

A New Method for Machining Concave Profile of the Worms' Thread

R.A. Abdullah and T.A. Abu Shreehah

Tafila Technical University, Department of Mechanical Engineering,

P.O. Box: 3588, Amman 11953, Jordan

Abstract: Research and development of wormgear drives have significantly focused on their geometrical accuracy, loadability tests, and their wear resistance and efficiency. The research has been going on in several directions. Individual method of problem solving is development new worm-gear sets and tools for their manufacturing. In the present study, worms with concave profile of their thread have been considered. To avoid the technological difficulties relevant to the application of special cutting tools for machining such gear sets, a rigid incongruent generating pair consisting of a standard hob and toroidal tool has been developed for processing the concave worm profile. The generating surface of the developed toroidal tool, which is essential for the tool manufacturing, was modeled on the basis of hob-toroidal tool interaction. The proposed method of modeling was divided into three steps: first, the common surface for both hobbing and toroidal tools has been found in terms of hyperboloid of revolution of one sheet, then, the matrix method of transforming the coordinates, from the hob-axis reference frame to the toroidal tool-axis reference frame, has been utilized, and finally, an equation described the generating surface of the toroidal tool has been derived and presented. By using the proposed model and the obtained final equation the worm thread surface machined by the mentioned tool can be defined and experimented.

Key words: Concave thread, generating surface, toroidal tool, worm-gear drive

INTRODUCTION

Gears are versatile mechanical components capable of performing many different kinds of power transmission or motion control (Sclater and Chirionis, 1996). The worm gear set or mesh is used for connecting skew shafts, usually 90°. It is characterized by high velocity ratios, high loading capacity associated with high kinematic accuracy, and relatively light mass and compact structure (Childs, 2004; Khurmi and Gupta, 2005).

During the last two decades many theoretical and experimental works have been directed towards analysis, development of new designs, and manufacturing of worm gear sets. These works improved the efficiency and working life time of worm gear sets. The Tooth Contact Analysis (TCA) and load distribution calculation of different types of worm gears have been studied by many researchers. For instance, Fang and Tsay (1996a, b, 2000), proposed mathematical models, bearing contact, Transmission Error (TE) and Contact Ratio (CR) of ZK, ZN and ZE-type worm gear set with various tooth profile modifications to investigate the effects on the meshing of worm gear sets. De Donno and Litvin (1999) proposed a new approach for design and generation of low-noise, stable bearing contact gear drive with cylindrical worm. The approach was based on application of an oversized

hob and varied plunging of worm generating tool. Argyris *et al.* (2000) proposed a computer program in Visual Basic language for simulation of meshing and contact of gear drives, the application of the program for design of worm gear drive, and advanced design of worm gear drive with reduced transmission errors and favorable bearing contact. Seol (2000) considered the design and simulation of meshing of a single enveloping worm-gear drive with a localized bearing contact in order to reduce the sensitivity of the worm-gear drive to misalignment. The developed approach has been applied for K type of single-enveloping worm-gear drives and the developed theory was illustrated with a numerical example. Vilmos (2006) presented a method for computer aided loaded tooth contact analysis in different types of cylindrical worm gears. The method covered both cases- that of the fully conjugated worm gears with theoretical line contact and of the mismatched worm gears with point contact. The contact analysis of the ZK-type worm gear set with a non-90° crossing angle was investigated by Liu *et al.* (2006). Chen and Tsay (2009) derived mathematical model for ZN-type hourglass worm gear set based on the theory of gearing and gear generation mechanism. Tooth surfaces of the ZN-type hourglass worm gear set were expressed in terms of design parameters of the worm-type hourglass hob cutter.

To increase the loading capacity of worm gears, an interesting method was developed by Hoehn *et al.* (2003) that allows for the determination and optimization of the idle and load contact patterns in the design stage. By this method, the contours of worm and worm wheel were simulated point to point, taking into account the boundary conditions of the manufacturing process. A computer program, based on finite elements, was developed by Vilmos (1996a) for the calculation of stress distributions in the worm thread and the gear tooth. By using this program, the influence of the design parameters of the worm gear drive and of the load position on stresses have been investigated. Also Vilmos (2006) investigated the influence of tooth errors and shaft misalignments on loaded tooth contact in different types of cylindrical worm gears.

On the other hand, to optimize the design of worm gears, Daizhong and Datong (2003) developed an integrated approach applicable for various types of worm gears and consists of three modules: numerical analysis, three-dimensional simulation and finite element analysis. Also, development of new geometry of different types of worm gear drives have been also studied by Vilmos (1994), Litvin and De Donno (1998), Litvin *et al.* (2000, 2002), Abu Shreehah and Abdullah (2006) and many others.

Manufacturing of different types of worms has also found interest by many researchers. Fang and Tsay (1996a) investigated the characteristics of a ZE-type worm gear cut with an oversize hob cutter. Bearing contacts and kinematic errors in the ZE type worm gear set due to regrinding of oversize hob cutters are also studied. The previously mentioned study of Vilmos (1996b) investigated a new type of cylindrical worm gear drive consisting of a worm ground by a grinding wheel of double arc profile and a worm gear processed by a hob whose generator surface corresponds to the worm surface. Yuehai *et al.* (2005) proposed a method for accurate grinding of single enveloping TI worm consisting of involute helical gear and enveloping Hourglass worm. Ahn *et al.* (2006) adopted a side-milling with a single-form side mill for efficient manufacturing of single and multi-start worms. Liu *et al.* (2006) investigated the ZK-type worm ground by a grinding wheel and the worm gear generated by a straight-edged flyblade. The work done by Hiltcher *et al.* (2006) highlighted the importance of gear and worm tool compatibility and the advantage of using a hob with a worm having a different type of profile. It presented a numerical tool developed to simulate worm and gear cutting and compute the kinematics error and unloaded contact pattern for optimizing manufacturing before any operation in the workshop.

Although the above mentioned studies and many others have significantly improved the characteristics of

worm gear sets, the present work is focused on further increase in the loading capacity of the worm-gearings used in agricultural machinery which can be achieved by optimizing the load contact pattern that ensures a uniform stress distribution in the worm thread and the gear tooth. Explicating this problem will make it possible to reduce the mass and enhance the compactness of the mechanisms of agricultural machines in which worm-gearing drives are used. Individual way of problem solving is development new worm-gear sets and tools for their manufacturing.

Among the existing shapes of worm threads, those with concave profiles occupy a singular place (Abdullah, 2003; Abdullah and Abu Shreehah, 2005; Abu Shreehah and Abdullah, 2004; Vilmos, 1994). These worms have loading capacity significantly higher than other classical worm gearings including involute, Archimedes, etc., (Dudley, 1962; Taec, 1975). However, such worm-gearing did not find wide spread in industry as a result of a number of technological difficulties pertained to the application of special cutting tools for machining both worms and worm-wheels. It is proved that these difficulties could be avoided if a rigid method of incongruent generating pair is used for formation of new worm-gearings (Nadein, 1978; Nadein and Kovreshkin, 1996; Nadein and Kovreshkin, 1997). Therefore, a rigid incongruent generating pair consisting of a disk type tool and of a standard gear hobbing cutter assigned for machining involute tooth wheels was considered by Nadein and Kovreshkin (1997a, b). One direction of the continuation of these researches is the new rigid incongruent generating pair development. It will consist of a standard hobbing-cutter and toroidal tool, i.e. the disk type tool used by Nadein and Kovreshkin (1997a, b) will be replaced by a toroidal one in the present study.

MATERIALS AND METHODS

Development of the toroidal tool: In order to develop a toroidal tool, its generating surface needs to be found. For this purpose, the followings are necessary:

- In obedience to requirements of the rigid method of the incongruent generating pair, it is essential to find a common surface for both hobbing and toroidal tools.
- To find a matrix of transformation from the system of coordinates connected with the hobbing-cutter to system of coordinates, connected with the toroidal tool;
- To find the equation of the generating surface of the toroidal tool.

Determination of the common surface for both hobbing and toroidal tools: For this purpose the generating surface of the hob will be thoroughly examined. Nadein (1978) proved that the generating

surface of the hob can be considered involute, with the sufficient accuracy for general machine building.

Let us examine the depression of the hob-thread in the axial cross-section. On Fig. 1 the arrangement of the coordinate system (S_0) connected to the hob is shown. The axis Z_0 coincides with the axis of the hob, while the axis r_0 is the line of symmetry of its thread-depression. Evolvent helicoid for the right and left sides of the hobbing-cutter depression is formed by lines of a_0 and b_0 which are tangent to the base cylinder at the points of A_0 and B_0 , respectively, and are also compose the angle of γ_{b0} with the plane, that is perpendicular to the axis of helicoid Z_0 . The lines of a_0 and b_0 pass through the points A_0 and B_0 of the cylinder pitch diameter of the hobbing-cutter, respectively.

According to Fig. 1 it is necessary for formation the surface of evolvent helicoid of a hob, that the body of the cutter moved along the axis Z_0 , and the lines a_0 and b_0 revolved around the same axis Z_0 .

The equation of a helicoid with a rolled surface that generated by a screw motion of a straight line was derived using the methodology which is fully described by Litvin and Fuentes (2004). Taking into account, that after the necessary mathematical transformations, the canonical equation of the surface formed by the lines of a_0 and b_0 , is obtained as:

$$\frac{X^2}{r_{b0}^2} + \frac{Y^2}{r_{b0}^2} - \frac{\left(Z \mp \frac{e_0}{2} \mp \tan \gamma_{b0} \sqrt{r_0^2 - r_{b0}^2} \right)^2}{(r_{b0} \tan \gamma_{b0})^2} = 1 \quad (1)$$

Where;

- r_{b0} : is the radius of the base-cylinder of the hobbing-cutter
- r_0 : is the pitch radius of the hob
- γ_{b0} : is the helix angle at the base cylinder of the hob-thread
- e_0 : is the axial width of the generating depression of the hob

Equation (1) describes, in space, two identical hyperboloids of revolution of one sheet which are symmetrically located relative to axis Z_0 . The top sign in Eq. (1) answers hyperboloid of revolution of one sheet formed by the line of a_0 , and bottom - to hyperboloid of revolution of one sheet formed by line b_0 .

As one-spacing hyperboloid of revolution can be formed by a straight line or a hyperbola. Let us find the equation of such hyperbolas in an axial cut plane $Z_0S_0Y_0$ of the one-spacing hyperboloids of rotation, where $X_0 = 0$:

$$r_0^2 = \left[\begin{array}{c} 0 \\ \pm \sqrt{\left[\frac{(r_0^2 - r_{b0}^2)}{\cos \gamma_{b0}} t \pm \sqrt{r_0^2 - r_{b0}^2} \right]^2 + r_{b0}^2} \\ \pm \frac{e_0}{2} \mp \frac{\tan \gamma_{b0}}{\cos \gamma_{b0}} (r_0^2 - r_{b0}^2) \times t \\ 1 \end{array} \right] \quad (2)$$

The top and bottom signs in Eq. (2) answer hyperbolas 2 and 3 which are the axial sections of hyperboloid of revolution of one sheet, formed by the lines of a_0 and b_0 , respectively (Fig. 1).

If the toroidal tool 4 (Fig. 2) arranges within the depression limits of the hobbing-cutter 1, then the arc parts of the of the hyperbolas 2 and 3 at the rotation 5 of the toroidal tool around its axis will form the generating surface of the tool. These parts of the hyperbolas' arcs will be defined on the basis of Eq. (2), as follows:

$$r_0^{(arc)} = \left[\begin{array}{c} 0 \\ \pm \sqrt{\left[\frac{(r_0^2 - r_{b0}^2)}{\cos \gamma_{b0}} t \pm \sqrt{r_0^2 - r_{b0}^2} \right]^2 + r_{b0}^2} \\ \pm \frac{e_0}{2} \mp \frac{\tan \gamma_{b0}}{\cos \gamma_{b0}} (r_0^2 - r_{b0}^2) \times t \\ 1 \end{array} \right] \quad (3)$$

The analysis of Fig.1 and the previous calculations show, that the involute helicoid of the hob is formed by the two lines a_0 and b_0 , and the toroidal tool's generating surface is formed by the arcs of hyperbolas 2 and 3, which are present in the axial sections of the one-spacing hyperboloids of the rotation formed correspondingly by the lines a_0 and b_0 . In this case, each point of hyperbolas belongs to appropriate linear generatrix of involute helicoid. The generating pair, which consists of a hobbing-cutter and the toroidal tool, has the common tangential surface i.e., the surface of the one-spacing hyperboloids of revolution, formed by the ruled generatrix of hobbing-cutter involute helicoid. Therefore, the generating surfaces of elements of the pair are incongruent. This meets the requirements of the rigid method of the incongruent generating pair concerning the selection of the generating surfaces.

Finding the matrix of transformation:

For determining the equation of the generating surface of the toroidal tool, the matrix method of transforming the coordinates will be used. Such systems of coordinates will be examined (Fig. 2):

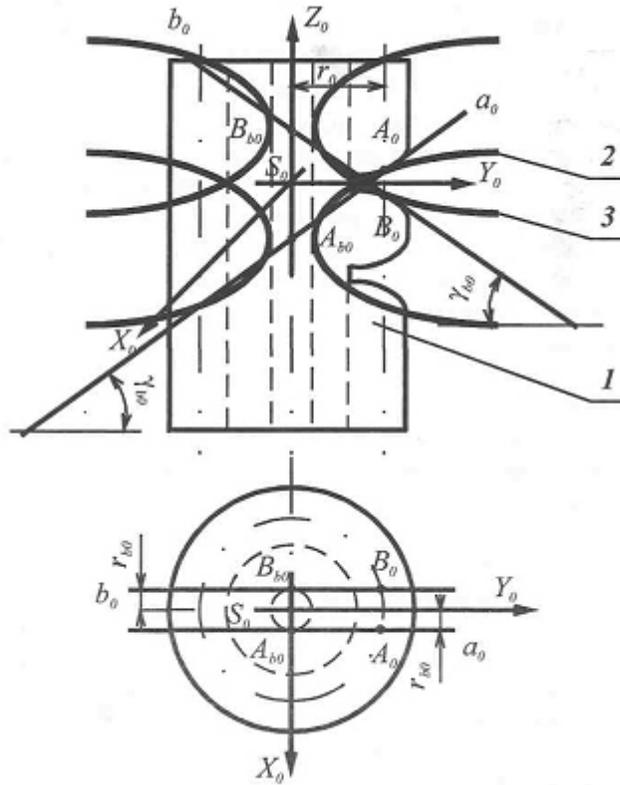


Fig. 1: Arrangement of system of coordinates: 1- the hob, 2- hyperbola in an axial section of hyperboloid of revolution of one sheet formed by the line of a_0 ; 3- hyperbola in an axial section of hyperboloid of revolution of one sheet formed by the line of b_0

S_0 - stationary system of coordinates, S_3 and S_4 stationary systems of coordinates relative to S_0 ; S_u - moving system of coordinates relative to S_0 .

The coordinate system S_3 is located so that axis r_3 coincides with the axis r_0 , axis Z_3 is parallel to Z_0 , and the axis of X_3 is parallel to X_0 . The origin of the coordinate system S_3 is located at a distance a from the origin of the system of coordinates S_0 in the positive direction of axis r_0 (Fig. 2).

The origin of the coordinate system S_4 coincides with the origin of the system of coordinates S_3 , and the axis r_4 coincides with the axis r_3 . The axes X_4 and Z_4 are revolved to an angle of γ_u around the axis of X_3 and Z_3 . When looking from the positive direction of the axis Z_4 , the angle γ_u lies between the axis X_3 and X_4 , or Z_3 , however Z_4 lies counterclockwise and has a positive value (Fig. 2).

The origin of the system of coordinates S_u coincides with the origin of the system of coordinates S_4 , and the axis Z_u coincides with the axis Z_4 . The system of S_u coordinates rotates around the axis Z_u so, that turning from X_4 to X_u and from Y_4 to Y_u within the angle of ϕ_u , it occurs counterclockwise when looking from the positive direction of the axis Z_u (Fig. 2).

The transformation matrix from the system of coordinates S_0 to the system of coordinates S_u is determined by the formula (Litvin and Fuentes, 2004):

$$M_{u0} = M_{u4} \cdot M_{43} \cdot M_{30} \quad (4)$$

where;

M_{30} : the transformation matrix from S_0 to S_3 system of coordinates

M_{43} : the transformation matrix from S_3 to S_4 the system of coordinates

M_{u4} : the transformation matrix from S_4 to S_u system of coordinates

The transformation matrix from S_0 to S_3 system of coordinates can be defined according to Fig. 2 as:

$$M_{30} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

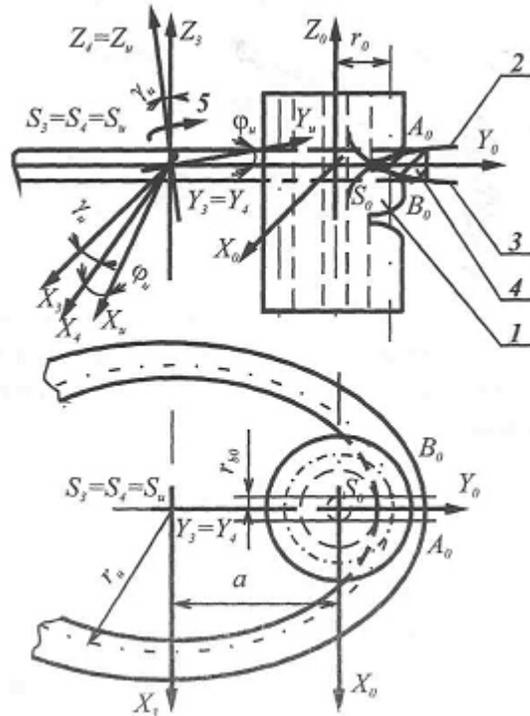


Fig. 2: The scheme of formation on the generating surface of the toroidal tool: 1- the hob; 2- the section of hyperbola in the axial section of the hyperboloid of revolution of one sheet, formed by the line a_0 ; 3- the section of hyperbola in the axial section of the one-spacing hyperboloid of revolution, formed by line b_0 ; 4- toroidal tool; 5- direction of rotation

The transformation matrix from coordinate system S_3 to S_4 is defined as:

$$M_{43} = \begin{bmatrix} \cos \gamma_u & 0 & -\sin \gamma_u & 0 \\ 0 & 1 & 0 & 0 \\ \sin \gamma_u & 0 & \cos \gamma_u & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Also, the transformation matrix from coordinate system of S_4 to S_u corresponding to Fig. 2 is defined as:

$$M_{u4} = \begin{bmatrix} \cos \varphi_u & -\sin \varphi_u & 0 & 0 \\ \sin \varphi_u & \cos \varphi_u & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

Consequently, by substituting Eq. (5), (6) and (7) in Eq. (4) and after the necessary mathematical transformations, the transformation matrix from coordinate system S_0 to S_u will be equaled to:

$$M_{u0} = \begin{bmatrix} \cos \varphi_u \cos \gamma_u & -\sin \varphi_u & -\cos \varphi_u \sin \gamma_u & -a \sin \varphi_u \\ \sin \varphi_u \cos \gamma_u & \cos \varphi_u & -\sin \varphi_u \sin \gamma_u & a \cos \varphi_u \\ \sin \gamma_u & 0 & \cos \gamma_u & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

RESULTS AND DISCUSSION

Finding the equation of the generating surface of the toroidal tool: The equation of the generating surface of the toroidal tool could be found in the following form:

$$r_u^{(u)} = M_{u0} r_0^{(arc)} \quad (9)$$

After the substitution (3) and (8) in the Eq. (9) and the necessary mathematical transformations, we obtain:

$$r_u^{(u)} = \left[-\sin \varphi_u \sqrt{\left[\frac{(r_0^2 - r_{b0}^2)}{\cos \gamma_{b0}} t \mp \sqrt{r_0^2 - r_{b0}^2} \right]^2 + r_{b0}^2 + a} \right]$$

$$\begin{aligned} & \cos \varphi_u \sin \gamma_u \left[\pm \frac{e_o}{2} \mp \frac{\tan \gamma_{bo}}{\cos \gamma_{bo}} (r_0^2 - r_{b0}^2) \times t \right] \\ & \cos \varphi_u \left[\sqrt{\left[\mp \frac{(r_0^2 - r_{b0}^2)}{\cos \gamma_{bo}} t \pm \sqrt{r_0^2 - r_{b0}^2} \right]^2 + r_{b0}^2 + a} \right] \\ & - \sin \varphi_u \sin \gamma_u \left[\pm \frac{e_o}{2} \mp \frac{\tan \gamma_{bo}}{\cos \gamma_{bo}} (r_0^2 - r_{b0}^2) \times t \right] \\ & - \cos \gamma_u \left[\pm \frac{e_o}{2} \mp \frac{\tan \gamma_{bo}}{\cos \gamma_{bo}} (r_0^2 - r_{b0}^2) \times t \right] \end{aligned} \quad (10)$$

Equation (10) describes the generating surface of the toroidal tool. The top sign in this equation replies to the left (L) side of the toroidal tool, which is located in the positive direction of axis Z_u , whereas the bottom sign - corresponds to right (R) side of the toroidal tool, which is located in the negative direction of axis Z_u (Fig. 2).

It is clear from Eq. (10) that the contour of the generating surface of the developed toroidal tool is a function of several factors: the base and pitch radiuses of the hobbing cutter; the axial width of the helix depression of the hob; the helix angle at the base cylinder of the hob-thread; the center distance measured between the axes of the hob and toroidal tool; and the position of the toroidal tool in relation with the hob (the angles of rotation of system of coordinates connected to the toroidal tool relative to that connected to the hob).

CONCLUSION

The principal conclusions drawn from this work are:

- The ever increasing demand for highly loaded gear power transmission, that comes from the trend to use power transmission systems with improved performances together with lightness and reduced volumes, lead engineers to look for new design and manufacturing solutions, able to overcome the ones traditionally used and known as reliable ones. In other words, it is necessary to use gears with smaller dimensions and to load them with higher torque and power, that is to say that it is necessary to increase the ratio power/mass (volume) of the system.
- Authors have suggested a new method for machining concave profile of worms' thread. The proposed method uses a rigid incongruent generating pair which consists of a standard hobbing-cutter and toroidal tool. The obtained scientific results can be

used to manufacture tools for worms and worm gears of new worm-gearing sets with concave profile of worms' thread.

- The generating surface of the developed toroidal tool intended for machining the concave worms' thread was defined in terms of matrix transformation from the system of coordinates connected with the hob to the system of coordinates connected with the toroidal tool as illustrated in Fig. 2. For this, the common surface for both hobbing and toroidal tools has been presented in the form of hyperboloids of revolution of one sheet formed by generatrix of involute helicoid of the hob in correspondence to Fig. 1.
- The generating surface of the toroidal tool can be defined through a simple Eq. (10). This simplifies the analysis, projection and geometrical estimation of wormgearing.
- There are many factors that influence at the mechanical behaviour of gears. The gear's mechanical phenomenons are greatly conditioned by teeth's profile (worm's thread in our case). This is in accordance with the fact that the teeth (thread) profile or geometry is one of several influential values for load distribution among simultaneously meshing tooth pairs. Also, this is in accordance with the fact that the load distribution is the most influential mechanical phenomenon for the deformation and stress state of gear pairs as well as for their load capacity. Thus, it is believed that the loading capacity of the worm gear sets can be increased when processing with the developed tool which may be explained in terms of uniform stress distribution in the worm thread and the gear tooth.
- To reveal it clearly, the advantages of the proposed toroidal tool and the considered new type of wormgearings with concave profiles of the worm thread need additional experimental investigation which intended to be performed by the authors in the future work.

ACKNOWLEDGMENT

The authors thank the administration of their affiliation: Tafila Technical University for its support in preparing this article.

REFERENCES

- Abdullah, R., 2003. Modeling of the worm gearing with concave shape turn, Proceedings of the Industrial Simulation Conference ISC' 2003, Valencia, Spain, pp: 188-192.
- Abdullah, R. and T. Abu Shreehah, 2005. Finishing the concave shape of the worm thread. Mach. Sci. Technol., 9: 589-599.

- Abu Shreehah, T. and R. Abdullah, 2004. Mathematical aspect for worm grinding using a toroidal tool. *J. Appl. Sci.*, 4(4): 526-530.
- Abdullah, R. and T. Abu Shreehah, 2006. Modification of geometry and technology of cylindrical worms. *Mach. Sci. Technol.*, 10: 539-547.
- Argyris, J., M. De Donno and F. Litvin, 2000. Computer program in visual basic language for simulation of meshing and contact of gear drives and its application for design of worm gear drive. *Comput. Meth. Appl. Mech. Eng.*, 189: 595-612.
- Chen, K. and C. Tsay, 2009. Mathematical model and worm wheel tooth working surfaces of the ZN-type Hourglass worm gear set. *Mech. Mach. Th.*, 44: 1701-1712.
- Childs, P., 2004. *Mechanical Design*. 2nd Edn., Elsevier Ltd.
- Ahn, J., D. Kang, M. Lee, H. Kim, S. Kim and K. Cho, 2006. Investigation of cutting characteristics in side-milling a multi-thread worm shaft on automatic lathe. *CIRP Ann-Manuf. Technol.*, 55(1): 63-66.
- Daizhong, S. and Q. Datong, 2003. Integration of numerical analysis, virtual simulation and finite element analysis for the optimum design of worm gearing. *J. Mater. Process. Tech.*, 138(1-3): 429-435.
- De Donno, M. and F. Litvin, 1999. Computerized design and generation of worm-gear drives with stable bearing contact and low transmission errors. *J. Mech. Design*, 121: 573-578.
- Dudley, D., 1962. *Gear Handbook, The Design, Manufacture and Application of Gears*. McGraw-Hill Publishing Company, New York.
- Fang, H. and C. Tsay, 1996a. Effects of the hob cutter regrinding and setting on ZE-type worm gear manufacture. *Int. J. Mach. Tools Manuf.*, 36(10): 1123-1135.
- Fang, H. and C. Tsay, 1996b. Mathematical model and bearing contacts of the ZK-type worm gear set cut by oversize hob cutters. *Mech. Mach. Theory*, 31(3): 271-282.
- Fang, H. and C. Tsay, 2000. Mathematical model and bearing contacts of the ZK-type worm gear set cut by oversize hob cutters. *Mech. Mach. Theory*, 35(12): 1689-1708.
- Hiltcher, Y., M. Guingand and J. Vaujany, 2006. Numerical simulation and optimization of worm gear cutting. *Mech. Mach. Theory*, 41(9): 1090-1110.
- Hoehn, B., K. Steingroever and M. Lutz, 2003. Determination and optimization of the contact, pattern of worm gears. *Gear Technol.*, March/April: 12-17.
- Khurmi, R., and J. Gupta, 2005. *A Textbook of Machine Design*. 13th Edn., S. Chand and Co Ltd., India.
- Litvin, F. and A. Fuentes, 2004. *Gear Geometry and Applied Theory*. 2nd Edn., Cambridge University Press.
- Litvin, F., G. Argentieri, M. De Donno and M. Hawkins, 2000. Computerized design, generation and simulation of meshing and contact of face worm-gear drives. *Comput. Method. Appl. M.*, 189(3): 785-801.
- Litvin, F. and M. De Donno, 1998. Computerized design and generation of modified spiroid worm-gear drive with low transmission errors and stabilized bearing contact. *Comput. Method. Appl. M.*, 162(1-4): 187-201.
- Litvin, F., A. Nava, Q. Fan and A. Fuentes, 2002. New geometry of face worm gear drives with conical and cylindrical worms: Generation, simulation of meshing, and stress analysis. *Comput. Method. Appl. M.*, 191(27-28): 3035-3054.
- Liu, C., J. Chen, C. Tsay and Y. Ariga, 2006. Meshing simulations of the worm gear cut by a straight-edged flyblade and the ZK-type worm with a non-90° crossing angle. *Mech. Mach. Theory*, 41(8): 987-1002.
- Nadein, V., 1978. Comparative tests of the worm gear cut by a rigid incongruent generating pair // perfection of designs of machines and methods of processing of parts. The collection of proceedings of Chelyabinsk Polytechnical Institute, Chelyabinsk, pp: 68-71.
- Nadein, V. and N. Kovreshkin, 1996. About one way of formation of worm gears with concave profile of the worm thread // Automation of designing and manufacture of products in mechanical engineering. Reports of the International Scientifically-Practical Conference, Lugansk, pp: 107.
- Nadein, V. and N. Kovreshkin, 1997a. To a question on increase of adaptability to manufacture and loading capacity of worm gearings. The Collection of Proceedings of the Kirovograd Institute of Agricultural Mechanical Engineering, Kirovograd, pp: 147-149.
- Nadein, V. and N. Kovreshkin, 1997b. Generating pair for cutting worm gearings with concave profile of the worm thread // Problems of quality and durability of tooth gearings and reducers. Reports of the International Scientifically-Technical Conference, The Kharkov State Polytechnical Institute, Kharkov, pp: 68-74.
- Seol, I., 2000. The design, generation, and simulation of meshing of worm-gear drive with longitudinally localized contacts. *J. Mech. Design*, 122(2): 201-206.
- Sclater, N. and N. Chirionis, 1996. *Mechanisms and Mechanical Devices*. 2nd Edn., McGraw-Hill.
- Taev, B., 1975. *Production of Toothed Gears*. Mashinostroene, Moscow, USSR.
- Vilmos, S., 1994. A new worm gear drive with ground double arc profile. *Mech. Mach. Theory*, 29(3): 407-414.
- Vilmos, S., 1996a. Displacements in worm gears with ground concave worm profile. *Mech. Mach. Theory*, 31(8): 1131-1140.

- Vilmos, S., 2006. Influence of tooth errors and shaft misalignments on loaded tooth contact in cylindrical worm gears. *Mech. Mach. Theory*, 41(6): 707-724.
- Vilmos, S., 1996b. Stress analysis in worm gears with ground concave worm profile. *Mech. Mach. Theory*, 31(8): 1121-1130.
- Yuehai, S., Z. Huijiang, B. Qingzhen and W. Shuren, 2005. Method of accurate grinding for single enveloping TI worm. *Sci. China Series E: Eng. Mater. Sci.*, 48(4): 430-440.