Analysis of the Temperature Effect on the Infinite Multiplication Factor for HEU-UAl₄ and LEU-UO₂ Lattices of GHARR-1

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Abstract:
The purpose of the study is to analyze the temperature effect on the infinite multiplication factor for light water moderated High Enriched Uranium (HEU)-UAl₄ and Low Enriched Uranium (LEU)-UO₂ lattices of the Ghana Research Reactor-1 (GHARR-1). To quantify the contribution of each component of the infinite multiplication factor with respect to temperature within the 20 to 140°C range, cell calculations were performed for the two MNSR typical lattices: the 90.2% enriched HEU-UAl₄ and 12.6% enriched LEU-UO₂ proposed fuel of the Ghana Research Reactor-1 (GHARR-1) using SCUBA, a locally developed FORTRAN 95 code for the calculations and analysis of temperature coefficients of GHARR-1. It was observed that at the beginning of life of the core, the temperature coefficient of the resonance escape probability and that of the thermal utilization factor, contributed significantly to the negative temperature coefficient of the infinite multiplication factor obtained for both fuels.

Key words: Core conversion, Ghana Research Reactor-1, infinite multiplication factor, reactor inherent safety, temperature effect, zero Burnup

INTRODUCTION

During reactor operation, numerous parameters such as temperature, pressure, power level, and flow are continuously monitored and controlled to ensure safe and stable operation of the reactor (DOE, 1993). The specific effects of variations in these parameters vary greatly depending upon reactor design. The most significant effect of a variation in temperature when a reactor is in operation is the temperature coefficient (DOE, 1993; Lamarsh and Baratta, 1982). The temperature coefficient which is the relative change of a physical property with respect to changes in temperature is considered as one of the most important physical quantities used to assess the inherent safety of a reactor (DOE, 1993). Temperature variation in a reactor changes the microscopic reaction rate and the density of the moderator and consequently, the infinite multiplication factor. This results in the addition of either positive or negative reactivity (DOE, 1993; Ott and Bezella, 1989) and hence changes in reactor power. The temperature dependence of the infinite multiplication factor is considerably important in connection with the transient behavior of a reactor (Ott and Bezella, 1989) and this has associated nuclear criticality safety issues and must therefore be investigated since it has a large influence on the estimation of operational reactivity balance.

In this research, the effect of temperature on the infinite multiplication factor has been analyzed in order to ascertain the polarity, magnitude and effects of fuel and moderator temperatures on its physical components for the safe operation of the reference 90.2% HEU-UAl₄ and the proposed 12.6% LEU-UO₂ fuel lattices of the Ghana Research Reactor-1 (GHARR-1) at zero burnup. The research was done as part of the core conversion studies currently on-going at the Department of Nuclear Engineering and Materials Science (NEMS) in the National Nuclear Research Institute (NNRI) of Ghana Atomic Energy Commission (GAEC).

Reactor description: GHARR-1 is a Chinese built Miniature Neutron Source Reactor (MNSR) with a maximum thermal neutron flux of $1 \times 10^{12}$ n cm$^{-2}$s$^{-1}$ and a thermal power of approximately 30 kW. The MNSR is used for Neutron Activation Analysis (NAA), training of Scientist and Engineers in nuclear science and technology and small scale radioisotope production (Anim-Sampong et al., 2007; Akaho et al., 2003). In Fig. 1 and 2, the horizontal and vertical cross sectional views of the GHARR-1 core are presented. A detailed description of the reactor is presented elsewhere (Alhassan et al., 2010; Akaho et al., 2003).

MATERIALS AND METHODS

The study was carried out in the year 2009 at the Graduate School of Nuclear and Allied Sciences of the University of Ghana, Ghana, West Africa.
A locally developed computer code named SCUBA, written in FORTRAN-95 programming language was used in the computation and analysis of the effect of the fuel and moderator temperature changes on the infinite multiplication factor based on Eq. (1) to (16). A detailed structure and development of the code has been presented elsewhere (Alhassan et al., 2010).

**Theoretical consideration:** For the maintenance of a self-sustaining chain reaction, it is necessary that every neutron produced in fission initiate another fission (DOE, 1993). The minimum requirement is for each nucleus undergoing fission to produce on the average, at least one neutron that will cause the fission of another nucleus. This condition is conveniently expressed in
terms of the multiplication factor, which is given by (Gladstone and Sesonke, 1981):

\[ k_e = \varepsilon f \eta \]

(1)

where, \( \varepsilon \) is the fast fission factor, \( p \) is the resonance escape probability, \( f \) is the thermal utilization factor and \( \eta \) is the thermal fission factor.

The Reactivity Temperature Coefficient (\( \alpha_r \)) which is defined as the change in reactivity with respect to temperature is expressed as (Ott and Bezella, 1989):

\[ \alpha_r = \frac{d\rho}{dT} = \frac{\frac{dk}{dT}}{k_{eff}} \]

(2)

where, reactivity (\( \rho \)) which is the fractional departure of a system from criticality is expressed as (Lamarsh and Baratta, 1982):

\[ \rho = \frac{k_{eff} - 1}{k_{eff}} \]

(3)

Differentiating equation (3) with respect to temperature (T) gives (Lamarsh and Baratta, 1982):

\[ \alpha_r = \frac{d\rho}{dT} = \frac{1}{k_{eff}} \frac{dk}{dT} - \frac{1}{k_{eff}^2} \frac{dk_{eff}}{dT} \]

\[ = \frac{1}{k_{eff}} \frac{dk_{eff}}{dT} - \frac{1}{k_{eff}^2} \frac{dk_{eff}}{dT} \]

(4)

where, \( k_{eff} \) is the effective multiplication number.

From the basic relationship between \( k_{eff} \) and \( k_4 \), \( P\_NL \), where \( k_4 \) is the infinite multiplication factor and \( P\_NL \) is the non-leakage probability; we obtain by differentiating Eq. (4):

\[ \alpha_r = \frac{1}{k_{eff}} \frac{dk_{eff}}{dT} - \frac{1}{k_{eff}} \frac{dk_{eff}}{dT} + \frac{1}{P\_NL} \frac{dP\_NL}{dT} \]

(5)

Equation (5) can be written as:

\[ \alpha_r = \alpha_{ke} + \alpha_L \]

(6)

where, the first term is the temperature coefficient of the infinite multiplication factor and the second term represent the leakage effect. By using the four factor formula, \( \alpha_{ke} \) can be expressed as follows:

\[ \alpha_{ke} = \frac{1}{k_e} \frac{dk_e}{dT} = \frac{1}{\varepsilon} \frac{d\varepsilon}{dT} + \frac{1}{p} \frac{dp}{dT} + \frac{1}{f} \frac{df}{dT} + \frac{1}{\eta} \frac{d\eta}{dT} \]

(7)

**Temperature effect on the fast fission factor:** For the calculation of the fast fission factor, it was assumed that, thermal fission occur only in U-235 whiles fast fission and epithermal absorptions occur in U-238 only. For a heterogeneous reactor, is given by (Bennet, 1972):

\[ \varepsilon = \frac{1 + 0.690 \left( \frac{N_{28}}{N_{H2O}} \right)}{1 + 0.563 \left( \frac{N_{28}}{N_{H2O}} \right)} \]

(8)

where, \( \frac{N_{28}}{N_{H2O}} \) is the ratio of the number of atoms of U-238 to that of water. To analyze the temperature effect on the fast fission factor, the moderator densities were varied with temperature and the corresponding number densities computed. The temperature coefficients for the various temperatures were calculated using Eq. (8) by applying polynomial of best fit and then differentiating and the results are plotted in Fig. 3.

**Temperature effect on the thermal utilization factor:**

The thermal utilization factor (\( f \)) by definition is given by (Akaho, 2007):

\[ f = \frac{\sum_f \phi^f \nu^f}{\sum_f \phi^f \nu^f + \sum_m \phi^m \nu^m + \sum_c \phi^c \nu^c} \]

(9)

where, the subscripts \( f, m, c \) represents fuel, moderator and cladding respectively. The moderator macroscopic cross section is given by:

\[ \sum_m = \frac{10^3 \rho_m N_A \sigma_m}{A_m} \]

(10)

where, \( \sigma_m \) is the microscopic cross section for the moderator, \( A_m \) is the atomic weight of the moderator, \( \rho_m \) is the density of the moderator and \( N_A \) is the Avogadro’s number. After substituting Eq. (10) into (9) and dividing through by \( N_{25} \), \( f \) becomes:

\[ f = \frac{\sigma_{25}^{25} + \left( \frac{N_{28}}{N_{25}} \right) \sigma_{28}^{28} + \left( \frac{N_{28}}{N_{25}} \right)^2 \sigma_{28}^{28} + \left( \frac{NH2O}{N_{25}} \right) \sigma_{H2O}^{H2O} + \left( \frac{NAL}{N_{25}} \right) \sigma_{Al}^{Al}}{\sigma_{25}^{25} + \left( \frac{N_{28}}{N_{25}} \right) \sigma_{28}^{28} + \left( \frac{NH2O}{N_{25}} \right) \sigma_{H2O}^{H2O} + \left( \frac{NAL}{N_{25}} \right) \sigma_{Al}^{Al}} \]

(11)

where, \( N_{25} \) and \( N_{28} \) are the number densities for U-235 and U-238, \( N_{Al} \) and \( N_{H2O} \) are the number densities for Al and
Temperature effect on the thermal fission factor: The thermal fission factor which is the number of fast neutrons produced per thermal neutron absorbed in the fuel is given by (Akaho, 2007):

\[ \eta = \frac{v N_{25} \sigma_{f25}}{N_{25} \sigma_{a25} + N_{28} \sigma_{a28}} \]  

(12)

where, \( v \) is the enrichment of the fuel, \( \sigma_{f25} \) is the fission microscopic cross sections for U-235, \( \sigma_{a25} \) and \( \sigma_{a28} \) are the absorption microscopic cross sections for U-235 and U-238 and, \( N_{25} \) and \( N_{28} \) are the number densities of U-235 and U-238, respectively.

The temperature dependence of the thermal fission factor (\( \eta \)) is contained in the microscopic cross sections of the various isotopes in the fuel. Since U-238 and U-235 are both non-1/v nuclides, the Westcott correction factor \( g(T) \) was used for the correction of the absorption cross sections.

For U-238 the absorption cross section is given as (Lamarsh and Baratta, 1982):

\[ \sigma_{a28}(T) = \frac{g(T)\sigma_{a28}(T_0)}{1.128} \left( \frac{T_0}{T} \right)^{1/2} \]  

(13)

where, \( \sigma_{a28}(T) \) and \( \sigma_{a28}(T_0) \) are the microscopic cross section of U-238 at temperature \( T \) and \( T_0 \) respectively, \( T_0 = 293 \) K and \( T = (\theta + 293) \) K with \( \theta \) in °C.

While for U-235, the absorption cross section was given by (Lamarsh and Baratta, 1982):

\[ \sigma_{a25}(T) = \frac{g(T)\sigma_{a25}(T_0)}{1.128} \left( \frac{T_0}{T} \right)^{1/2} \]  

(14)

where, \( \sigma_{a25}(T) \) and \( \sigma_{a25}(T_0) \) are the microscopic cross section of U-235 at temperature \( T \) and \( T_0 \) respectively. SCUBA calculates using Eq. (12) to (14) for both the 12.6% UO\(_2\) and 90.2% UAl\(_2\) enrichments for temperatures between 20 and 140°C.

Temperature effect on the resonance escape probability: With the assumption that all the resonances are resolved and do not overlap, the resonance escape probability, \( P \) for a heterogeneous reactor is given mathematically by (Zweitfel, 1985):

\[ p = \exp\left[ \frac{N_{a2} V^A I_{\text{eff}}}{\xi \sum_p} \right] \]  

(15)

with,

\[ \xi \sum_p = \frac{V^A \left( \zeta F \sum_p + \zeta_m \sum_{pM} \right) + V^C \sum_p + V^M \sum_p}{V^C + V^M + V^C} \]  

(16)

where, \( \xi \sum_p \) is the average slowing down power in the non-resonant region, \( \sum_M \) is the potential cross section of the moderator, \( \sum_{pM} \) is the potential cross section of region A, \( \xi_m \) is the loss energy decrement of the moderator, \( \sum_{AM} \) is the scattering macroscopic cross section of the moderator, \( N_{23} \) is the density of atoms/nuclei of U-238, \( V^A \), \( V^M \) and \( V^C \) are the volume of fuel region, moderator and clad respectively and \( I_{\text{eff}} \) is the effective Resonance Integral from the Strawbridge correlation.

The temperature effect on the resonance escape probability was calculated by varying the fuel temperature and computing the corresponding cross sections. The temperature coefficient for the various fuel temperatures were calculated using Eq. (15) by differentiating the polynomial of best fit and the results are plotted in Fig. 5.

RESULTS

The results of the factors of the temperature coefficient of the infinite multiplication factor calculated and averaged over the temperature range of 20 to 140°C for the LEU and HEU fuels of GHARR-1 and experimental results are compared in Table 1.

The factors of the temperature coefficient of the infinite multiplication factor plotted against temperature (of 20 to 140°C) for the LEU and HEU fuels of GHARR-1 are presented in Fig. 3, 4, 5, 6 and 7.

DISCUSSION

In Table 1, the temperature coefficient of the infinite multiplication factor averaged over the 20 to 140°C temperature range have been presented. The results obtained from the code showed the values of the temperature coefficient of the thermal utilization factor and that of the resonance escape probability to be -0.070...
Table 1: Temperature effect of $k_4$ in a UAl$_4$ and UO$_2$ lattice of GHARR-1

<table>
<thead>
<tr>
<th>Fuel lattice</th>
<th>HEU-UAl$_4$ Experimental (20 to 45 °C)/mk</th>
<th>HEU-UAl$_4$, This work (20 to 140 °C)/mk</th>
<th>LEU-UO$_2$, This work (20 to 140 °C)/mk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/p $\left( \frac{dP}{dT} \right)$</td>
<td>Negligible</td>
<td>-0.00056</td>
<td>-0.0058</td>
</tr>
<tr>
<td>1/f $\left( \frac{df}{dT} \right)$</td>
<td>-0.092</td>
<td>-0.070</td>
<td>-0.160</td>
</tr>
<tr>
<td>1/\eta $\left( \frac{d\eta}{dT} \right)$</td>
<td>-</td>
<td>+0.00056</td>
<td>+0.028</td>
</tr>
<tr>
<td>1/k$_4$ $\left( \frac{d(k_4)}{dT} \right)$</td>
<td>-0.092</td>
<td>-0.073</td>
<td>-0.131</td>
</tr>
</tbody>
</table>

From Fig. 3, it can be observed that the temperature coefficient of the fast fission factor increases linearly with temperature. For the HEU-UAl$_4$ fuel, the coefficient is -0.092 mk, and for the LEU-UO$_2$ fuel, it is -0.160 mk. These results are in reasonable agreement with literature. The temperature coefficient of the thermal utilization factor was found to be negligible for the 90.2% HEU fuel. These results are therefore in reasonable agreement. The contribution of the temperature coefficient of the fast fission factor and the thermal fission factor were found to be positive but less dominant.
temperature increase and its contribution to the temperature coefficient of the multiplication factor was found to be positive for both fuels but more significant for the LEU fuel. This can be attributed to the moderator density effect; an increase in temperature results in a reduction of the moderation ratio and hence leads to an increase in the fast to thermal ratio. Since fast fission can only occur in U-238 nuclei whiles fission in U-235 nuclei is limited to only thermal neutrons, LEU fuel with a higher amount of U-238 is fissioned by some of the fast neutrons and thereby making available more neutrons in the reactor core.

The temperature effect of the thermal utilization factor was found to be negative for both fuels and decreases linearly for both fuels as shown in Fig. 4. The negative contribution of the temperature coefficient of the thermal utilization factor is related to the water density effect; an increase in moderator temperature reduces the number of neutron collisions per unit volume and so causing the neutron spectrum to harden (Alhassan, 2009; Mahmood et al., 2008). This alters the balance between the fission and absorption rates and reduces the thermal fission factor and hence the temperature coefficient of the thermal utilization factor. The hardened spectrum results in an increase in resonance absorption for the low-lying resonances of U-238 which are more dominant in LEU-UO2 than the HEU-UAl4 fuel hence that of the LEU fuel was seen to drop more sharply.

The linear nature of the plots in Fig. 5 illustrates the fact that, the temperature of the fuel reacts immediately to changes in reactor power for both fuels. With increasing temperature, U-238 resonances broaden thereby increasing the absorption of neutrons (Doppler broadening). Because of the strong U-238 capture cross section in the LEU fuel, the temperature effect on the resonance escape probability is more dominant in the 12.6% LEU-UO2 core than the HEU-UAl4 fuel hence that of the LEU fuel was seen to drop more sharply.

As can be observed from Fig. 6 that, the temperature coefficient of the thermal fission factor was positive and decreases almost linearly for LEU fuel whiles increasing gradually for the HEU fuel for the whole temperature range. This temperature effect is mainly due to the spectral shift effect which is more dominant for the LEU fuel because the thermal fission factor is enrichment dependent. Also, the thermal energy spectrum changes and broadens with temperature and hence alters the balance between the fission and absorption rates in the core.

From Fig. 7, it is observed that the temperature coefficient of the infinite multiplication factor decreases for the entire temperature range (20-140°C). The negative contribution comes from the temperature coefficient of the thermal utilization factor and resonance escape probability whiles the temperature coefficient of the fast fission factor and the thermal fission factor both contributed positively.

CONCLUSION

The analyses of the temperature effect on the infinite multiplication factor for the light water moderated 90.2% High Enriched Uranium (HEU)-UAl4 and the proposed 12.6% Low Enriched Uranium (LEU)-UO2 lattices of the Miniature Neutron Source Reactor (MNSR) for a fresh core replacement has been carried out using SCUBA a locally developed FORTRAN 95 code. The results showed the predominance of the contribution of the temperature coefficients of the resonance escape probability and the thermal utilization factor to the negative temperature coefficient of the infinite multiplication factor. The contribution of the temperature coefficient of the resonance escape probability which was negligible with the HEU fuel will contribute significantly to the inherent safety ability of the proposed LEU fuel for GHARR-1.

ACKNOWLEDGMENT

Authors want to thank friends and colleagues at the Ghana Research Reactor-1 Centre, National Nuclear Research Institute, Ghana Atomic Energy Commission for their suggestions and encouragement.

REFERENCES


