Research Journal of Applied Sciences, Engineering and Technology 2(8): 727-733, 2010
ISSN: 2040-7467

Submitted date: September 15, 2010 Accepted date: October 14, 2010 Published date: December 10, 2010

On Common Fixed Point Theorems in Fuzzy Metric Spaces Satisfying Integral Type Inequality

[1]Manish Kumar Manish, [2]Priyanka Sharma and [1]D.B. Ojha
[1]Department of Mathematics, R.K.G.I.T., U.P.T.U Delhi-Meerut road Ghaziabad, U.P., 201003, India
[2]S.I.E.T., Uttar Pradesh Technical University, Ghaziabad , U.P., India

Abstract: In this study, we established some common fixed point theorems for occasionally weakly compatible mappings in fuzzy metric spaces satisfying integral type inequality.

Key words: Contractive condition of integral type, fuzzy metric space, occasionally weakly compatible mappings

INTRODUCTION

In recent years, several common fixed point theorems for contractive type, mappings have been established by several authors Jachymski (1995), Jungck (1993) and Pant (1996). Using the concept of reciprocal continuity which is weaker from of continuity of mappings Pant (1996), Popa (1997, 1999) proved some fixed point theorems satisfying certain implicit relation. Recently Aliouche (2005) established a general common fixed point theorem for pair of reciprocally continuous mappings satisfying an implicit relation Pathak *et al.* (2007).

After introduction of fuzzy sets by Zadeh (1965) and Kramosil and Michalek (1975), introduced the concept of fuzzy metric space, many authors extended their views as some George and Veeramani (1994), Grabiec (1988), Subrahmanyam (1995) and Vasuki (1999). Pant and Jha (2004) obtained some anologus results proved by Balasubramaniam *et al.* (2002) Recent literature on fixed point in fuzzy metric space can be viewed in Aage (2009a, b, 2010), Imdad and Javid (2006), Pant and Jha (2004) and Cho (2006).

In this study, we established common fixed point theorems for more general commutative condition i.e., occasionally weakly compatible mappings in fuzzy metric space satisfying integral type inequality.

MATERIALS AND METHODS

Definition: (Zadeh, 1965)
A fuzzy set in A in X a function with domain Xand values [0, 1].

Definition: (George and Veeramani, 1994)
A binary operation $*$: $[0, 1]\times [0, 1]\rightarrow [0, 1]$ is a continuous t - norm if it satisfies the following conditions:

- $*$ is communicative and associative
- $*$ is continuous
- $a * 1 = a$ for all $a \in [0, 1]$
- $a*b \le c*d$ whenever $a\le c$ and $b\le d$ and $a,b,c,d \in [0, 1]$

Examples of continuous t-norm are:

- $a*b = ab$
- $a*b = \min\{a, b\}$

Definition: (Vasuki, 1999)
A 3-tuple $(X, M, *)$ is called a fuzzy metric space, if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$. Satisfying the following conditions for each $v,y,z \in X$, $s,t > 0$:

- $M (x, y, t) > 0$
- $M (x, y, t) = 1$ if and only if $x = y$
- $M (x, y, t) = M (y, x, t)$
- $M(x, y, t) * M (y, z, s) \le M (x, z, t+s)$
- $M (x, y, .): (0, \infty) \rightarrow [0, 1]$ is continuous

Then M is called a fuzzy metric on X. Then $M (x, y, t)$ denotes the degree of nearness between x and y with respect to t.

Example: (Induced fuzzy metric (Vasuki, 1999)
Let (X, d) be a metric space. Denote $a*b = ab$ for all $a,b \in [0, 1]$ and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d (x, y, t) = t / t+d (x, y)$$

Then $(X, M_d, *)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d as the standard intuitionistic fuzzy metric.

Corresponding Author: Manish Kumar Mishra, Department of Mathematics, R.K.G.I.T., U.P.T.U Delhi-Meerut road Ghaziabad, U.P., 201003, India

Definition: (Vasuki, 1999)
Let (X, M_d, *) be a fuzzy metric space. Then:

- a sequence $\{x_n\}$ in X is said to converges to x in X if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that M (x_n, x, t) $> 1 - \epsilon$ for all $n \geq n_0$
- a sequence $\{x_n\}$ in X is said to be Cauchy if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that M (x_n, x_m, t) $> 1 - \epsilon$ for all n, $m \geq n_0$
- A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete

Definition: (Jungck, 1988)
A pair of self-mappings (f, g) of a fuzzy metric space (X, M, *) is said to be:

- weakly commuting if M(fgx, gfx, t) $\geq$ M(fgx, gfx, t) for all $x \in X$ and $t > 0$
- R-weakly commuting if there exists some $R > 0$ such that M(fgx, gfx, t) $\geq$ M(fx, gx, t/R) for all $x \in X$ and $t > 0$

Definition: (Balasubramaniam *et al.*, 2002)
Two self mappings f and g of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty}$ M(fgx, gfx, t) = 1 whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition: (Rhoades and Jungck, 1998):
Two self maps f and g of a fuzzy metric space (X, M, *) are called reciprocally continuous on X if:

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$$

whenever $\{x_n\}$ is a sequence in X such that:

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$$

for some x in X.

Lemma a: Let (X, M, *) be a fuzzy metric space. If there exists $q \in (0, 1)$ such that M (x, y, qt) $\geq$ M (x, y, t) for all x, $y \in X$ and $t > 0$, then x = y.

Definition: (Vasuki, 1999)
Let X be a set, f, g self maps of X. A point x in X is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition: (Rhoades and Jungck, 2006)
A pair of maps S and T is called weakly compatible pair if they commute at coincidence points. The concept occasionally weakly compatible is introduced by Al-Thagafi and Naseer (2008). It is stated as follows:

Definition: Two self maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

Al-Thagafi and Naseer (2008) shown that occasionally weakly is weakly compatible but converse is not true.

Example: Let R be the usual metric space. Define S T: R $\to$ R by Sx = x and Tx = x^3 for all $x \in R$. Then Sx = Tx for x = 0, 2 but ST0 = TS0 and ST2 $\neq$ TS 2. S and T are occasionally weakly compatible self maps but not weakly compatible.

Lemma b: (Al-Thagafi and Naseer, 2008)
Let X be a set, f, g owc self maps of X. If f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

Definition: If $\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that:

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some $t \in X$. Clearly, weakly commuting maps are compatible, but the implication is not reversible.

Let (X, d) be a complete metric space, $\alpha \in [0, 1]$, f: X $\to$ X a mapping such that for each x, $y \in X$:

$$\int_0^{d(fx,fy)} \varphi(t)dt \leq \alpha \int_0^{d(x,y)} \varphi(t)dt$$

where ϕ: $R^+ \to R$ is a lebesgue integrable mapping which is summable, nonnegative and such that, for each $\epsilon > 0$:

$$\int_0^{\varepsilon} \phi(t)dt > 0$$

Then f has a unique common fixed $z \in X$ such that for each $x \in X$:

$$\lim_{n\to\infty} f^n x = z$$

Rhoades (2003), extended this result by replacing the above condition by the following:

$$\int_0^{d(fx,fy)} \phi(t)dt \le \alpha \int_0^{\max\left\{d(x,y),d(y,fy),\frac{1}{2}\left[d(x,fy)+d(y,fx)\right]\right\}} \phi(t)dt$$

Ojha *et al.* (2010) Let (X, d) be a metric space and let f: X → X, F: X → CB(X) be a single and a multi-valued map respectively, suppose that f and F are occasionally weakly commutative (OWC) and satisfy the inequality:

$$\int_0^{J^p(Fx,Fy)} \phi(t)dt \le \alpha \int_0^{\max\left[\begin{array}{l} ad(fx,fy)d^{p-1}(fx,Fx), ad(fx,fy)d^{p-1}(fx,fy), \\ ad(fx,Fx)d^{p-1}(fy,Fy), cd^{p-1}(fx,Fy)d(fy,Fx) \end{array}\right]} \phi(t)dt$$

for all x, y in X,where $p \ge 2$ is an integer $a \ge 0$ and $0 < c < 1$ then f and F have unique common fixed point in X.

RESULTS

Theorem: Let (X, M, *) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exists $q \in (0, 1)$ such that:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \le \int_0^{\min\left\{M(Sx,Ty,t),M(Sx,Ax,t),M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\right\}} \phi(t)dt \qquad (1)$$

for all $x, y \in X$ and for all $t > 0$, then there exists a unique point $w \in X$ such that Aw = Sw = w and a unique point $z \in X$ such that Bz = Tz = z. Moreover, z = w, so that there is a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc , so there are points $x, y \in X$ such that Ax = Sx and By = Ty. We claim that Ax = By. If not, by inequality (1):

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \le \int_0^{\min\left\{M(Sx,Ty,t),M(Sx,Ax,t),M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\right\}} \phi(t)dt$$

$$= \int_0^{\min\{M(Ax,By,t),M(Ax,Ax,t),M(By,By,t),M(Ax,By,t),M(By,Ax,t)\}} \phi(t)dt = \int_0^{M(Ax,By,t)} \phi(t)dt$$

Therefore, Ax = By i.e., Ax = Sx = By = Ty. Suppose that there is a another point z such that Ax = Bz then by (1) we have Az = Sz = By = Ty, so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S.

By Lemma b w is the only common fixed point of A and S. Similarly there is a unique point $z \in X$ such that z = Bz = Tz.

Assume that $w \ne z$. We have:

$$\int_0^{M(w,z,qt)} \phi(t)dt = \int_0^{M(Aw,Bz,qt)} \phi(t)dt$$

$$\le \int_0^{\min\left\{M(Sw,Tz,t),M(Sw,Az,t),M(Bz,Tz,t),M(Aw,Tz,t),M(Bz,Sw,t)\right\}} \phi(t)dt$$

$$= \int_0^{\min\{M(w,z,t),M(w,z,t),M(z,z,t),M(w,z,t),M(z,w,t)\}} \phi(t)dt = \int_0^{M(w,z,t)} \phi(t)dt$$

Therefore we have z = w by Lemma b and z is a common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (1).

Theorem: Let (X, M, *) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exists $q \in (0, 1)$ such that:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \le \int_0^{\psi\left(\min\left\{M(Sx,Ty,t),M(Sx,Ax,t),M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\right\}\right)} \phi(t)dt \qquad (2)$$

for all x, y (---) X and $\psi: [0, 1] \to [0, 1]$ such that $\psi(t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of A, B, S and T.

Proof: The proof follows from above Theorem.

Theorem: Let (X, M, *) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exists $q \in (0, 1)$ such that:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \le \int_0^{\psi\left\{M(Sx,Ty,t),M(Sx,Ax,t),M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\right\}} \phi(t)dt \qquad (3)$$

for all $x, y \in X$ and $\psi: [0, 1]^5 \to [0, 1]$ such that $\psi(t, 1, 1, t, t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, there are $x, y \in X$ such that Ax = Sx and By = Ty. We claim that Ax = By. By inequality (3) we have:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \le \int_0^{\psi\left\{M(Sx,Ty,t),M(Sx,Ax,t),M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\right\}} \phi(t)dt$$

$$= \int_0^{\psi\left\{M(Ax,By,t),1,1,M(Ax,By,t),M(By,Ax,t)\right\}} \phi(t)dt$$

$$\ge \int_0^{\psi\left\{M(Ax,By,t),M(Ax,Ax,t),M(By,By,t),M(Ax,By,t),M(By,Ax,t)\right\}} \phi(t)dt > \int_0^{M(Ax,By,qt)} \phi(t)dt$$

a contradiction, therefore Ax = By, i.e., Ax = Sx = By = Ty. Suppose that there is a another point z such that Az = Sz then by (3) we have Az = Sz = By = Ty, so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and T. By Lemma b w is a unique common fixed point of A and S. Similarly there is a unique point $z \in X$ such that z = Bz = Tz. Thus zis a common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (3).

Theorem (A): Let (X, M, *) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exists $q \in (0, 1)$ for all $x, y \in X$ and $t > 0$:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \ge \int_0^{\left\{M(Sx,Ty,t),*M(Ax,Sx,t),*M(By,Ty,t),M(Ax,Ty,t)\right\}} \phi(t)dt \qquad (4)$$

then there exists a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} are owc, there are points $x, y \in X$ such that Ax = Sx and By = Ty. We claim that Ax = By. By inequality (4), we have:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \geq \int_0^{\{M(Sx,Ty,t),*M(Ax,Sx,t),*M(By,Ty,t),M(Ax,Ty,t)\}} \phi(t)dt$$

$$= \int_0^{\{M(Ax,By,t),*M(Ax,Ax,t),*M(By,By,t),M(Ax,By,t)\}} \phi(t)dt$$

$$\geq \int_0^{\{M(Ax,By,t),*1*1*M(Ax,By,t)\}} \phi(t)dt \geq \int_0^{M(Ax,By,t)} \phi(t)dt$$

Thus we have Ax = By, i.e., Ax = Sx = By = Ty. Suppose that there is a another point z such that Az = Sz then by (4) we have Az = Sz = By = Ty, so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. Similarly there is a unique point $z \in X$ such that z = Bz = Tz. Thus is a common fixed point of A, B, S and T.

Corollary (A): Let (X, M, *) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exists $q \in (0, 1)$ for all $x, y \in X$ and $t > 0$:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \geq \int_0^{\{M(Sx,Ty,t),*M(Ax,Sx,t),*M(By,Ty,t),M(By,Sx,2t)*M(Ax,Ty,t)\}} \phi(t)dt \quad (5)$$

then there exists a unique common fixed point of A, B, S and T.

Proof: We have:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \geq \int_0^{\{M(Sx,Ty,t),*M(Ax,Sx,t),*M(By,Ty,t),M(By,Sx,2t)*M(Ax,Ty,t)\}} \phi(t)dt$$

$$\geq \int_0^{\{M(Sx,Ty,t),*M(Ax,Sx,t),*M(By,Ty,t),*M(Sx,Ty,t),*M(Ty,By,t)*M(Ax,Ty,t)\}} \phi(t)dt$$

$$\geq \int_0^{\{M(Sx,Ty,t),*M(Ax,Sx,t),*M(By,Ty,t),*M(Ax,Ty,t)\}} \phi(t)dt$$

and therefore from Theorem 3.4, A, B, S and T have a common fixed point.

Corollary: Let (X, M, *) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exists $q \in (0, 1)$ for all $x, y \in X$ and $t > 0$:

$$\int_0^{M(Ax,By,qt)} \phi(t)dt \geq \int_0^{M(Sx,Ty,t)} \phi(t)dt \quad (6)$$

then there exists a unique common fixed point of A, B, S and T.

Proof: The Proof follows from Corollary (A)

Theorem: Let (X, M, *) be a complete fuzzy metric space. Then continuous self mappings S and T of X have a common fixed point in if and only if there exists a self mapping A of X such that the following conditions are satisfied

- $AX \subset TX \cap SX$
- the pairs {A, S} and {B, T} are weakly compatible
- there exists a point $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$\int_0^{M(Ax,Ay,qt)} \phi(t)dt \ge \int_0^{M(Sx,Ty,t)*M(Ax,Sx,t)*M(Ay,Ty,t)*M(Ax,Ty,t)} \phi(t)dt \qquad (7)$$

Then A, S and T have a unique common fixed point.

Proof: Since compatible implies owc, the result follows from Theorem (A).

Theorem: Let (X, M, *) be a complete fuzzy metric space and let A and B be self-mappings of X. Let the pairs A and B be owc. If there exists $q \in (0, 1)$ for all $x, y \in X$ and $t > 0$:

$$\int_0^{M(Sx,Sy,qt)} \phi(t)dt \ge \alpha\int_0^{M(Ax,Ay,t)} \phi(t)dt + \beta\int_0^{\min\{M(Ax,Ay,t),M(Sx,Ax,t),M(Sy,Ay,t)\}} \phi(t)dt \qquad (8)$$

for all $x, y \in X$, where $\alpha, \beta > 0$, $\alpha + \beta > 1$. Then A, and S have a unique common fixed point.

Proof: Let the pairs {A, S} be owc, so there is a point $x \in X$ such that $Ax = Sx$. Suppose that there exist another point $y \in X$ for which, $Ay = Sy$. We claim that $Sx = Sy$. By inequality (8) we have:

$$\int_0^{M(Sx,Sy,qt)} \phi(t)dt \ge \alpha\int_0^{M(Ax,Ay,t)} \phi(t)dt + \beta\int_0^{\min\{M(Ax,Ay,t),M(Sx,Ax,t),M(Sy,Ay,t)\}} \phi(t)dt$$

$$= \alpha\int_0^{M(Sx,Sy,t)} \phi(t)dt + \beta\int_0^{\min\{M(Sx,Ay,t),M(Sx,Sx,t),M(Sy,Sy,t)\}} \phi(t)dt = (\alpha+\beta)\int_0^{M(Sx,Sy,t)} \phi(t)dt$$

a contradiction, since $\alpha + \beta > 1$. Therefore, $Sx = Sy$. Therefore $Ax = Ay$ and Ax is unique. From Lemma b, A and S have a unique fixed point.

CONCLUSION

In this study, we proved some common fixed point theorem in fuzzy metric spaces satisfying integral type inequality with two corollaries with appropriate example.

REFERENCES

Aage, C.T. and J.N. Salunke, 2009a. Common fixed point theorems in fuzzy metric spaces. Int. J. Pure Appl. Math., 56(2): 155-164.

Aage, C.T. and J.N. Salunke, 2009b. Some fixed point theorems in fuzzy metric spaces. Int. J. Pure Appl. Math., 56(3): 311-320.

Aage, C.T. and J.N. Salunke, 2010. On fixed point theorems in fuzzy metric spaces using a control function. Int. J. Open Prob. Compt. Math., 3(2).

Aliouche and A. Djondi, 2005. A general Common fixed point theorem reciprocally continuous mapping satisfying an implicit relation. AJMAA, 2(2): 1-7.

Al-Thagafi, and S. Naseer, 2008. Generalized I-nonexpansive selfmaps and invariant approximations. Acta Mathematica Sinica, English Series, May, 24(5): 867876.

Balasubramaniam, P., S. Muralisankar and R.P. Pant, 2002. Common fixed points of four mappings in a fuzzy metric space. J. Fuzzy Math., 10(2): 379-384.

Cho, S.H., 2006. On common fixed point in fuzzy metric space. Int. Math. Forum, 1(10): 471-479.

George and P. Veeramani, 1994. On some results in fuzzy metric spaces. Fuzzy Sets Sys., 64: 395-399.

Grabiec, M., 1988. Fixed points in fuzzy metric spaces. Fuzzy Sets Sys., 27: 385-389.

Imdad, M. and A. Javid, 2006. Some common fixed point theorems in fuzzy metric spaces. Math. Comm., 11: 153-163.

Jachymski, J., 1995. Common fixed point theorems for some families of maps. Indian J. Pure. Appl. Math., 25: 925-937.

Jungck, G., 1988. Compatible mappings and common fixed points (2). Internat. J. Math. Math. Sci., 11: 285-288.

Jungck, G., K.B. Moon, S. Park and B.E. Rhoades, 1993. On generalization of the Meri-keeler type contraction maps. J. Math. Anal. Appl., 180: 221-222.

Kramosil, and J. Michalek, 1975. Fuzzy metric and statistical metric space. Kybernetika, 11: 336-344.

Ojha, D.B., K.M. Manish and K. Udayana, 2010. A common fixed point theorem satisfying integral type for occasionally weakly compatible maps. Res. J. Appl. Sci. Eng. Technol., 2(3): 239-244.

Pathak, H.K. and R. Tiwari and M.S. Khan, 2007. Common fixed point theorem satisfying integral type implicit relations. Appl. Math. E- notes, 7: 222-228.

Pant, R.P., 1996. Common fixed point theorems of sequences of mappings. Ganita, 47: 43-49.

Pant, R.P. and K. Jha, 2004. A remark on common fixed points of four mappings in a fuzzy metric space. J. Fuzzy Math., 12(2): 433-437.

Popa, V., 1997. Fixed point theorems for implicit contractive mappings. Stnd. Cercet, Stiint. Ser, Math. Univ. Bacon., 7: 127-133.

Popa, V., 1999. Some fixed point theorems for compatible mappings satisfying an implicit relation. Demonstration Math, 32(1): 157-163.

Rhoades, B.E. and G. Jungck, 1998. Fixed point for set valued functions without continuity. Indian J. Pure Appl. Math., 29(3): 771-779.

Rhoades, B.E., 2003. Two fixed point theorem for mapping satisfying a general contractive condition of integral type. Int. J. Math. Sci., 3: 4007-4013.

Rhoades, B.E. and G. Jungck, 2006. Fixed point theorems for occasionally weakly compatible mappings. Fixed Point Theory, 7(2): 287-296.

Subrahmanyam, P.V., 1995. Common fixed point theorem in fuzzy metric spaces. Inform. Sci., 83: 105-112.

Vasuki, R., 1999. Common fixed points for R-weakly computing maps in fuzzy metric spaces. Indian J. Pure Appl. Math., 30(4): 419-423.

Zadeh, L.A., 1965. Fuzzy sets. Inform. Control, 8: 338-353.