Research Journal of Applied Sciences, Engineering and Technology 2(7): 673-678, 2010
ISSN: 2040-7467

Submitted Date: September 10, 2010 Accepted Date: October 14, 2010 Published Date: October 25, 2010

Generalized Fuzzy Metric Spaces with Properties

[1]Guangpeng Sun and [2]Kai Yang
Department of Applied Mathematics, Sichuan University, Chengdu Sichuan, P.R. China

Abstract: In this study, the notion of Q-fuzzy metric space is introduced and some properties are obtained. Two new common fixed point theorems are proved in Q-fuzzy metric spaces under some suitable conditions.

Key words: Fixed point, generalized fuzzy metric, Q-fuzzy metric

INTRODUCTION

The concept of fuzzy sets, introduced by Zadeh (1965), plays an important role in topology and analysis. Since then, there are many authors to study the fuzzy sets with applications. Especially, Kramosil and Michalek (1975) put forward a new concept of fuzzy metric space. George and Veermani (1994) revised the notion of fuzzy metric space with the help of continuous t-norm. As a result, many fixed point theorems for various forms of mappings are obtained in fuzzy metric spaces. Dhage (1992) introduced the definition of D-metric space and proved many new fixed point theorems in D-metric spaces. Recently, Mustafa and Sims (2006) presented a new definition of G-metric space and made great contribution to the development of Dhage theory.

On the other hand, Lopez - Rodrigues and Romaguera (2004) introduced the concept of Hausdorff fuzzy metric in a more general space.

In this study we introduce the notion of Q-fuzzy metric space, which can be considered as a generalization of fuzzy metric space. We show some new fixed point theorems in such Q-fuzzy metric spaces. The results presented in this paper improve and extend some known results due to Sharma (2002) and Rhoades (1996).

MATERIALS AND METHODS

We recall some definitions and properties for G-metric spaces given by Mustafa, Z. and B. Sims, (2006).

Definition 1: [M.S] Let X be a nonempty set,and let $G:X\times X\times X\to R^+$ be a function satisfying:

- $G(x, y, z) = 0$ if $x = y = z$
- $0<G(x, x, y)$ for all $x, y\in X$ with $x \neq y$
- $G(x, x, y)\leq G(x, y, z)$ for all $x, y \in X$ with $z\neq y$
- $G(x, y, z) = G(p\{x, y, z\})$ (symmetry) where p is a permutation function,
- $G(x, y, z)\leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality)

Then the function G is called a generalized metric, or more specifically a G-metric on X, and the pair (X, G) is a G-metric space.

Definition 2: [M.S] A G-metric space (X, G) is said to be symmetric if:

- $G(x, y, y) = G(x, x, y)$ for all $x, y \in X$

Following are examples of symmetric and non empty G-metric spaces, respectively.

Example: Let (X, d) be a metric space. Define $G: X\times X\times X\to R^+$, by $G(x, y, z) = d(x, y) + d(y, z) + d(z, x)$.

Example: Let $X = \{a,b\}$, let $G(a,a,a) = G(b,b,b) = 0, G(a,a,b) = 1, G(a,b,b) = 2$ and extend G to all of $X\times X\times X$ by symmetry in the variables. Then it is easily verified that G is a G-metric, but $G(a,a,b)\neq G(a,b,b)$.

Definition 3: [M.S] Let (X,G) be a G-metric space, then for $x_0 \in X$, $r>0$, the G-ball with centre x_0 and radius r is. $B_G(x_0,r) = \{y\in X \mid G(x_0,y,y)<r\}$.

Proposition: [M.S] Let (X,G) be a G-metric space, then for any $x_0 \in X$ and $r>0$, we have:

(1) if $G(x_0,x,y)< r$ then $x,y \in B_G(x_0, r)$
(2) if $y \in B_G(x_0,r)$ then there exists a $\delta>0$ such that $B_G(y,r)<B_G(x_0,r)$

Proof: (1) follows directly from (c), while (2) follows from (e) with $\delta = r-G(x_0,y,y)$.

It follows that if for every $x\in A\subset X$ there exists $r>0$ such that $B_G(x,r)\subset A$, then subset A is called open subset of X.

Definition 4: [M.S] Let (X,G) be a G-metric space.

(α) Subset A of X is said to be G-bounded if there exists $r>0$ such that $G(x,y,y)<r$ for all $x,y \in A$

Corresponding Author: Guangpeng Sun Department of Applied Mathematics, Sichuan University, Chengdu Sichuan, P.R. China

(β) Sequence $(x_n)\subseteq X$ is called a G-Cauchy sequence if for every $\in>0$,there exists $N\in N$ such that $G(x_n,x_m,x_l)<\in$ for all $n,m,l\geq N$

(γ) A sequence $(x_n)\subseteq X$ is said to be convergent to x if for every $\in>0$, there exists $N\in N$ such that $G(x_n,x_m,x)<\in$ for all $m,n\geq N$. The G-metric space (X,G) is said to be complete if every G-Cauchy sequence is convergent

Remark: Let X be a G-metric space, define $d_G : X\times X\to R^+$ by $d_G(x,y) = G(x,y,y)+ G(y,x,x)$. Then (X,d_G) is a metric space, therefore, a G-Cauchy sequence also be a Cauchy sequence.

Proposition: [M.S] Let (X,G) be a G-metric space, then for any $x,y,z,a\in X$ it follows that:

(1) $G(x,y,z)\leq G(x,x,y)+G(x,x,z)$
(2) $G(x,y,y)\leq 2G(y,x,x)$
(3) $G(x,y,z)\leq G(x,a,z)+ G(a,y,z)$

The properties above are readily derived from the axioms.

Proposition: [M.S] Let X be a G-metric space then for a sequence $(x_n)\subseteq X$ and point $x\in X$ the following are equivalent:

(1) $(x_n)$is convergent to x
(2) $d_G(x_n,x)\to 0$ as $n\to\infty$, i.e., (x_n) converges to x relative to the metric d_G
(3) $G(x_n,x_n,x)\to 0$ as $n\to\infty$
(4) $G(x_n,x,x)\to 0$ as $n\to\infty$
(5) $G(x_m,x_n,x)\to 0$ as $m,n\to\infty$

Proof: The equivalent (1) of and (2) is obvious. That (2) implies (3) and (4) follows from the definition of d_G. (3). implies (4) is a consequence of (2) of the proposition above, while (4) entails (5) follows from (1) of proposition. Finally, that (5) implies (2) follows from the definition of $d_G(x,y)$ and (2) of proposition above.

Lemma: Let (X,G) be a G-metric space. If r>0, then ball $B_G(x,r)$ with center $x\in X$ and radius r is open ball.

Proof: Let $z\in B_G(x,r)$. If set $G(x,z,z) = \delta$ and $r' = r-\delta$, then we prove that $B_G(x,r)$ .Let $y\in B_G(z,r')$,by the proposition of G-metric we have $G(x,y,y,)\leq G(x,z,z) + G(z,y,y)<\delta + r-\delta = r$. Hence $B_G(z,r')\subseteq B_G(x,r)$. That is ball $B_G(x,r)$ is open ball.

Definition 5: A binary operation $* :[0.1]\times[0.1]\to[0.1]$ is a continuous t-norm if it satisfies the following conditions:

- * is associative and commutative,
- * is continuous,
- $a*1 = a$ for all $a\in[0.1]$
- $a*b \leq c*d$ whenever $a\leq c$ and $b\leq d$, for each $a,b,c,d \in [0,1]$

Definition 6: A 3-tuple (X,Q,*) is called a Q-fuzzy metric space if X is an arbitrary (non-empty) set, * is a continuous t-norm, and Q is a fuzzy set on $X^3 \times (0,\infty)$, satisfying the following conditions for each $x,y,z,a \in X$ and t,s>0:

- $Q(x,x,y,t)>0$ and $Q(x,x,y,t)\leq Q(x,y,z,t)$ for all $x,y,z \in X$with $z \neq y$
- $Q(x,y,z,t) = 1$ if and only if $x = y = z$
- $Q(x,y,z,t) = Q(p(x,y,z),t)$, (symmetry) where p is a permutation function,
- $Q(x,a,a,t)* Q(a,y,z,s) \leq Q(x,y,z,t+s)$,
- $Q(x,y,z,.): (0,\infty)\to[0,1]$ is continuous

A Q-fuzzy metric space is said to be symmetric if $Q(x,y,y,t) = Q(x,x,y,t)$ for all $x,y \in X$.

Example: Let X is a nonempty set and G is the G-metric on X. Denote $a*b = a.b$ for all $a,b\in[0,1]$. For each t>0:

$$Q(x,y,z,t) = \frac{t}{t+G(x,y,z)}$$

Then (X,Q,*) is a Q-fuzzy metric.

Let (X,Q,*) be a Q-fuzzy metric space. For t>0, the open ball B_Q (x,r,t) with center $x\in X$ and radius $0<r<1$ is defined by:

$$B_Q(x,r,t) = \{y\in X:Q(x,y,y,t)>1-r\}$$

A subset A of X is called open set if for each $x\in A$ there exist t>0 and $0<r<1$ such that $B_Q(x,r,t)\subseteq A$

A sequence $\{x_n\}$ in X converges to x if and only if $Q(x_m,x_n,x,t)\to 1$ as $n\to\infty$, $m\to\infty$ for each t>0. It is called a Cauchy sequence if for each $0<\in<1$ and t>0,there exist $n_0\in N$ such that $Q(x_m,x_n,x_l)>1-\in$ for each $l,n,m\geq n_0$.The Q-fuzzy metric space is called to be complete if every Cauchy sequence is convergent. Following similar argument in G-metric space, the sequence $\{x_n)$ in X also converges to x if and only if $Q(x_n,x_n,x,t)\to 1$ as $n\to\infty$,for each t>0 and it is a Cauchy sequence if for each $0<\in<1$ and t>0, there exist $n_0\in N$ such that $Q(x_m,x_n,x_n)>1-\in$ for each n, $m\geq n_0$.

Lemma: If (X,Q,*) be a Q-fuzzy metric space, then Q(x,y,z,t) is non-decreasing with respect to t for all x,y,z in X.

Proof: By Definition 4, let a = x, we get $Q(x,x,x,t) * Q(x,y,z,s) \leq Q(x,y,z,t+s)$, that is $Q(x,y,z,t+s)\geq Q(x,y,z,t)$. Throughout this study, we assume that $\lim_{t\to\infty} Q(x,y,z,t) = 1$ and that N is the set of all natural numbers.

Lemma: Let (X,Q,*) be a Q-fuzzy metric space. (a) If there exists a positive number 0<k<1 such that :

$$Q(y_{n+2},y_{n+1},y_{n+1},kt) \geq Q(y_{n+1},y_n,y_n,t),\ t>0,\ n \in N$$

then $\{y_n\}$ is a Cauchy sequence in X. (b) if there exists $k\in(0,1)$ such that $Q(x,y,kt)\geq Q(x,y,t)$ for all $x,y\in X$ and t>0, then x = y.

Proof: By the assume $\lim_{t\to\infty} Q(x,y,z,t) = 1$ and the property of non-decreasing, it is easy to get the results.

Definition 7: Let (X,Q,*) be a Q-fuzzy metric space. The following conditions are satisfied:

$$\lim_{n\to\infty} Q(x_n,y_n,z_n,t_n) = Q(x,y,z,t)$$

whenever, $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$, $\lim_{n\to\infty} z_n = z$ and $\lim_{n\to\infty} Q(x,y,z,t_n) = Q(x,y,z,t)$; then Q is called continuous function on $X^3\times(0,\infty)$

Lemma: Let (X,Q,*) be a Q-fuzzy metric space. Then Q is a continuous function on $X^3\times(0,\infty)$.

Proof: Since $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$, $\lim_{n\to\infty} z_n = z$ and $\lim_{n\to\infty} Q(x,y,z,t_n) = Q(x,y,z,t)$ there is $n_0\in N$ such that $|t-t_n| <\delta$ for $n\geq n_0$ and $\delta< \frac{t}{2}$.

We have known that Q(x,y,z,t) is non-decreasing with respect to t,so we have:

$$Q(x_n, y_n, z_n, t_n) \geq Q(x_n, y_n, z_n, t-\delta)$$

$$\geq Q(x_n, x, x, \tfrac{1}{3}\delta) * Q(x, y_n, z_n, t- \tfrac{4\delta}{3})$$

$$\geq Q(x_n, x, x, \tfrac{1}{3}\delta) * Q(y_n, y, y, \tfrac{1}{3}\delta)*Q(y, x, z_n, t- \tfrac{5\delta}{3})$$

$$\geq Q(x_n, x, x, \tfrac{1}{3}\delta) * Q(y_n, y, y, \tfrac{1}{3}\delta)* Q(z_n, z, z, t, \tfrac{1}{3}\delta)* Q(z, y, x, t-2\delta)$$

and $Q(x, y, z, t+2\delta) \geq Q(x, y, z, t_n +\delta)$

$$\geq Q(x, x_n, x_n, \tfrac{1}{3}\delta)*Q(x_n, y, z, t_n + \tfrac{2}{3}\delta)$$
$$\geq Q(x, x_n, x_n, \tfrac{1}{3}\delta)*Q(y, y_n, y_n, \tfrac{1}{3}\delta)* Q(y_n, x_n, z, t_n + \tfrac{1}{3}\delta)$$

$$\geq Q(x, x_n, x_n, \tfrac{1}{3}\delta) * Q(y, y_n, y_n, \tfrac{1}{3}\delta)* Q(z, z_n, z_n, \tfrac{1}{3}\delta)* Q(z, y, x, t_n)$$

Let $n \to \infty$,by continuity of the function Q with respect to t, we can get $Q(x, y, z, t + 2\delta)\geq Q(z, y, x, t) \geq Q(Q(z, y, x, t - 2\delta))$ Therefore Q is continuous function on $X^3 \times (0, \infty)$.

Definition 8: Let f and g be two self-mappings of a Q-fuzzy metric space (X, Q, *). If f and g satisfy the following condition: there exists a sequence $\{x_n\}$ such that:

$$\lim_{n\to\infty} Q(fx_n,u,u,t) = \lim_{n\to\infty} Q(gx_n,u,u,t) = 1$$

for some $u \in X$ and t>0. We say that f and g have the property (Q.E).

Definition 9: Let f and g be self maps on a Q-fuzzy metric space (X, Q, *). Then the mappings are said to be weakly compatible if they commute at their coincidence point, that is, fx = gx implies that fgx = gfx.

Definition 10: Let f and g be self maps on a Q-fuzzy metric space (X, Q, *). The pair (f, g) is said to be compatible if

$$\lim_{n\to\infty} Q(fgx_n, gfx_n, gfx_n, t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that:

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$$

for some $z \in X$.

RESULTS

We first generalize a classic theorem in Q-fuzzy metric space.

Theorem: Let f,g S and T be self-mappings of a complete symmetric Q-fuzzy metric space (X, Q, *) with $t*t \geq t$, if the mappings satisfy the following conditions:

- $f(x) \subseteq S(x)$, $g(x) \subseteq T(x)$
- (f, T) or (g, S) satisfy the property (Q.E),
- (f, T) and (g, S) are weakly compatible
- There exists $k \in(0,1)$ such that for every $x,y \in X$ and t>0

$$Q(fx, gy, gz, kt) \geq Q(Tx, Sy, Sz, t)* Q(fx, Sy, Sz, t) * Q(Tx, gy, gz, t)$$

Then f, g, S and T have a unique common fixed point in X.

Proof: Suppose (f, T) satisfy the property (Q.E), hence there exists a sequence $\{x_n\}$ such that:

$$\lim_{n\to\infty} Q\left(fx_n, u, u, t\right) = \lim_{n\to\infty} Q\left(Tx_n, u, u, t\right) = 1$$

for some u∈X and t>0;

Since f(x) ⊆ S(x), there exists a sequence $\{y_n\}$ such that $fx_n = Sy_n \Rightarrow \lim_{n\to\infty} Q\left(Sy_n, u, u, t\right) = 1$,

Therefore,

$$Q(fx_n, gy_n, gy_{n+1}, kt) \geq Q(Tx_n, Sy_n, Sy_{n+1}, t)* Q(fx_n, Sy_n, Sy_{n+1}, t) * Q(Tx_n, gy_n, gy_{n+1}, t)$$

$$\geq Q(Tx_n, u, u, ½t) * Q(u, Sy_n, Sy_{n+1}, ½t)*Q(fx_n, u, u, ½t)* Q(u, Sy_n, Sy_{n+1}, ½t)*Q(Tx_n, gy_n, gy_{n+1}, t)$$

There exists δ>0 such that k+δ<1, for k ∈ (0, 1). On making n → ∞ and by the symmetry of Q-fuzzy metric space,
We have:

$$\lim_{n\to\infty} Q\left(fx_n, gy_n, gy_{n+1}, kt\right) \geq 1*1*1*1*$$

$$\lim_{n\to\infty} Q\left(Tx_n, fx_n, fx_n\left[1-(k+\delta)\right]t\right)* \lim_{n\to\infty} Q\left(fx_n, gx_n, gy_{n+1}(k+\delta)t\right) \geq 1*1*1*1*1*$$

$$\lim_{n\to\infty} Q\left(fx_n, gy_n, gy_{n+1}, (k+\delta)t\right)$$

Hence

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Tx_n = u$$

Let (X, Q, *) is a complete Q-fuzzy metric space,there exists x_0 ∈ X such that:

$$Tx_0 = u \Rightarrow Q(fx_0, gy_n, gy_{n+1}, kt) \geq Q(fx_0, gy_n, gy_{n+1}, kt) \geq Q(Tx_0, Sy_n, Sy_{n+1}, t)*Q(fx_0, Sy_n, Sy_{n+1}, t)* Q(Tx_0, gy_n, gy_{n+1}, t)*$$

If n → ∞,we can get $Q(fx_0, u, u, kt) \geq 1* Q(fx_0, u, u, t) * 1$. By the property of non-decreasing with respect to t, it is easy to see that $fx_0 = Tx_0 = u$.

As f(x) ⊆ S(x), there exists y_0 such that $fx_0 = Sy_0$. Suppose $Sy_0 \neq gy_0$. Then:

$$Q(fx_n, gy_0, gy_0, kt) \geq Q(Tx_n, Sy_0, Sy_0, t)* Q(fx_n, Sy_0, Sy_0, t)*Q(Tx_n, gy_0, gy_0, t) \geq Q(Tx_n, u, u, t)*Q(fx_n, u, u, t)*Q(Tx_n, gy_0, gy_0, t)$$

Letting n → ∞, we have:

$$Q(u, gy_0, gy_0, kt) \geq Q(u, u, u, t)* Q(u, u, u, t)*Q(u, gy_0, gy_0, t)$$

which is a contradiction. So $gy_0 = Sy_0 = u$.

Now, by (f, T) and (g, S) are weakly compatible, we can get that:

$ffx_0 = fTx_0 = Tfx_0 = TTx_0$ and $ggy_0 = gSy_0 = Sgy_0 = SSy_0$

Suppose fu ≠ u. Then $Q(fu, u, u, kt) = Q(fu, gy_0, gy_0, kt) \geq Q(Tu, Sy_0, Sy_0, t)*Q(fu, Sy_0, Sy_0, t)*Q(Tu, gy_0, gy_0, t) \geq Q(Tu, u, u, t)*Q(fu, u, u, t)*Q(Tu, u, u, t)$

$$\geq \lim_{n\to\infty} Q(Tu, Tx_n, Tx_n, t)*Q(fu, u, u, t)* \lim_{n\to\infty} Q(Tu, Tx_n, Tx_n, t) \quad (1)$$

By $fu = ffx_0 = fTx_0 = Tfx_0 = TTx_0 = Tu$ and t*t≥t, it is easy to see that (1) yields a contradiction and so fu = u = Tu.

Now following the similar argument, we can get gu = u = Su. So f,g S and T have a fixed common point u.

Let v ≠ u be another common fixed point of f,g,S and T. Then:

$$Q(v, u, u, kt) = Q(fv, gu, gu, kt) \geq Q(Tv, Su, Su, t)*Q(fv, Su, Su, t)*Q(Tv, gu, gu, t) = Q(v, u, u, t)*Q(v, u, u, t)*Q(v, u, u, t)$$

By t*t ≥ t, we can get Q(v, u, u, kt) ≥Q(v, u, u, t) is a contradiction. Thus v = u.

Hence f,g,S and T have a unique common fixed point in X.

Remark: This theorem is obtained in symmetric Q-fuzzy metric space and the constant k should exist, which limits its application. Hence it is interesting to make some improvement.

Theorem: Let f,g S and T be self-mappings of a complete Q-fuzzy metric space (X, Q, *) with t*t > t, if the mappings satisfy the following conditions:

- f(x) ⊆ T(x), g(x) ⊆ S(x)
- Suppose (f, S) satisfy the property (Q.E)
- (f, S) and (g, T) are weakly compatible
- Q(fx, gy, gz, t) ≥

$$\phi\left[\min\begin{pmatrix} Q(Sx, Ty, Tz, t) & Q(Sx, gy, gz, t) \\ Q(fx, Ty, Tz, t) & Q(fx, Sx, Sx, t) \end{pmatrix}\right]$$

for all x,y ϵ X and t>0 where ϕ: [0, 1] → [0, 1] is a continuous and increasing function with ϕ(s) > s for 0<s<1 and ϕ(1) = 1.

Then f,g,S and T have a unique common fixed point in X.

Proof: Let (f, S) satisfy the property (Q.E). By the definition of (Q.E), we can get:

$$\lim_{n\to\infty} Q(fx_n, u, u, t) = \lim_{n\to\infty} Q(Sx_n, u, u, t) = 1$$

for some u ϵ X and every t>0;

Because Q-fuzzy metric space is complete and f(x)⊆ T(x), there exists a sequence $\{y_n\}$ such that $f(x_n) = T(y_n)$, which implies:

$$\lim_{n\to\infty} Q(Ty_n, u, u, t) = 1$$

Now $Q(fx_n, gy_n, gy_{n+1}, t) \geq$

$$\phi\left[\min\begin{pmatrix} Q(Sx_n, Ty_n, Ty_{n+1}, t) \\ Q(fx_n, Ty_n, Ty_{n+1}, t) \\ Q(Sx_n, gy_n, gy_{n+1}, t) \\ Q(fx_n, Sx_n, Sx_n, t) \end{pmatrix}\right]$$

By the definition of Q-fuzzy metric space, we can get:

$Q(x, y, z, t) \geq Q(x, u, u, ⅓t)*Q(u, y, z, ⅔t)$
$\geq Q(x, u, u, ⅓t)*Q(y,u, u, ⅓t)*Q(z, u, u, ⅓t)$
Thus

$$\lim_{n\to\infty} Q(fx_n, gy_n, gy_{n+1}, t) \geq$$

$$\lim_{n\to\infty} \phi\left[\min\begin{pmatrix} Q(Sx_n, Ty_n, Ty_{n+1}, t) \\ Q(fx_n, Ty_n, Ty_{n+1}, t) \\ Q(Sx_n, gy_n, gy_{n+1}, t) \\ Q(fx_n, Sx_n, Sx_n, t) \end{pmatrix}\right]$$

$$\geq \lim_{n\to\infty} \phi\left[\min\begin{pmatrix} 1*1*1 & Q(Sx_n, gy_n, gy_{n+1}, t) \\ 1*1*1 & 1*1*1 \end{pmatrix}\right]$$

If $gy_n \neq u$, then $\lim_{n\to\infty} Q(fx_n, gy_n, gy_{n+1}, t)$

$$\geq \lim_{n\to\infty} \phi\left[Q(Sx_n, gy_n, gy_{n+1}, t)\right]$$

$$> \lim_{n\to\infty} Q(Sx_n, gy_n, gy_{n+1}, t)$$

is a contradiction by the Lemma above. Therefore:

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n$$
$$= \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = u$$

Let (X, Q, *) is a complete Q-fuzzy metric space, there exists x_0 ϵ X such that $Sx_0 = u$,
Hence $Q(fx_0, gy_n, gy_{n+1}, t) \geq$

$$\phi\left[\min\begin{pmatrix} Q(Sx_0, Ty_n, Ty_{n+1}, t) \\ Q(fx_0, Ty_n, Ty_{n+1}, t) \\ Q(Sx_0, gy_n, gy_{n+1}, t) \\ Q(fx_0, Sx_0, Sx_0, t) \end{pmatrix}\right]$$

On making n → ∞, $Q(fx_0, u, u, t) \geq$

$$\phi\left[\min\begin{pmatrix} Q(u,u,u,t) & Q(u,u,u,t) \\ Q(fx_0,u,u,t) & Q(fx_0,u,u,t) \end{pmatrix}\right]$$

which can imply $fx_0 = u$ with ϕ(s) > s for 0<s<1.

As f(x) ⊆ T(x), there exists y_0 such that $fx_0 = Ty_0$. Suppose $Ty_0 \neq gy_0$, Now $Q(fx_n, gy_0, gy_0, t) \geq$

$$\phi\left[\min\begin{pmatrix} Q(Sx_n, Ty_0, Ty_0, t) \\ Q(fx_n, Ty_0, Ty_0, t) \\ Q(Sx_n, gy_n, gy_0, t) \\ Q(fx_n, Sx_n, Sx_n, t) \end{pmatrix}\right]$$

If n → ∞, $Q(u, gy_0, gy_0, t) \geq$

$$\phi\left[\min\begin{pmatrix} Q(u,u,u,t) & Q(u,gy_0,gy_0,t) \\ Q(u,u,u,t) & Q(u,u,u,t) \end{pmatrix}\right]$$

by the continuity of Q and ϕ.

Hence $Q(u, gy_0, gy_0, t) \geq \phi(Q(u, gy_0, gy_0, t)) > Q(u, gy_0, gy_0, t)$ is a contradiction. So $Ty_0 = gy_0$.

Now, by (f, T) and (g, S) are weakly compatible, we can get:

$ffx_0 = fSx_0 = Sfx_0 = SSx_0$ and $ggy_0 = gTy_0 = Tgy_0 = TTy_0$,

Then Q(fu, u, u, t)

$$\lim_{n\to\infty} Q(fu, gy_n, gy_{n+1}, t) \geq$$

$$\phi\left[\min\begin{pmatrix} Q(Su, Ty_n, Ty_{n+1}, t) \\ Q(fu, Ty_n, Ty_{n+1}, t) \\ Q(Su, gy_n, gy_{n+1}, t) \\ Q(fu, Su, Su, t) \end{pmatrix}\right]$$

⇒ fu = u = Su. Similarly, we can get Tu = gu = u.

Let v be another common fixed point of f,g,S and T. Then

Q(v, u, u, t) = Q(fv, gu, gu, t)

$$\geq \phi\left[\min\begin{pmatrix} Q(Sv, Tu, Tu, t) & Q(Sv, gu, gu, t) \\ Q(fv, Tu, Tu, t) & Q(fv, Sv, Sv, t) \end{pmatrix}\right]$$

It implies v = u. Hence f,g,S and T have a unique common fixed point in X.

CONCLUSION

In this study we introduce the notion of Q-fuzzy metric space, which can be considered as a generalization of fuzzy metric space. We also show two new fixed point theorems in such Q-fuzzy metric spaces. In fact, the results presented in this paper improve and extend some known results. The previous results such as Rhoades (1996) are obtained in fuzzy metric space, while we can get fixed point theorems in more general space.

ACKNOWLEDGMENT

The author must be thankful of the referees and Professor NanJing Huang for giving valuable comments and suggestions.

REFERENCES

Dhage, B.C., 1992. Generalised metric spaces and mappings with fixed point. Bull. Calcutta Math. Soc., 84(4): 329-336.

George, A. and P. Veermani, 1994. On some results in fuzzy metric spaces. Fuzzy Sets Sys., 64: 395-399.

Kramosil. J and J. Michalek, 1975. Fuzzy metric and statistical metric spaces. Kybernetica, 11: 326-334.

Lopez-Rodrigues, J. and S. Romaguera, 2004. The Hausdorff fuzzy metric on compact sets. Fuzzy Sets Sys., 147: 273-283.

Mustafa, Z. and B. Sims, 2006. A new approach to generalized metric spaces. J. Nonlinear Convex Anal., 7: 289-297.

Rhoades, B.E., 1996. A fixed point theorem for generalized metric spaces. Int. J. Math. Sci., 19: 145-153.

Sharma, S., 2002. Common fixed point theorems in fuzzy metric spaces. Fuzzy Sets Sys., 127: 345-352.

Zadeh, L.A., 1965. Fuzzy sets. Inform. Control, 8: 338-353.