

Equalisation of Transient Temperature Profile Within the Fuel Pin of a Miniature Neutron Source Reactor (MNSR) During Total Loss of Coolant

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Abstract: Transient temperature distributions in cylindrical fuel element of Ghana Research Reactor-1 (GHARR-1) Miniature Neutron Source Reactor (MNSR) following sudden total loss of cooling have been investigated. The loss of cooling in the reactor core resulting from a blockage of the inner orifice of coolant flow channels was assumed to occur during normal operations and led to sudden shut down of the reactor. The objective was to analyse the transient behaviour by solving analytically the heat transfer equation using Bessel functions and also develop from first principle the transient temperature equations for the fuel element. Results obtained during a sudden total loss of cooling showed a high transient temperature distribution at the centre of the fuel element, with the surface of the fuel clad recording the least temperature. The transient temperature distribution decreased from the centre of the fuel element to the surface of the fuel clad and followed a parabolic decay pattern which after increase in time followed an equalisation pattern. During sudden shut down, since there was no heat generated and decay heat, the rate at which the fuel element was cooled was directly proportional to time.

Key words: Bessel functions, fuel clad, heat transfer, loss of cooling, parabolic, temperature profile distribution

INTRODUCTION

In steady state operation of the reactor, fuel element temperature distribution is determined by thermal balance between heat generated and heat transferred to the coolant, since the neutron flux level is limitless. Provided that there is adequate heat-removal systems, the temperature in the core does not exceed specific safety limits, and damage to the fuel pins and other reactor materials would be prevented (Akaho and Maaku, 2002; Akaho, 2006).

Safety analysis ensures that the material integrity of the outer orifice of the coolant and walls of the reactor vessel, are maintained so as to sustain the high temperatures of the coolant by transient heat removal from the system (SAR, 2004).

The fuel pin thermal operating limits are therefore set such that, the temperatures do not exceed the melting point of the fuel clad. Which allow no radioactive leakage or structural failure during steady state and/or any anticipated operational transient.

Heat removal from the Ghana Research Reactor -1 (GHARR-1) is accomplished by natural convective flow of coolant during steady state operation as shown in Fig. 1 (Akaho and Maaku, 1996). Since measurement of

steady fluid flow variables such as temperature, density, velocity and pressure are not amenable to direct measurements, modelling and simulations have been conducted in this study.

Transient phenomenon in nuclear reactors: Transient phenomenon in nuclear reactors describe any significant deviation from Steady State operation and/or from the normal value (either high or low value) of one or more of the important operating parameters, such as system temperatures and pressures, thermal power level, coolant flow rate, equipment failure, etc. If the transient is minor and within permissible operating limits of the reactor, the controls of the reactor adjust automatically to compensate for the deviation. If the thermal power should exceed the limit value or if abnormal conditions which might endanger the system should arise, the reactor protection system would cause the reactor trip (or 'scram'). The purpose of the protection system is to shut the reactor down or maintain safe operation condition, in the event of a system transient or malfunction that might cause damage to the reactor core, most likely from overheating.

For an uncooled fuel element, the heat from sustained fission reaction would raise the temperature to the point where damage might occur, and therefore the rates of heat

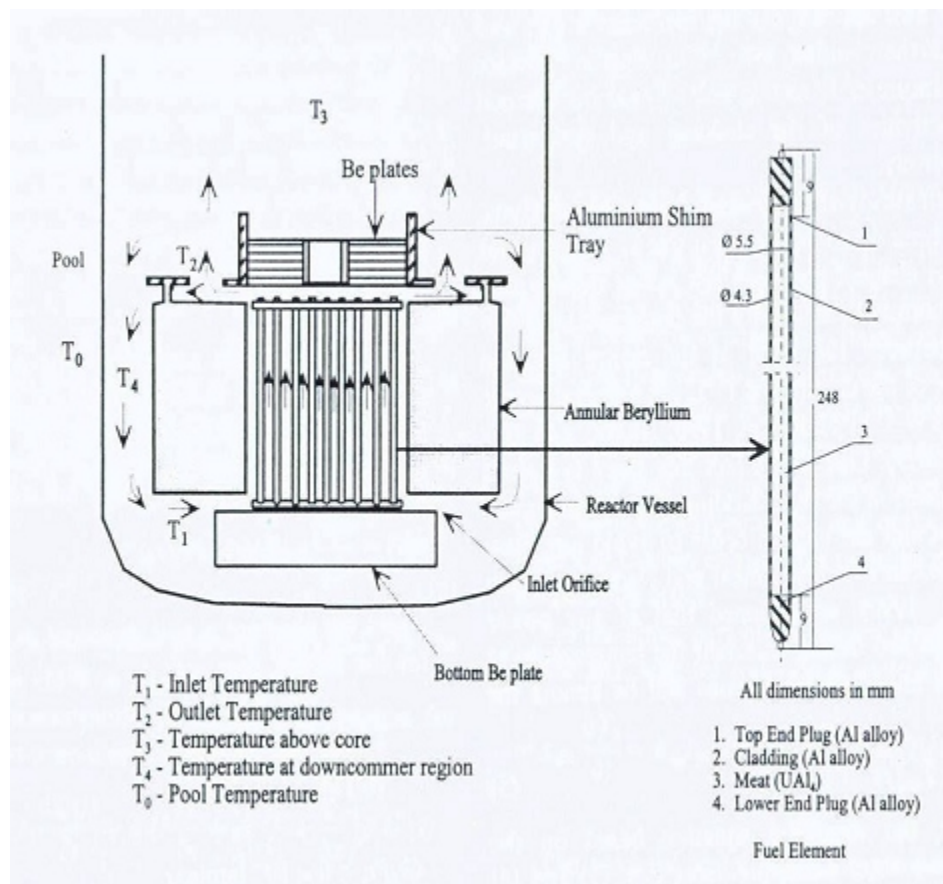


Fig. 1: Schematic diagram of natural convective flow of coolant water within the Reactor vessel and enlarged view of fuel element (Yue-Wen, 1989).

generation and of heat removal must be balanced during reactor operation.

The maximum operation power is limited by temperature within the reactor which are set by changes in the properties of the fuel elements, the coolant flow, moderator, allowable thermal stress in some parts of the system, and other thermal effects (Wheeler, 1976). By thermal analysis design, maximum permissible temperature are established to ensure the cooling system is adequate under all anticipated operating conditions, by estimating the magnitude and distribution of the heat sources in the reactor, and determining the temperature differences along various paths of heat flow within the system.

In this research, the mathematical formulation of transient heat generation in the GHARR-1 core was investigated to obtain the transient temperature distribution in the fuel element, originating from loss of total coolant leading to reactor shut down which was caused by total blockage of the reactor coolant inlet orifice.

MATERIALS AND METHODS

The study was done in the year 2009 at the Graduate School of Nuclear and Allied Sciences of the University of Ghana, in Ghana, West Africa.

The transient conditions dealt with the Ghana research reactor-1 being operated at maximum power (30 kW) during steady state and suddenly shutdown by total loss of coolant without any subsequent internal heat generated from the fuel pins, and no presence of residual heat or decay heat. The mathematical equations of transient temperature distribution were developed, analyzed, and solved by analytical techniques using Bessel functions.

Assumptions for transient analysis: In GHARR-1 fuel there is no gap between the fuel pellet and the surface of the cladding, the heat generated by nuclear fission is conducted through the fuel meat to the fuel cladding, and transferred to the coolant by convection (Winterton, 1981; Todreas *et al.*, 2001). A radial heat conduction model was

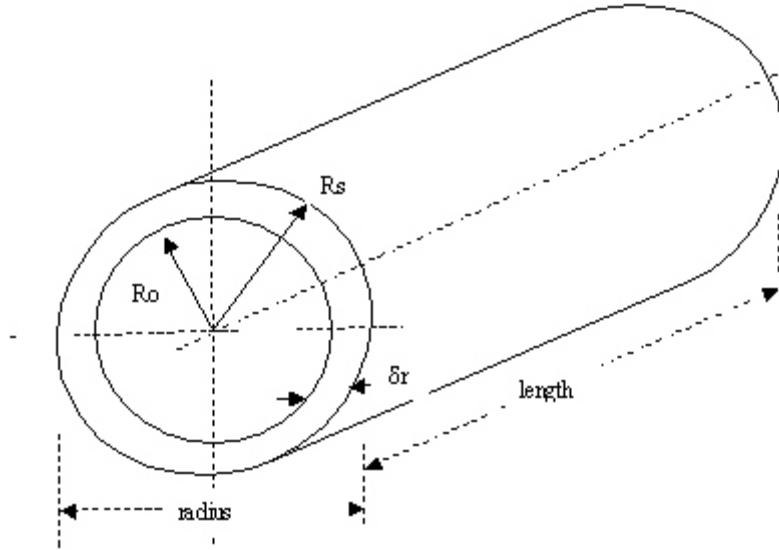


Fig. 2: A cross section of the fuel pin

analyzed to determine the fuel heat flux and transient temperature distribution within the fuel element following sudden loss of total coolant and sudden shut down of reactor. The thermal energy and transient temperature distribution was modelled based on the following assumptions (Hainoun and Alissa, 2005):

- Axial heat conduction was negligible
- Volumetric heat generation was uniformly distributed axially over the fuel pellet cross-section
- Fuel element length to diameter ratio was large, hence axial temperature gradient was small, and axial distribution of temperature remains constant
- Two dimensional cylindrical model was adopted for calculating the temperature distribution in the core
- Thermal conductivity in the fuel pellet and the cladding depended on the temperature
- Volumetric heat capacity and density of fuel pellet and cladding were temperature dependent

Evaluating radial heat conduction within the fuel pellet:The radial flow of heat in a cylindrical fuel tube was obtained by considering a heat balance on an annular element, of thickness and unit length Fig. 2.

The rate of heat flow into the element considering the circumference of the fuel rod at radius r expressed as:

$$\frac{\partial Q}{\partial t} = -kA \frac{\partial T}{\partial r} = -2\pi rk \frac{\partial T}{\partial r}$$

and the rate of heat flow out of the element at was:

$$\frac{\partial Q}{\partial t} = 2\pi k(r + dr) \frac{\partial T}{\partial r}$$

$$Q(r) = 2k\pi \left(r \frac{\partial T}{\partial r} + dr \frac{\partial T}{\partial r} \right) \quad (1)$$

$$\frac{\partial Q(r)}{\partial r} = 2\pi k \left(\frac{\partial T}{\partial r} + \frac{\delta^2 T}{\delta r^2} \right) (r + dr)$$

$$\frac{\partial Q(r)}{\partial r} = 2\pi k(r + dr) \left(\frac{\partial T}{\partial r} + \frac{\delta^2 T}{\delta r^2} dr \right) \quad (2)$$

the net rate at which heat was conducted into the fuel element was obtained ignoring higher powers of two:

$$\frac{\partial Q(r)}{\partial r} = 2k\pi r \frac{\delta^2 T}{\delta r^2} dr + 2k\pi dr \frac{\partial T}{\partial r} \quad (3)$$

The heat generated caused the fuel element to warm up, at a rate $\delta T/\delta t$. Introducing the rate at which heat was generated within the fuel element, $2\pi r dr H$ into Eq. (3) and dividing through by $2\pi r dr$, yields:

$$\alpha \frac{\partial T}{\partial t} = k \left(\frac{\delta^2 T}{\delta r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + H$$

$$\alpha \frac{\partial T}{\partial t} = \frac{k}{r} \frac{\partial}{\partial r} r \frac{\partial T}{\partial r} + H \quad (4)$$

For a period after the reactor has been shut down, the decay heat could be ignored, $H = 0$. Introducing the thermal diffusivity, $\alpha = k/\rho c$ then Eq. (4) becomes:

$$\frac{\rho c}{\rho c} \frac{\partial T}{\partial t} = \frac{\alpha \rho c}{r \rho c} \frac{\partial}{\partial r} r \frac{\partial T}{\partial r} + 0 \tag{5}$$

The radial temperature distribution decayed as function of time as is evident in Eq. (6):

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial t} + \frac{\partial^2 T}{\partial r^2} = \nabla^2 T \tag{6}$$

Analytical solution of temperature distribution by Bessel functions: The general solution of Eq. (6) was derived, and the specific solution for GHARR-1 fuel element was provided. For a circular plate of unit radius with insulated plane faces, if the initial temperature was $F(r)$, and the rim was kept at zero temperature, then the temperature at any time was $T(r, t)$. The boundary conditions for the boundary value problem were:

$$T(1,t) = 0, \quad T(r,0) = F(r)$$

By separation of variables, $T(r,t) = P(r)S(t) = PS$, Eq. (6) was transformed and yields after dividing through by αPS :

$$\frac{S''}{\alpha S} = \frac{P''}{P} + \frac{1}{r} \frac{P'}{P} = -\beta^2$$

The first ODE, $S'' = -\alpha \beta^2 S$ has a general solution $S = C_1 e^{-\alpha \beta^2 t}$, while $P'' + \frac{1}{r} P' + \beta^2 P = 0$ is a Bessel function of first kind and zero order ($n = 0$) with solution $P = AJ_0(\beta r) + DY_0(\beta r)$, such that:

$$T(r,t) = C e^{-\alpha \beta^2 t} [AJ_0(\beta r) + DY_0(\beta r)]$$

Since $T = PS$ is bounded at $r = 0$, $D = 0$ and the second term became irrelevant, i.e. in the $\lim_{r \rightarrow 0} Y_0(\beta r) \rightarrow -\infty$ and as $Y_0(\beta r)$ is undefined for $x \leq 0$ $D = 0$. Hence

$$T(r,t) = A_1 e^{-\alpha \beta_1^2 t} J_0(\beta_1 r) \text{ where } A_1 = AC.$$

From the first boundary conditions, $T(1, t) = 0$, $A_1 e^{-\alpha \beta_1^2 t} J_0(\beta_1 a) = 0$ for which $J_0(\beta_1 a)$ has positive roots, $\beta_1, \beta_2, \beta_3, \dots, \beta_n$.

so that $T(r,t) = A e^{-\alpha \beta_n^2 t} J_0(\beta_n r)$, $n = 1, 2, 3, 4, \dots$ and by superposition, the solution was written as:

$$T(r,t) = \sum_{n=1}^{\infty} A_n e^{-\alpha \beta_n^2 t} J_0(\beta_n r)$$

The second boundary condition was used to find A_n , since $T(r,0) = F(r) = \sum_{n=1}^{\infty} A_n J_0(\beta_n r)$ and for $n = 0$ and by the defining property of Bessel function:

$$A_n = \frac{2}{J_1^2(\beta_n)} \int_0^1 r F(r) J_0(\beta_n r) dr \tag{9}$$

the general solution to transient conduction Eq. (6) becomes:

$$T(r,t) = \sum_{n=1}^{\infty} \left\{ \left[\frac{2}{J_1^2(\beta_n)} \int_0^1 r F(r) J_0(\beta_n r) dr \right] e^{-\alpha \beta_n^2 t} J_0(\beta_n r) \right\}$$

Specific solution of $T(r, t)$ for GHARR-1 fuel rod (LOCA)

For the fuel element $r = a$, $\frac{dT}{dr} = 0$ and by transforming:

$$\lambda_n r = \beta_n \frac{r}{a}, \quad \lambda_n = \frac{\beta_n}{a}$$

the $T(r, t)$ in Eq. (10) was expressed in (2) dimensionless parameters, $\frac{kt}{a^2}$ (time dependant) and r/a radius dependence, which would be valid, regardless of the size or composition of the cylinder.

The initial heat generated within the fuel rod during steady state prior to the shut down of the reactor is manifested as temperature $f(r) = \frac{R}{4\pi k} \left(1 - \frac{r^2}{a^2} \right)$, where

R is the rate of heat generated per unit radius and k is the thermal conductivity.

From Eq. (10), integrating from 0 to $r = a$ and $\lambda_n = \beta_n / a$

$$I = \int_0^1 r f(r) J_0(\beta_n r) dr \quad n = 0, 1, 2, 3$$

$$I = \int_0^a r f(r) J_0\left(\beta_n \frac{r}{a}\right) dr \quad n = 0, 1, 2, 3..$$

$$= \int_0^a r \frac{R}{4\pi k} \left(1 - \frac{r^2}{a^2} \right) J_0\left(\beta_n \frac{r}{a}\right) dr$$

$$= \frac{R}{4\pi k a^2} \left[\int_0^a a^2 r J_0\left(\beta_n \frac{r}{a}\right) - \int_0^a r^3 J_0\left(\beta_n \frac{r}{a}\right) dr \right]$$

By standard integral of Bessel functions (Zhang *et al.*, 1996):

$$\beta \int x^y J_{y-1}(\beta x) dx = x^y J_y(\beta x) + c$$

and

$$I - \frac{R}{4\pi k a^2} \left[\frac{a^4}{\beta^4} J_1(\beta) - \frac{a^4}{\beta^2} J_3(\beta) + \frac{2a^4}{\beta} J_5(\beta) \right] - \frac{R a^2}{4\pi k} J_1(\beta) \tag{11}$$

Now J_2 was related to J_0 generally as:

$$2nJ_n(z) = zJ_{n-1}(z) + zJ_{n+1}(z)$$

So for $n = 1$ and $z = \beta n$

$$2J_1(\beta_n) = \beta_n J_0(\beta_n) + \beta_n J_2(\beta_n) \text{ or}$$

$$J_2(\beta_n) = -J_0(\beta_n) + \frac{2}{\beta_n} J_1(\beta_n)$$

and since the βn were roots of $J_1(\beta) = 0$.

Hence

$$T(r,t) = \sum_{n=1}^{\infty} \frac{2}{J_1^2(\beta_n/a)} \int_0^a r f(r) J_0\left(\beta_n \frac{r}{a}\right) dr e^{-\beta_n^2 a^2 t/a^2} J_0\left(\beta_n \frac{r}{a}\right)$$

$$T(r,t) = \frac{R a^2}{4\pi k} \sum_{n=1}^{\infty} \frac{2}{J_1^2(\beta_n/a)} J_2(\beta_n) e^{-\beta_n^2 a^2 t/a^2} J_0\left(\beta_n \frac{r}{a}\right) \tag{12}$$

The radial transient temperature distribution in the cylindrical fuel rod Eq. (11) decayed from the centre ($r = r_0$) to clad ($r = a$), such that $T(r,t) = f(r_0,t)$ and was expressed by adding a step function $\frac{2}{a^2} \int_0^a r f(r) dr$ to the series Eq. (11) (Basmadjian and Farnood, 2006; Stroud and Dexter, 2003). Substituting the integral Eq. (15) and replacing $J_1^2(\beta_n)$ in addition:

$$T(r,t) = \frac{2}{a^2} \int_0^a r f(r) dr - \sum_{n=1}^{\infty} e^{-\beta_n^2 a^2 t/a^2} \frac{2 J_0(\beta_n r/a)}{J_1^2(\beta_n/a)} \int_0^a r f(r) J_0(\beta_n r/a) dr \tag{13}$$

Where, the integrals $\frac{2}{a^2} \int_0^a \frac{R}{4\pi k} r \left(1 - \frac{r^2}{a^2}\right) dr = \frac{R}{8\pi k}$

and. $\int_0^a r f(r) J_0(\beta_n r/a) dr = \frac{2}{\beta_n^2} \frac{R}{4\pi k} J_0(\beta_n)$

Substituting the integrals into Eq. (13) the solution became:

$$T(r,t) = \frac{R}{4\pi k} \left(\frac{1}{2} - 4 \sum_{n=1}^{\infty} e^{-\beta_n^2 a^2 t/a^2} \frac{J_0(\beta_n r/a)}{J_0(\beta_n)} \frac{1}{\beta_n^2} \right) \tag{14}$$

The solution of the temperature distribution in a dimensionless form was expressed as:

$$\frac{T}{R/4\pi k} = \frac{1}{2} - 4 \sum_{n=1}^{\infty} e^{-\beta_n^2 a^2 t/a^2} \frac{J_0(\beta_n r/a)}{J_0(\beta_n)} \frac{1}{\beta_n^2} \tag{15}$$

where, β_n is the positive root of the Bessel function, T is the temperature J_0 is the Bessel function of zero order, R is the heat generated per unit length, a is the radius of the fuel rod, is the thermal diffusivity of the fuel, a is the time, t is variance radius, and r is the thermal conductivity of the fuel.

Equation (15) represents the derived equation to determine the temperature profile in the fuel pin of a MNSR reactor during LOCA.

RESULTS AND DISCUSSION

Data of analytical solution: The data obtained was from computing the temperature profile within the fuel pellets from variance radius r/a (from a range of 0 to 0.0275) for the time period of 5 sec (from $t = 1$ to 5) during transient (LOCA) using the derived Eq. (15).

Graphical representation of data obtained: The graphical presentation of the analytical obtained data from the analytical solution if presented in the graphs of Fig. 3 to 8.

Figure 3 represents the plot of r/a against $\frac{T}{R/4\pi k}$

for $t=1$, the centre of the fuel elements zero ($r = 0$) where the transient temperature was maximum with a value of approximately 0.89, but reduces to 0.15 at the surface of the fuel clad, since most of the heat generated was concentrated in the centre of the fuel element.

Figure 4 represents the graph at $t = 2$ and shows that at the centre of the fuel elements ($r = 0$) the transient temperature was maximum with a value of approximately 0.70 but reduced to 0.32 at the fuel clad surface.

Figure 5 represents the graph at $t = 3$, from the graph the centre of the fuel elements zero ($r = 0$) depicts that the

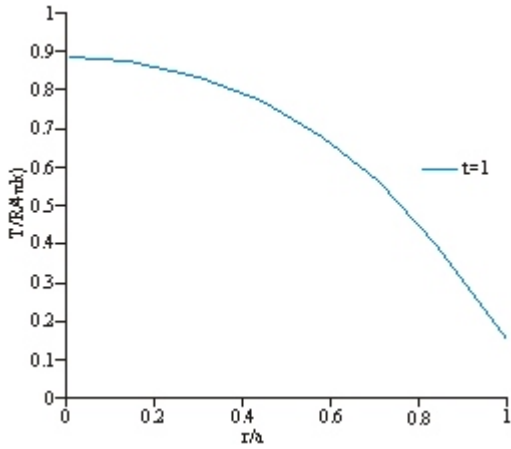


Fig. 3: Progressive transient temperature profile within the fuel pin during LOCA for time, t = 1

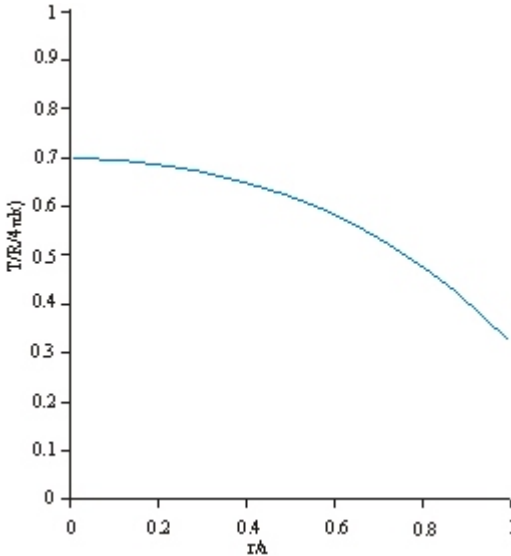


Fig. 4: Progressive transient temperature profile within the fuel pin during LOCA for time, t = 2

transient temperature profile was maximum with a value approximately 0.60, but reduces at the surface of the fuel clad to approximately 0.42. The ratio of temperature profile at the centre of the fuel element and that of the clad reduces as compared to that of the graph at time $t = 1, 2$.

Figure 6 represents the graph at $t = 4$, from the graph the centre of the fuel elements zero ($r = 0$) depicts that the transient temperature profile was maximum with a value approximately 0.532, but reduces at the surface of the fuel clad to approximately 0.47.

Figure 7 represents the graph of $t = 5$, from the graph the centre of the fuel elements zero ($r = 0$) depicts that the

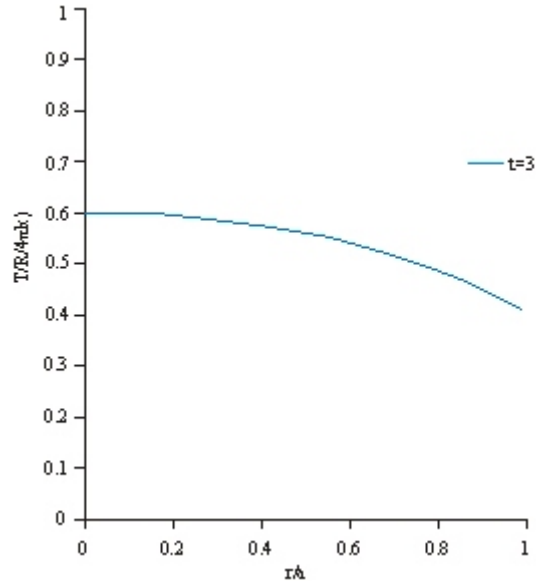


Fig. 5: Progressive transient temperature profile within the fuel pin during LOCA for time, t = 3

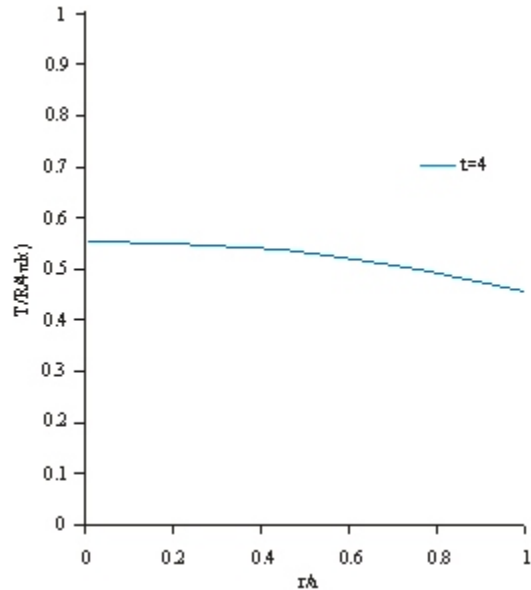


Fig. 6: Progressive transient temperature profile within the fuel pin during LOCA for time, t = 4

transient temperature profile was maximum with a value approximately 0.52, but reduces at the surface of the fuel clad to approximately 0.4854, and almost equalises.

In general all the plots indicates a parabolic thermal decay for $t = 1$ to 5, the temperature at the centre of the fuel rod reduced, while that of the surface clad increased. Initial equalization was established, and the temperature distribution within the fuel element was uniform (Fig. 8).

Table 1: Analytical data of transient temperature profile for various values of time

r/a	$J_0(\alpha_n r/a)$	$J_0(\alpha_n)$	t = 1	t = 2	t = 3	t = 4	t = 5
0.0073	0.9999	-0.8919	0.8517	0.6595	0.5723	0.5328	0.5149
0.0655	0.9938	-0.8919	0.8495	0.6585	0.5719	0.5326	0.5148
0.1382	0.9722	-0.8919	0.8419	0.6551	0.5703	0.5319	0.5145
0.2109	0.9347	-0.8919	0.8287	0.6491	0.5676	0.5307	0.5139
0.2836	0.8803	-0.8919	0.8096	0.6404	0.5637	0.5289	0.5131
0.3564	0.8081	-0.8919	0.7842	0.6289	0.5585	0.5265	0.512
0.4291	0.7166	-0.8919	0.752	0.6143	0.5518	0.5235	0.5107
0.5018	0.604	-0.8919	0.7124	0.5964	0.5437	0.5198	0.509
0.5745	0.4686	-0.8919	0.6648	0.5748	0.5339	0.5154	0.507
0.6473	0.3084	-0.8919	0.6085	0.5492	0.5223	0.5101	0.5046
0.72	0.1211	-0.8919	0.5426	0.5193	0.5088	0.504	0.5018

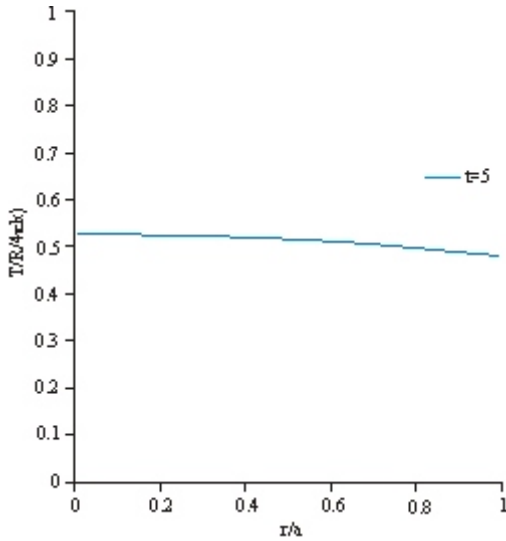


Fig. 7: Progressive transient temperature profile within the fuel pin during LOCA for time, t = 5

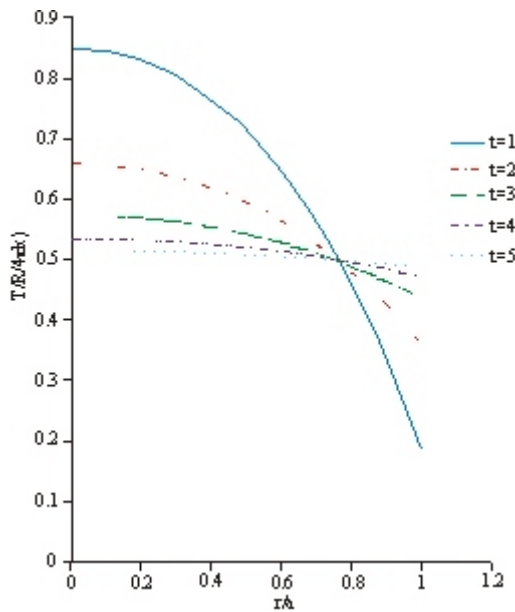


Fig. 8: Progressive equalisation of transient temperature profile distribution within the fuel pin following LOCA

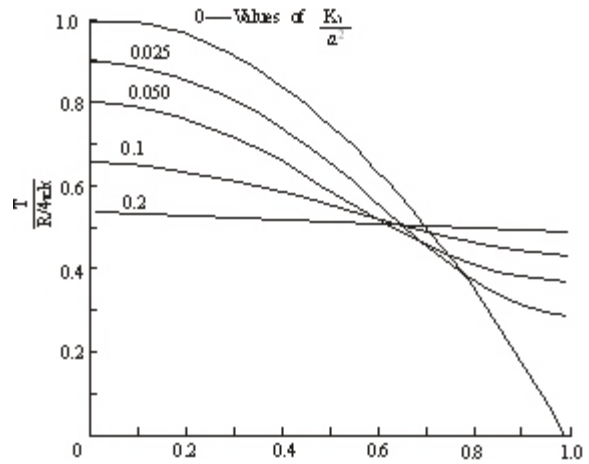


Fig. 9: Progressive equalisation of temperature within a fuel pellet following sudden total loss of cooling

DISCUSSION

The solution of transient situation of total loss of coolant was derived from the general transient equation

by equating the internal heat source $\frac{Q(r)}{k} = 0$ to obtain

Eq. (12) and solving analytically using Bessel functions of first kind and zero order to obtain the final Eq. (15). Analytical data was obtained and graphed, respectively from the final Eq. (15) using Excel to obtain the temperature profile distribution flow pattern within the fuel element during sudden total loss of cooling. The results obtained from Eq. (15) were derived from the

graph of $\frac{T}{R/4\pi k}$ against the temperature profile r/a .

Figures 1 to 8 are graphical representation of results obtained from Table 1; the results obtained shows the curves for the temperature distribution following sudden total loss of cooling, for various values of the dimensionless time $\alpha t/a^2$. For the times shown the infinite series in Eq. (15) converges very rapidly with a parabolic decay pattern, and in fact only the first few terms are needed. The values of $t=1$ and increasing values of the time the first few terms in the series dominates the behaviour with a time constant of $a^2/\alpha\beta_1^2$. From the

graph, the temperature at the centre of the fuel element is highest but during the transient phenomenon whereby there was no residual heat or decay heat the temperature at the centre of the fuel element decreases with increasing time; while from the graph the temperature at the surface of the fuel clad was least but increased with increasing time during the transient phenomenon. The time for the surface temperature to rise within 3% of its final value was obtained as $t = 5$, $t = 1$ the temperature profile at the centre of the fuel element was obtained as 0.89 and that of the clad was 0.2, but at $t = 5$ the temperature profile at the centre of the fuel element was obtained as 0.56 and as it equalizes the temperature profile at the surface of the clad was obtained as 0.4854. Incidentally the graph indicates that during the sudden complete loss of cooling as time increases the initial heat within the fuel element diffuses radially and the temperature within the centre of the fuel element decreases while that of the fuel clad increases. Thus the temperature turns to equalize radially within the fuel element.

Previous work done by Othman (2007) and results obtained from literature for a Pressurized Water Power Reactor (PWR) on the temperature distribution in fuel pellet following sudden total loss of cooling is represented in Fig. 9. Comparing the graphical representation of results Fig. 7 obtained in this research for the MNSR reactor with that of literature of a nuclear power reactor.

CONCLUSION

The transient solution for total loss of coolant for the GHARR-1 MNSR was developed and solved analytically by Bessel functions. The results obtained from the study indicated that the transient temperature gradient during total loss of coolant leading to shutdown, with the fuel element equalises or normalises because of absence of residual heat during shutdown. Therefore, in the absence of residual heat during total loss of coolant leading to sudden shutdown the material integrity of the fuel element will not be compromised.

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