESULTS

Two variable-pulse-width techniques are proposed to control the output voltage of a single phase inverter. SBPW technique is used to improve the inverter operation so as to be nearly continuous and with minimum generation of harmonics. The research has developed a software-model instead of a hardware-mode to generate the gating signals of the inverter drive. This software-model gives an opportunity to modify the number of pulses per half cycle without changing the width of the original pulse. This keeps the same operation for the inverter, but with high performance of inverter operation.

Various results are obtained for the new pulse-width techniques. In the SBPW, the width of the center pulse is lengthening and for the boundary pulses are shortening. This technique produces a waveform for the inverter output voltage closer to the sinusoidal form.

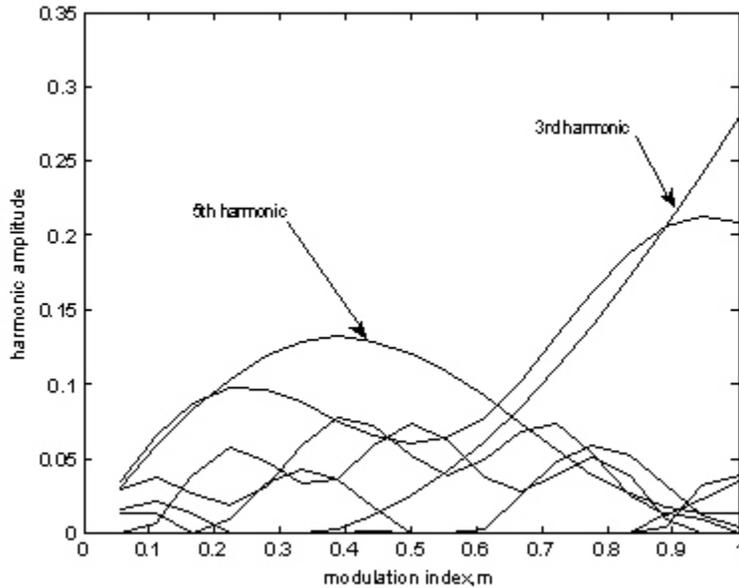


Fig. 6: The Harmonic profile of Single Boundary Pulse Width when $p = 5$

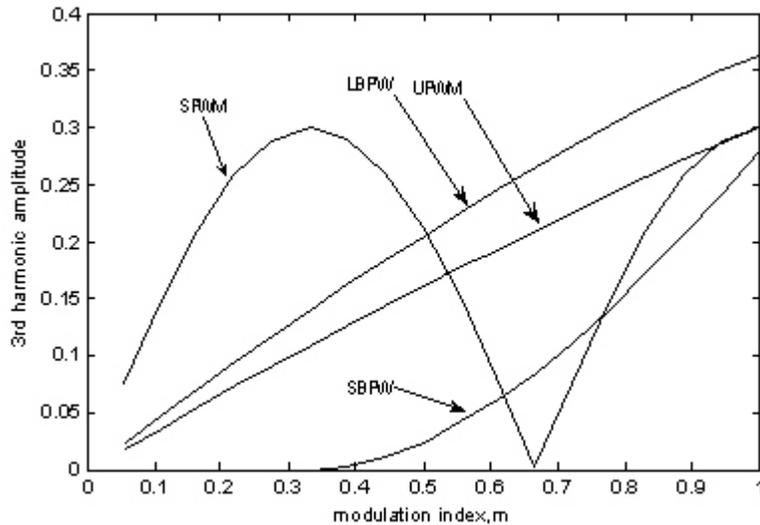


Fig. 7: Harmonic amplitude for the proposed pulse-width and conventional techniques when $p = 5$

Consequently, an improved harmonic profile is obtained as shown in Fig. 6.

From the harmonic profile results, it can be seen that the dominant harmonic component is the third one. A comparison between the new pulse-width techniques and the conventional methods, for the third harmonic component is shown in Fig. 7. It is clear that the SBPW technique gives the superior performance compared to that of conventional methods (SPWM and UPWM) and the other proposed method (LBPW), especially for the low range of the drive conduction angle. This is due to the

ability of the SBPW technique for making the inverter output voltage closer to the sinusoidal form as discussed earlier.

The number of pulses (p) per half cycle has a great effect on the inverter performance. Therefore, the performance of the inverter operation is also evaluated in terms of the number of the pulses (p) per half cycle. The third harmonic amplitudes for SBPW and LBPW techniques, respectively are shown in Fig. 8 and 9. Results show that the harmonic amplitudes decreased as the number of pulses per half cycle is increased. The

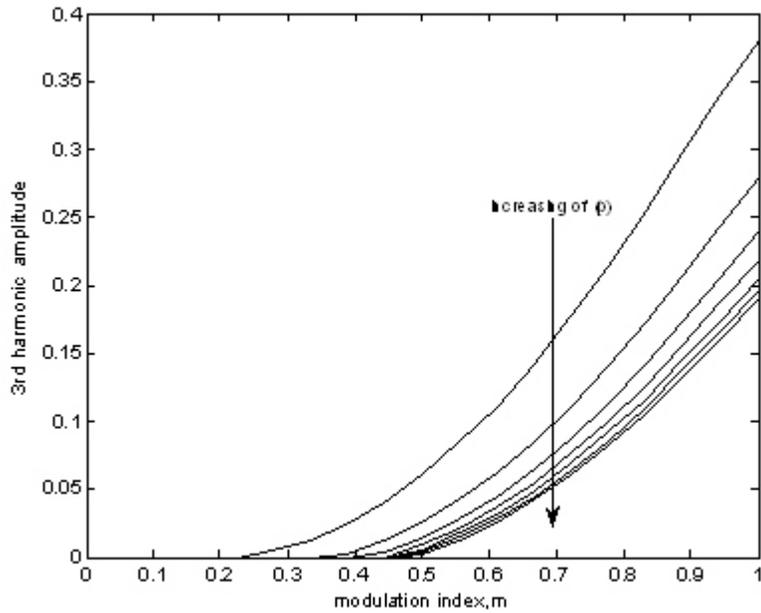


Fig. 8: The Third harmonic amplitudes for SBPW, when $p = 3, 5, 7, \dots, 15$

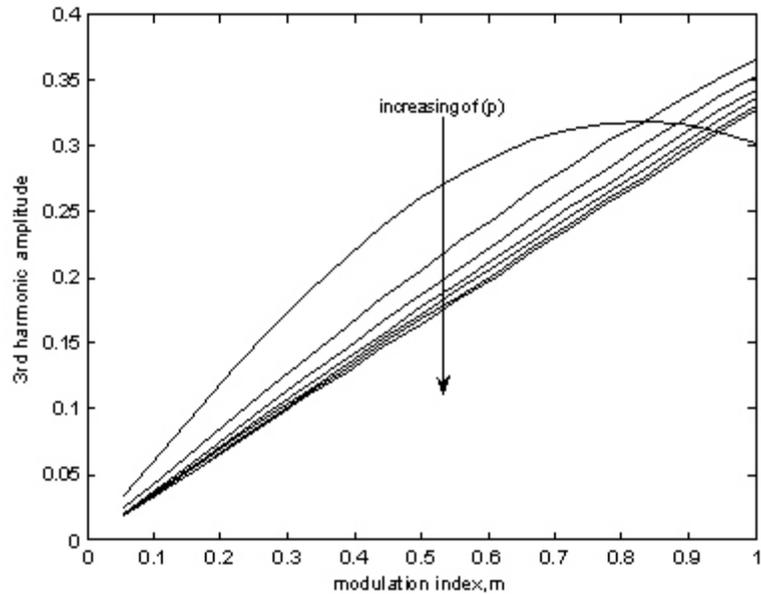


Fig. 9: The third harmonic amplitudes for LBPW, $p = 3, 5, \dots, 15$

optimal performance is obtained with the SBPW technique, and at low range of modulation index gives nearly zero third harmonic amplitude.

CONCLUSION

The research has introduced new pulse-width techniques to control the inverter output voltage. The

proposed techniques applied the pulse-width-modulation method with various alterations in the output pulses of the inverter. The two proposed methods are: SBPW and LBPW. A software model was developed to determine the conduction angles and their gating angles for the inverter drive. The advantage of this software-model is that the number of pulses and their widths per half cycle can be modified to provide an optimum inverter operation. The

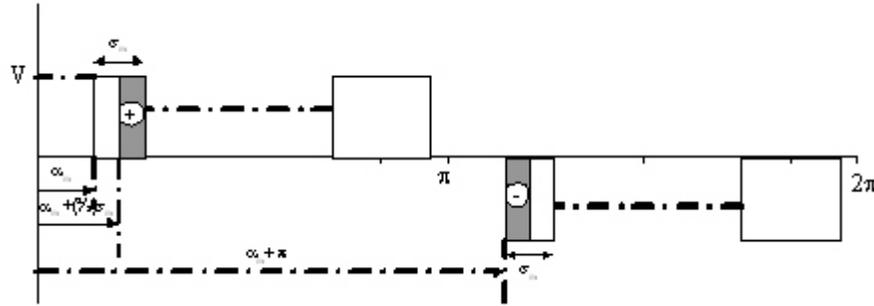


Fig. 10: Positive and negative pulses of the inverter output voltage

harmonic amplitudes of in inverter output voltage for SBPW was reduced compared to that of conventional methods. The SBPW technique makes the inverter output voltage near to the sinusoidal form with less discontinuity. Therefore, the results have shown that the SBPW performance is superior to the conventional methods, especially for large number of pulses per half cycle.

Appendix A: In Fig. 10 assume that there is a pulse of order (m) has a pulse width equal's σ_m and its gating angle is α_m . The Fourier coefficient b_n of this pulse including positive and negative parts is derived below. The integral limit is taken from $(\alpha_m + (1/2)\sigma_m)$ to $(\alpha_m + (1/2)\sigma_m + \pi)$ to include the positive and negative parts of the inverter output voltage:

$$b_n = \frac{2V}{\pi} \left[\int_{\alpha_m + \frac{\sigma_m}{2}}^{\alpha_m + \sigma_m} \sin n(\omega t) d\omega t - \int_{\alpha_m + \pi}^{\alpha_m + \pi + \frac{\sigma_m}{2}} \sin n(\omega t) d\omega t \right]$$

$$b_n = \frac{2V}{\pi} \left[\frac{1}{n} (-\cos n \alpha t) \Big|_{\alpha_m + \frac{\sigma_m}{2}}^{\alpha_m + \sigma_m} + \frac{1}{n} (\cos n \omega t) \Big|_{\alpha_m + \pi}^{\alpha_m + \pi + \frac{\sigma_m}{2}} \right]$$

$$b_n = \frac{2V}{\pi n} \left[\cos n \left(\alpha_m + \frac{\sigma_m}{2} \right) - \cos n (\alpha_m + \sigma_m) + \cos n \left(\alpha_m + \pi + \frac{\sigma_m}{2} \right) - \cos n (\alpha_m + \pi) \right]$$

and;

$$b_n = \frac{2V}{\pi n} [\cos n \alpha_m - \cos n (\alpha_m + \sigma_m)] \quad (A2)$$

Then, the Fourier coefficient b_n is:

$$b_n = \sum_{k=1}^{2p} \frac{2V}{\pi n} [\cos n \alpha_m - \cos n (\alpha_m + \sigma_m)] \quad (A3)$$

SYMBOLS

p	Number of pulses per half cycle
A_c	Carrier signal amplitude
A_r	Reference signal amplitude
T_s	Time of half cycle
m	Modulation index
δ	Original single-pulse per half cycle
σ	Uniform pulse width
R	Number of pulses per single group
α	Gating angle of the pulse
C_{next}	Center of the next pulse
C_{prior}	Center of the previous pulse
$v(t)$	Inverter output voltage
n	Harmonic order

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