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A Qualitative Evaluation of Correlated Data from a Noisy Dynamic Process

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Abstract: This study proposes a prediction scheme for qualitative evaluation of noisy dynamic process variables from a set of correlated data. The process models of a noisy power generating plant to, which this study directly applies incorporate a priori knowledge of noise contamination in the measured terminal voltage and load frequency. A conceptual simplicity of parameter by a least squares computational algorithm is evident in the analysis. Convergence of the iteration was enhanced by postulation of a filter to enable the additive noise to be reduced to a white noise sequence which yielded unbiased estimates of the measured data.

Key words: Damped isolation system, electric field sensitive fluid, digital computer simulation, vibration attenuation

INTRODUCTION

Many of the design techniques based on modern control theory (Isaksson *et al*., 2001) assume that values are available for all the state variables in the system of interest. However, in most practical situations, it is not feasible to measure all the state variables and furthermore, the measurements that are available often contain significant amounts of random noise and/or systematic errors in these situations (Lim, 2002; Karimi *et al*., 2004). on-line state estimation techniques are usually employed to produce estimates of the true process values from noisy measurements and their values are instantly calculated. This problem (Hsiao and Wang, 2000; Ahmed and Chang, 1993) is of fundamental interest to operators of discrete time dynamic plants. Several aspects concerning state space formulation which rely on current measurements of the process variables and which control the ease with which generation is adjusted in response to demand, have been addressed in the existing literature (Karimi *et al*., 2004). An important deficiency in the state space description of the problem (Hansen, 2000) is that estimates of all the process variables must be known. Unfortunately only a few of the states are monitored instantaneously because of sensor cost and time delays caused by the need for intermittent processing of data. In absence of correct measurement, any change in the control input can hardly influence the dynamics of interconnected subsystems.

From experience, it is difficult to establish directly the true value of the terminal voltage and the associated load frequency in power generating plants, for example, since all measurements are unavoidable subject to errors (Hyland, 1994; Skelton and Delorenzo, 1983). Elements within the system, which contribute to these errors, are numerous; they range from the problems of infrastructure to errors introduced in transmission of data especially from a remote collation site. The response of any system to a given input function is clearly dependent on the nature of the input as well as the dynamic characteristics of the system. In practice, these errors are collated as additional inputs and thereby modify the response of the system in a random manner. Such inputs, generally referred to as noise when they consist of random fluctuations about a mean value, have a negligible effect in many situations, and can also be the variable causing the observed alteration in the system datum operating point, and hence in the parameter of the system equations (Kammar and Dumont, 2001; Lin *et al*., 1988; Obinabo and Ojieabu, 2007).

State estimation of the process causes a 'smoothing' of a given set of data and eliminates errors in observation (Juang *et al*., 1993), recording transmission (horta *et al*., 1993) and conversion and other types of random errors (Hansen, 1998; Hwang and Ma, 1993) which may somehow become introduced in the data except, of course, bias errors due to a fixed offset, say in the measurements which cannot be eliminated by smoothing techniques. In the light of these, therefore, this paper provides a basis for search for a solution to the filtering problem for optimal operating conditions in a given dynamic process.

MATERIALS AND METHODS

The approach was one of finding the coefficients of the prediction model (Obinabo and Ojieabu, 2007):

$$\hat{y}\left(\frac{k}{k-1}\right) = \frac{B(z^{-1})}{C(z^{-1})}u(k) + \frac{C(z^{-1}) - A(z^{-1})}{c(z^{-1})}y(k) \quad (1)$$

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such that the mean square prediction error:

$$V(\theta) = \sum_{k=1}^{N} y(k) - \hat{y}\left(\frac{k}{k-1}\right)^2 = \sum_{k=1}^{N} \varepsilon^2(k) \quad (2)$$

was as small as possible. By doing so, the assumption of Gaussian distribution of the noise sequence $\varepsilon(k)$ was be relaxed, while on the other hand, the stochastic properties (Lim and Gawronski, 1996; Lin *et al.*, 1988) of the estimates were lost. In the model given in (2), the residues were derived as:

$$\varepsilon(k) = y(k) - \left(\sum_{i=1}^{n} \frac{b_i z^{-1}}{1 + a_i z^{-1}}\right) u(t) \quad (3)$$

An autoregressive residual modeling has been proposed (Esho, 1993; Grimble, 1981; Dalley *et al.*, 1989) and applied to discrete models until the resulting random error series was white. When this method was applied to data μ_f and μ_f obtained by filtering u and y with the noise model, the estimated β's were no longer biased provided the model for the residual e was adequate.
The discrete model (4) was developed as:

$$y(k) = \frac{Bz^{-1}}{Az^{-1}} u(k) + \frac{Dz^{-1}}{Cz^{-1}} \eta(k) \quad (4)$$

Multiplying through by Az^{-1} gives:

$$Ay = Bu + (AD/C)\,\eta \quad (5)$$

$$\text{i.e., } Ay = Bu + e \quad (6)$$

where $e = ADC^{-1}\,\eta(k)$ is correlated noise. (7)
In matrix form we write:

$$\gamma = \phi\beta + e \quad (8)$$

Using least squares:

$$\hat{\beta} = (\phi^T \phi)^{-1} \phi^T \gamma$$
$$= (\phi^T \phi)^{-1} \phi^T (\phi\beta + e)$$
$$= \beta + (\phi^T \phi)^{-1} \phi^T e \quad (9)$$

$$\phi^T \phi(\hat{\beta} - \beta) = \phi^T e \quad (10)$$

Taking the expected values of both sides of this equation:

$$E\left[\phi^T \phi(\hat{\beta} - \beta)\right] = E\left[\phi^T e\right] \quad (11)$$

Since $y(k)$ depends on $e(k)$ then $\phi = [u{:}y]$ and e are correlated.

Thus: $E[\phi^T e] \neq 0$ (12)

Therefore, the least square estimates of $\hat{\beta}$ are biased.
From Eq. (9):

$$E\left[\hat{\beta}\right] = \beta + (\phi^T \phi)^{-1} E\left[\phi^T e\right] \quad (13)$$

An unbiased estimate of β could only be obtained when the noise sequence $e(k)$ was reduced to a white noise sequence. In order to reduce the noise sequence to a white noise sequence, we considered (5) again given by:

$$Ay = Bu + (AD/C)\,\eta$$

Now we postulated a filter F such that (Hansen, 1998; Obinabo and Ojieabu, 2007)

$$F(z^{-1}) = 1 + f_1 z^{-1} + f_2 z^{-1} + \ldots \quad (14)$$

and obtained:

$$A(Fy) = B(Fu) + F(AD/C)\,\eta \quad (15)$$

Now with the filter chosen as: $F = C/AD$ (16)
Equation (15) became:

$$A(Fy) = B(Fu) + \eta \quad (17)$$

and the additive noise $e = (AD/C)\,\eta$ became reduced to a white noise sequence η. The least square estimate $\hat{\beta}$ was then unbiased. Fu and Fy were the filtered inputs and outputs.

$$u^F = Fu; \qquad y^F = Fy \quad (18)$$

The generalized least squares procedure is summarized as follows:

- Form matrices φ, Y and β, and we obtained the simple least squares estimate from the relation:

$$\hat{\beta} = (\phi^T \phi)^{-1} \phi^T Y \quad (19)$$

These estimates were biased because of the correlated noise data.

- Then, we analyzed the residuals using the system equation:

$$Ay = Bu + ADC^{-1}\,\eta \quad (20)$$

Residuals = $\hat{e} = \hat{A}y - \hat{B}u = \hat{A}DC^{-1}\eta$ (21)

This was followed by fitting an autoregressive model to the residuals using the least squares:

$$\hat{\hat{F}}\hat{e} = \eta \quad (22)$$

This resulted in:

$$\hat{\hat{F}}\rangle\rangle\left(ADC^{-1}\right)^{-1} \quad (23)$$

- We then filtered the input and the output with $\hat{\hat{F}}$ so that:

$u^F = Fu$; and $y^F = Fy$ (24)

From which the matrices:

$\phi^F = [u^F : y^F]$ and $Y^F = [y^F]$ (25)

were formed. Using the filtered inputs and outputs, the least squares estimate was obtained as:

$$\hat{\beta} = \left(\left(\phi^F\right)^T \phi^F\right)^{-1}\left(\phi^F\right)^T Y^F \quad (26)$$

We repeated from step 2 until convergence was achieved.

Research example: Consider a representation for an interconnected electric power plant shown in Fig. 1. the representation was of the form indicated in (2) directly with the block diagram.

The equation of this system was obtained as:

$y(k) = Bz^{-1} / Az^{-1}\, u(k) + \eta$ (27)

or simply: $y = (B/A)\, \mu + \eta$ (28)

which was rewritten as:

$Ay = Bu + A\eta$ (29)

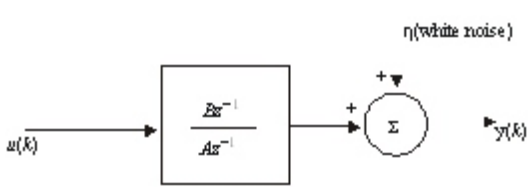

Fig. 1: Interconnected electric power plant

where $A\eta$ was correlated noise. We then applied the generalized least squares to obtain unbiased estimates. We did this first by postulating a filter Fz^{-1} to obtain:

$$A(Fy) = B(Fu) + FA\,\eta \quad (30)$$

Then, we choose F = 1/A so that the correlated noise $FA\eta$ became reduced to a white noise sequence η and the least squares estimate was unbiased.

RESULTS AND DISCUSSION

The general state estimation problem was defined in terms of estimating in some sense, the sequence of the states based on the observed finite sequence subject to the constraints introduced by the dynamic model (2) and the measurement process. The smoothing problem was concerned with estimating the sequence with all data available similar to the classical problem of curve filtering. The filtering problem (Kammar and Dumont, 2001; Ljung, 1979) was computed from the current state of the process and was based on the function of past observation. Our result was parametric in nature with the dependent variables representing unknown parameters in the assumed known structural model.

By applying the established results to the power plant considered in this study, and taking into account the interconnection network Eq. (4), a system of equations in terms of nodal voltages and currents, was reduced to the form:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (31)$$

where A and B were constant matrices of appropriate dimensions. The state variable description of the problem (31) was used to estimate the noise contamination in the process.

The response to the input u(0) = 1.0 of the power generating plant considered above was derived as:

$y(k) = (bz^{-1} / 1 + az^{-1})\, u(k)$ (32)

which resulted in the data sequence shown below:

K	1	2	3	4
y(k)	0.5	-0.4	0.32	-0.256

From (32), we obtained:

$(1 + az^{-1})\, y(k) = bz^{-1}\, u(k)$

$$Y = \begin{bmatrix} -0.400 \\ 0.320 \\ -0.256 \end{bmatrix}, \quad \phi = \begin{bmatrix} -0.50 & 1 \\ 0.40 & 0 \\ -0.32 & 0 \end{bmatrix}$$

$$\hat{\beta} = \left(\phi^T \phi\right)^{-1} \phi^T Y$$

$$\phi^T \phi = \begin{bmatrix} -0.5 & 0.4 & -0.32 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.50 & 1 \\ 0.40 & 0 \\ -0.32 & 0 \end{bmatrix} = \begin{bmatrix} 0.512 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

$$(\phi^T \phi)^{-1} = \frac{1}{0.262} \begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.512 \end{bmatrix}$$

$$\phi^T Y = \begin{bmatrix} -0.5 & 0.4 & -0.32 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -0.400 \\ 0.320 \\ -0.256 \end{bmatrix} = \begin{bmatrix} 0.409 \\ -0.4 \end{bmatrix}$$

$$\hat{\beta} = \frac{1}{0.262} \begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.512 \end{bmatrix} \begin{bmatrix} 0.41 \\ -0.4 \end{bmatrix} = \frac{1}{262} \begin{bmatrix} 0.210 \\ 0.001 \end{bmatrix}$$

giving a = 0.8 and b = 0.0038 as the least squares estimates of the unknown parameters. This result compares favourably with the data reported in the existing literature.

CONCLUSION

We have shown in this study that parameter estimations could be sought by recourse to the assumption that some or all of the parameters may be unknown even though the structure of the differential equation characterizing the system as well as the initial and boundary conditions may be available. This assumption reduced the problem to one of optimal control whereby the 'best' estimate was sought using the measured values of the input and output of the system. Because our research data invariably contained errors, solution of the problem utilized concepts of probability and statistics in which the problem addressed state estimation because noise was known to be correlated with the measured data. This was estimated at the same time as the parameters. It was in connection with this condition that filtration became mandatory in the processing of the acquired data.

The output of the process measuring instrument was an approximation of the true value. Our computation of the estimates relied on convergence of the iteration employed, which was accomplished through sequential filtration of the estimate. There were no useful convergence results available in the existing literature for the plant situation considered in this study which do not operate on the basis of data filtration. The plant considered in this study was represented by a stable, linear, parameter-dependent state space model.

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