

Performance Analysis of Modified Nonsubsampled Contourlet Transform for Image Denoising

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Abstract: In this study, we develop modified Nonsubsampled Contourlet Transform (NSCT). The construction of NSCT is based on new nonsubsampled pyramid structure and Nonsubsampled Directional Filters (NSDF). The result is improved in flexible multiage, multidirectional and shift invariant image decomposition that can be effectively implemented through Matlab. The modified NSCT, it proposed to distinguish noise and edge effectively. So we assess the performance of the modified NSCT in image denoising applications. In this application the NSCT compares favorably to other existing method.

Key words: Contourlet transform, modified nonsubsampled contourlet transform, image denoising, nonsubsampled directional filter banks, multidirectional decomposition

INTRODUCTION

During the acquiring and transferring of image, there always exists noise, so in order to improve the quality of image, we must do some work to delete noise. Since eliminating noise and preserving the edge is the main problem in image denoising by NSCT. With this NSCT we loose the information in high frequency bands. So that the authors are focused on the edge and texture detail. A more advanced method such as wavelets, NSWT, curvelets (Cand and Donoho, 2004) and contourlet transform (Cunha, 2005) eliminates the noise in redundant images. The redundant images represents more flexible to design. In this paper application as denoising and contour detection, a redundant representation outperform a non redundant.

Another important feature of a transform is its stability with respect to shifts of the input signal. An example that will discuss the importance of shift in variance is image denoising by thresholding where the lack of shift invariance phenomena around singularities (Coifman, 1995). Thus, most state-of-the-art wavelet denoising algorithms (Chang, 2000) use an expansion with less shift sensitivity than the standard maximally decimated wavelet decomposition the most common being the nonsubsampled wavelet transform (NSWT) computed with a trous algorithm (Shensa, 1992) now it is implemented using Mat lab software.

In addition to shift-invariance, it has been recognized that an efficient image representation has to account for the geometrical structure pervasive in natural scenes. In this direction, several representation schemes have

recently been proposed (Donoho, 1999). The contourlet transform is a multidirectional and multiscale transform that is constructed by combining the Laplacian pyramid (Do and Vetterli, 2005) with the directional filter bank (Bamberger and Smith, 1992) proposed. The pyramidal filter bank structure of the contourlet transform has very little redundancy, which is important for compression applications. However, designing good filters for the contourlet transform is a difficult task.

So In this study, we propose an over complete transform that we call the modified Nonsubsampled Contourlet Transform (NSCT). Our main motivation is to construct a flexible and efficient transform targeting applications where redundancy is not a major issue (e.g., denoising). The NSCT is a fully shift-invariant, multiscale, and multidirectional expansion that has a fast implementation. The proposed construction leads to a filter-design problem that to the best of our knowledge has not been addressed elsewhere. The design problem is much less constrained than that of contourlets. This enables us to design filters with better frequency selectivity thereby achieving better subband decomposition. So the NSCT has proven to be very efficient in image denoising. So first let us see the contourlet transform and its construction.

In the literature (Cunha *et al.*, 2005; Donoho, 1999; Simoncelli *et al.*, 1992; Pennec and Mallat, 2005; Cand and Donoho, 2004; Liang, 2008; Portilla *et al.*, 2003; Rosiles and Smith, 2003; Sendur and Selesnick, 2002; Luo and Wu, 2008; Shensa, 1992; Wakin *et al.*, 2006; Do and Vetterli, 2005; Qing *et al.*, 2008; Coifman and Donoho, 1995; Bamberger and Smith, 1992; Chang

et al., 2000), the modified nonsubsampled contourlet transform is not present. In this work an attempt is made to implement modified NSCT using mat lab.

CONTOURLET TRANSFORM

Contourlet transform can offer a sparse representation for piecewise smooth images. By first applying a multiscale transform and then applying a local direction transform to gather the nearby basis function at the same scale into linear structures. With this insight, a double filter bank structure in which at first the Laplacian Pyramid (LP) is used to capture the point discontinuities, And followed by a Directional Filter Bank (DFB) to link point discontinuities into linear structure.

Laplacin pyramid: The basic idea of the LP is the following. First, derive a coarse approximation of the original signal, by low pass filtering and down sampling. Based on this coarse version, predict the original (by up sampling and filtering) and then calculate the difference as the prediction error. Usually, for reconstruction, the signal is obtained by simply adding back the difference to the prediction from the coarse signal. The process can be iterated on the coarse version. Analysis and usual synthesis of the LP are shown in Fig. 1a and b, respectively. The outputs are a coarse approximation c and a difference d between the original signal and the prediction. The process can be iterated by decomposing the coarse version repeatedly

Directional Filter Banks (DFB): We introduced a 2-D directional filter bank (DFB) that can be maximally decimated while achieving perfect reconstruction. The DFB is efficiently implemented via an 1-level tree structured decomposition that leads to 2^l sub bands with wedge-shaped frequency partition as shown in Fig. 2.

The original construction of the DFB in involves modulating the input signal and using diamond-shaped filters. Furthermore, to obtain the desired frequency partition, an involved tree expanding rule has to be followed. As a result, the frequency regions for the resulting sub bands do not follow a simple ordering as shown in Fig. 2 based on the channel indices. The new DFB avoids the modulation of the input image and has a simpler rule for expanding the decomposition tree. We focus on the analysis side of the DFB since the synthesis is exactly symmetric. Intuitively, the wedge-shaped frequency partition of the DFB is realized by an appropriate combination of directional frequency splitting by the fan QFB's and the "rotation" operations done by resampling. To obtain a four directional frequency partitioning, the first two decomposition levels of the DFB are given in Fig. 3. We chose the sampling matrices

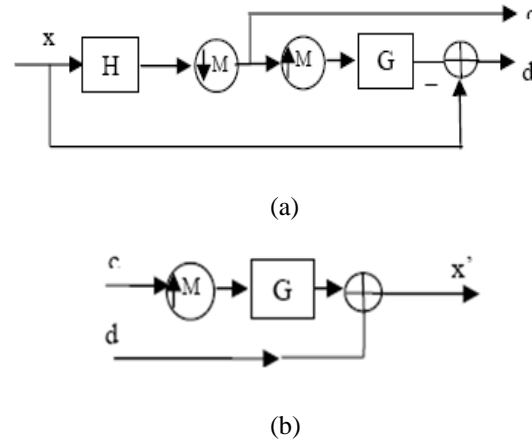


Fig. 1: Laplacian pyramid scheme (a) Analysis (b) Synthesis

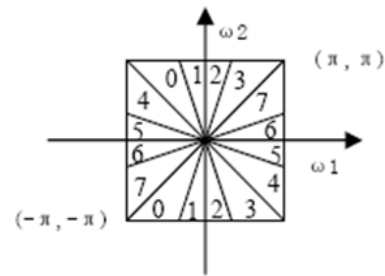


Fig. 2: Frequency portioning obtained with NSCT

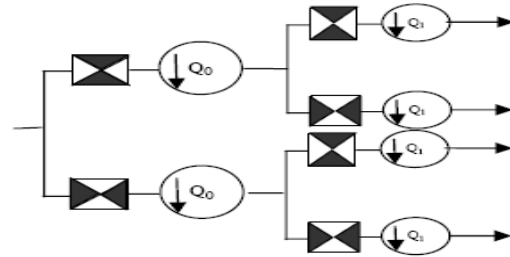


Fig. 3: The first two levels of the DFB. At each level, QFB's with fan filters are used. The black regions represent the ideal frequency supports of the filters

in the first and second level to be Q_0 and Q_1 , respectively, so that the overall sampling after two levels is $Q_0 Q_1 = 2 \cdot 12$, or down sampling by two in each dimension.

Directional Filter Bank (DFB): So the contourlet allows form any number of DFB decomposition levels l_j to be applied at each LP level j and then can realize the property of anisotropy scaling. Figure 4 shows a multiscale and directional decomposition using a combination of a LP and a DFB. Band pass images from the LP are fed into a DFB so that directional information can be captured. The scheme can be iterated on the coarse

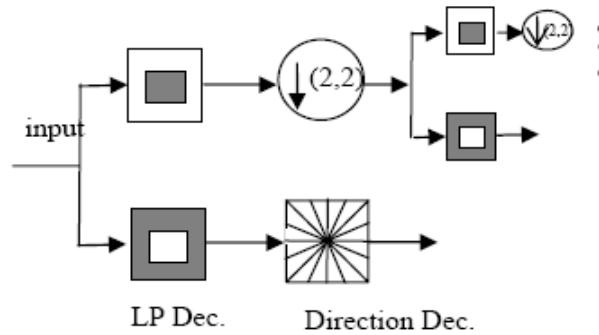


Fig. 4: The nonsubsampling contourlet

image. The combined result is a double iterated filter bank structure, named contourlet filter bank, which decomposes images into directional sub bands at multiple scales.

An effective transform captures the essence of a given signal or a family of signals with few basis functions. The set of basis functions completely characterizes the transform and this set can be redundant or not, depending on whether the basis functions are linear dependent. By allowing redundancy, it is possible to enrich the set of basis functions so that the representation is more efficient in capturing some signal behavior. In addition, redundant representations are generally more flexible and easier to design. In applications such as denoising, enhancement, and contour detection, a redundant representation can significantly outperform a no redundant one.

Due to down sampling and up sampling, the contourlet transform is shift-variant and the lack of shift invariance causes pseudo-Gibbs phenomena around singularities. To achieve the shift-invariance, the nonsubsampling contourlet transform is applied which built upon nonsubsampling pyramids and Nonsubsampling DFB. Compare with the common contourlet transform, the NSCT has such advantage as follows. First each subband has the same size, so it is easier to gain the relationship among the sub-bands. Second the resolution can be kept since the original data is not decimated, at the same time the contourlet coefficients has more redundant information which helps to distinguish the noise from feature.

CONSTRUCTION OF NSCT

Figure 5a displays an overview of the proposed NSCT. The structure consists in a bank of filters that splits the 2-D frequency plane in the subbands illustrated in Fig. 5b. Our proposed transform can thus be divided into two shift-invariant parts:

- A nonsubsampling pyramid structure that ensures the multiscale property

- A nonsubsampling DFB structure that gives directionality

Nonsubsampling Pyramid (NSP): The multiscale property of the NSCT is obtained from a shift-invariant filtering structure that achieves subband decomposition similar to that of the Laplacian pyramid. This is achieved by using two-channel nonsubsampling 2-D filter banks. Fig.6 illustrates the proposed Nonsubsampling Pyramid (NSP) decomposition with $J = 3$ stages. Such expansion is conceptually similar to the one dimensional (1-D) NSWT computed with the Matlab and has $J+1$ redundancy, where J denotes the number of decomposition stages. The ideal pass band support of the low-pass filter at the j th stage is the region $[-(\pi/2^j), (\pi/2^j)]^2$. Accordingly, the ideal support of the equivalent high-pass filter is the complement of the low-pass, i.e., the region $[-(\pi/2^{j-1}), (\pi/2^{j-1})]^2 / [-(\pi/2^j), (\pi/2^j)]^2$. The filters for subsequent stages are obtained by upsampling the filters of the first stage. This gives the multiscale property without the need for additional filter design. The proposed structure is thus different from the separable NSWT. In particular, one band pass image is produced at each stage resulting in $J+1$ redundancy. By contrast, the NSWT produces three directional images at each stage, resulting in $3J+1$ redundancy. The 2-D pyramid is obtained with a similar structure. Specifically, the NSFB is built from low-pass filter $H_0(Z)$. One then set $H_1(Z) = 1 - H_0(Z)$, and the corresponding synthesis filters $G_0(Z) = G_1(Z) = 1$. A similar decomposition can be obtained by removing the down samplers and upsamplers in the Laplacian pyramid and then upsampling the filters accordingly. Those perfect reconstruction systems can be seen as a particular case of our more general structure. The advantage of our construction is that it is general and as a result, better filters can be obtained.

Nonsubsampling Directional Filter Bank (NSDFB): The directional filter bank is constructed by combining critically-sampled two-channel fan filter banks and resampling operations. The result is a tree-structured filter

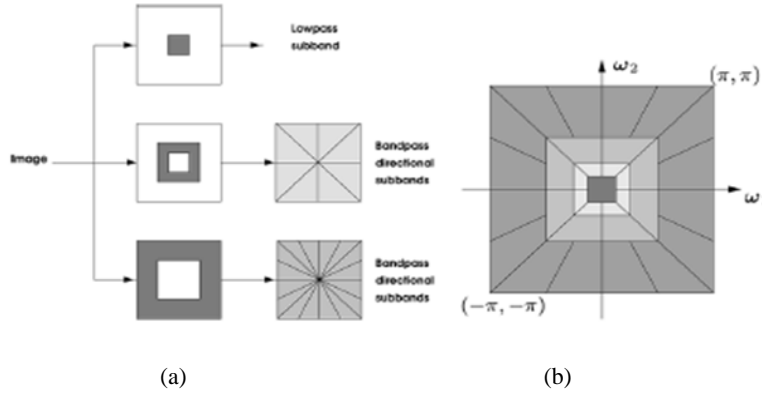


Fig. 5: Nonsubsampled contourlet transform. (a) NSFB structure that implements the NSCT; (b) Idealized frequency partitioning obtained with the proposed structure

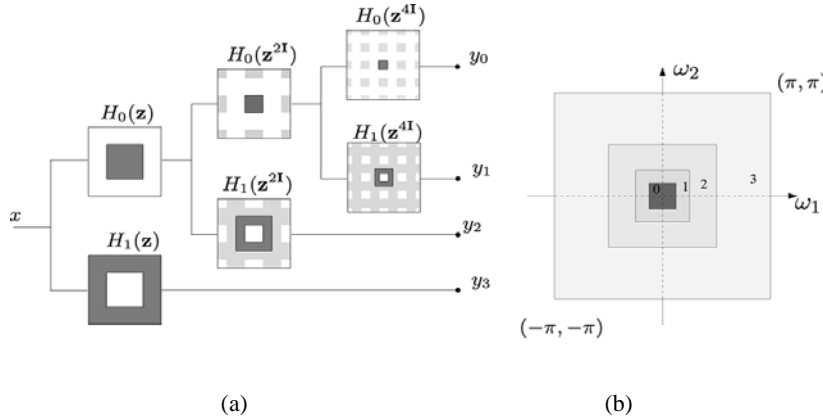


Fig. 6: Proposed nonsubsampled pyramid is a 2-D multiresolution expansion similar to the 1-D NSWT. (a) Three-stage pyramid decomposition. The lighter gray regions denote the aliasing caused by upsampling; (b) Subbands on the 2-D frequency plane.

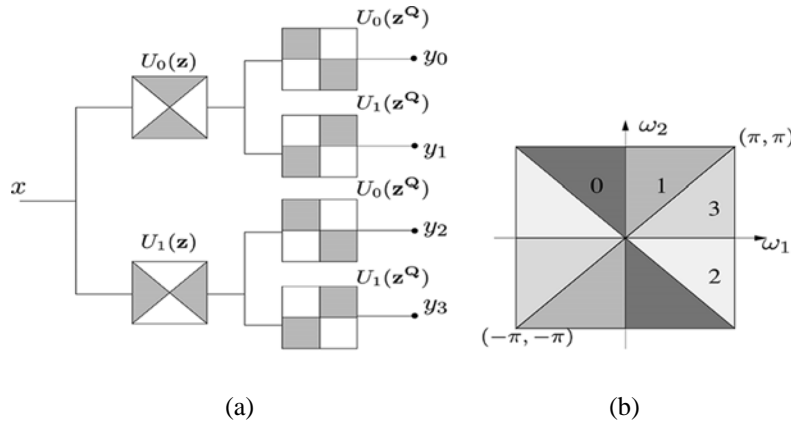


Fig. 7: Four-channel nonsubsampled directional filter bank constructed with two-channel fan filter banks. (a) Filtering structure; (b) Corresponding frequency decomposition

bank that splits the 2-D frequency plane into directional wedges. A shift-invariant directional expansion is obtained with a nonsubsampled DFB (NSDFB). The NSDFB is constructed by eliminating the down samplers and up samplers in the DFB. This is done by switching off the down samplers/up samplers in each two-channel filter

bank in the DFB tree structure and up sampling the filters accordingly. This result in a tree composed of two-channel NSFBS. Figure 7 illustrates four channel decomposition. Note that in the second level, the up sampled fan filters $U_i(Z^Q)$, $i = 0, 1$ have checker-board frequency support, and when combined with the filters in

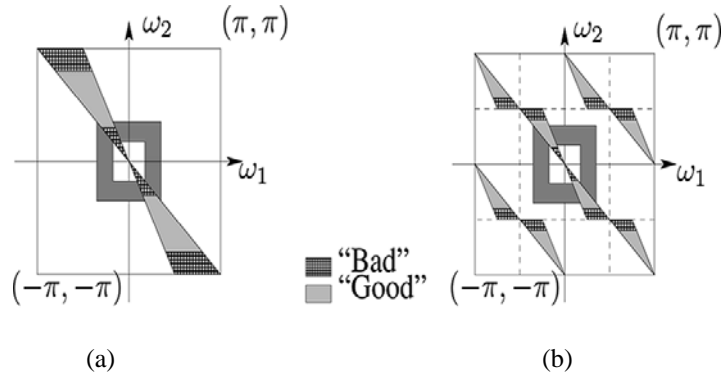


Fig. 8: Need for upsampling in the NSCT. (a) With no upsampling, the high pass at higher scales will be filtered by the portion of the directional filter that has "bad" response; (b) Upsampling ensures that filtering is done in the "good" region

the first level give the four directional frequency decomposition shown in Fig. 7. The synthesis filter bank is obtained similarly. Just like the critically sampled directional filter bank, all filter banks in the nonsubsampling directional filter bank tree structure are obtained from a single NSDFB with fan filters. Moreover, each filter bank in the NSDFB tree has the same computational complexity as that of the building-block NSFB.

Combining the Nonsubsampling pyramid and nonsubsampling directional filter bank in the NSCT:

The NSCT is constructed by combining the NSP and the NSDFB as shown in Fig. 5a. In constructing the NSCT, care must be taken when applying the directional filters to the coarser scales of the pyramid. Due to the tree-structure nature of the NSDFB, the directional response at the lower and upper frequencies suffers from aliasing which can be a problem in the upper stages of the pyramid. This is illustrated in Fig. 8a, where the pass band region of the directional filter is labeled as "Good" or "Bad." Thus, we see that for coarser scales, the high-pass channel in effect is filtered with the bad portion of the directional filter pass band. This results in severe aliasing and in some observed cases a considerable loss of directional resolution.

We remedy this by judiciously upsampling the NSDFB filters. Denote the k -th directional filter by $U_k(z)$. Then for higher scales, we substitute $U_k(z^{2^m}I)$ for $U_k(z)$ where m is chosen to ensure that the good part of the response overlaps with the pyramid pass band. Figure 8b illustrates a typical example. Note that this modification preserves perfect reconstruction. In typical five-scale decomposition, we up sample by $2I$ the NSDFB filters of the last two stages. Filtering with the upsampled filters does not increase computational complexity. Specifically, for a given sampling matrix and a 2-D filter $H(z)$, to obtain the output $y(n)$ resulting from filtering $x(n)$ with $H(z^s)$, we use the convolution formula:

$$Y[n] = \sum h[k]x[n-sk]$$

This is the result of Matlab. Therefore, each filter in the NSDFB tree has the same complexity as that of the building-block fan NSFB. Likewise, each filtering stage of the NSP has the same complexity as that incurred by the first stage. Thus, the complexity of the NSCT is dictated by the complexity of the building-block NSFBs. If each NSFB in both NSP and NSDFB requires L operations per output sample, then for an image of N pixels the NSCT requires about BNL operations where B denotes the number of subbands. For instance, if $L = 32$, a typical decomposition with four pyramid levels, 16 directions in the two finer scales, and eight directions in the two coarser scales would require a total of 1536 operations per image pixel.

The modified method of NSCT on image denoises:

Since the NSCT can decompose image in multi-scale and multi-direction. We know that the edge can be kept best when the anisotropic filter's long axes is in accord with the edge, and with the angle between the edge and the anisotropic filter's long axes becomes filter's long axes is placed at different orientations and which resulting in the many directional sub bands. In these directional sub bands, every one represents a direction and the edge in this direction has a largest gray scale comparing with which in any other direction since the edge in this direction is in accord with the filter's long axes and the largest gray scale is denoted as the largest contourlet coefficients in corresponding position.

On the base of the idea that after the transforming of contourlet, the image contourlet coefficients is larger than the noise contourlet coefficients, we can compare the contourlet coefficients at the same locations among the directional sub bands, since the largest coefficients in the same location represent the gray scale is largest and the edge direction is in accord with the filter's long axes and

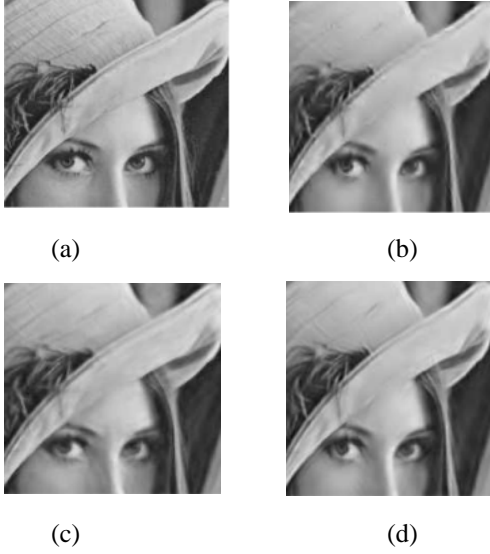


Fig. 9: Image denoising with the NSCT. Where the noisy intensity is 20 (a) Original Lena image (b) Denoised with the NSWT, PSNR = 32:40 dB (c) Denoised with the curvelet transform and hard thresholding, PSNR = 32:52 dB. (d) Denoised with the NSCT, PSNR = 33:04 dB

which means the edge is kept best. Through choosing the largest contourlet coefficients in each corresponding location, we choose the best kept image edge and by comparing the contourlet coefficients the small noise coefficients are replaced by the large image edge, so it removes the noise without the need to set a threshold

We summarize our denoising method using the NSCT in the following algorithm:

- Compute the NSCT of the input image for N levels and the direction of every level can be chosen as needed.
- As we know, the noises are in high frequency bands which lie in the first two or three levels, so we just need to adjust the contourlet coefficients in these first two or three levels. For each level, we compare the directional subband coefficients in the same location and replace the small one with the large one. For example, at the n level, where $n = 1, 2, 3$

$$d(j,k,n) = C_n^l(j,k); \quad \text{if } d(j,k,n) < C_n^l(j,k) \\ = d(j,k,n); \quad \text{otherwise}$$

where l denotes the direction and $l=1, 2, \dots, L-1$. And we use the $d(j,k,n)$ to replace the $C_n^l(j,k)$ so all the contourlet coefficients represent the best kept edge.

- Reconstruct the denoising image from the modified NSCT coefficients, and then we can get a denoised image.

Table 1: Show the denoising performance of the NSCT

| Comparison to other transform | | | | | |
|-------------------------------|-------|-------|----------|-------|---------------|
| Lena | Noisy | NSWT | CURVELET | CT | MODIFIED NSCT |
| 10 | 28.13 | 35.26 | 35.17 | 32.91 | 35.69 |
| 20 | 22.13 | 32.40 | 32.52 | 29.30 | 33.04 |
| 30 | 18.63 | 30.66 | 31.10 | 28.01 | 31.05 |
| 40 | 16.13 | 28.37 | 29.14 | 26.85 | 30.10 |
| 50 | 14.20 | 27.41 | 28.16 | 25.87 | 29.10 |

NUMERICAL EXPERIMENTS

We compare the denoising results by the proposed method with modified NSCT. In this paper we choose a Lena image to satisfy our method. Figure 9 shows our results. Figure 9a represents the original Lena image. Figure 9b represents the denoised image using NSWT with PSNR 32.40dB. Figure 9c shows denoised image using with PSNR 32.52 dB. Figure 9d shows result of our approach. From these figures we can show that NSCT has more efficient. And result our approach has less noise and increase its PSNR values. Several noise images are tested to improve the effect and results are listed in Table 1.

CONCLUSION

In this study we have developed a fully shift-invariant version of the contourlet transform, the NSCT. The design of the NSCT is reduced to the design of a nonsubsampled pyramid filter bank and a nonsubsampled fan filter bank. We exploit this new less stringent filter-design problem using approach, thus dispensing with the need for 2-D factorization. We also developed the 2-D NSF. This structure, when coupled with the filters designed via mapping, provides a very efficient implementation that under some additional conditions can be reduced to 1-D filtering operations. Applications of our proposed transform in image denoising. In denoising, we studied the performance of the NSCT when coupled with a hard thresholding, soft thresholding estimator. For hard thresholding; our results indicate that the NSCT improves the better performance than competing transform such as the existed NSWT and curvelets. Concurrently, our results are competitive to other denoising methods. In particular, our results show that a fairly simple estimator in the NSCT domain yields comparable performance to state-of-the-art denoising methods that are more sophisticated and complex. In image denoising, the results obtained with the NSCT are superior to those of the NSWT both visually and with respect to objective measurements.

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