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Models and Application of Stability Decision under Uncertain Environment

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Abstract: The stabilization of decision is very important when information is incomplete or environment changes. The stabilization of decision under uncertain information is considered in this paper. In the first, the concepts of global stability decision, absolute stability decision, relativistic stability decision and expected stability decision are put out. Then models of global stability decision, absolute stability decision and expected stability decision and judge condition of relativistic stability decision are given. Next stability decision models of supplier choose problem are built. At last numerical example is given.

Key words: Optimal model, robust optimization, stability decision, supplier choose problem

INTRODUCTION

Decision often faces two challenges: first problem is the information is incomplete and the parameters may be only given an estimate range. The second problem is that the event often varies. Some changes can be predicted in advance, but more changes can not be accurately predicted. The direct way to solve these problems is to adjust the decision according to the change of information or events. Some decisions can be adjusted, such as the daily decision behavior should be flexibility. But some decisions can not be always changed, such as laws and regulations, urban planning, infrastructure development, product design, facility location, etc. Changes in these problems' decision will bring high cost. In this case, adjustments of decision should be avoided, so decision is required highly adaptive to changes of parameters, that is, the stability of decision.

In general, the effect duration time of decision the longer and the cost of making adjustments the higher, the stability requirements will be more intense. The stability of decision just is not good in China. Short-term and irrational of decision often happen and decision frequently changes. The policies and regulations frequently change, undermines their authority. The life span of urban architecture is very short and an overpass will be dismantled using no more a decade, these caused great waste of resources. Therefore, how to enhance the stability of decision and reduce unnecessary losses has particular practical significance to China. The cause of instability includes the problems of decision system and decision making method, here we consider mainly the problems of decision methods.

There now have some the decision methods which consider stability of decision, such as sensitivity analysis of mathematical programming (Diao *et al*., 2007) and the What-if analysis (Wang, 1999). Sensitivity analysis is to search the range of the same optimal solution, but generally the range is relatively small. What-if analysis considers all the possible parameters correspond to what the optimal solution. These methods take into account the relationship between the change of parameters and the optimal solution, but did not answer how to select decision in the face of so many possible parameters. Robust optimization method recently developed can solve the problem (Beyer and Sendhoff, 2007). This method takes into account how to make decisions meeting all change of the parameters, its basic idea is that decision is required to satisfy all possible changes of the parameters, or making decision under the regret value of the best change of the parameters (Ben-Tal and Nemirovski, 2002). Robust optimization considering the stability of decision, and thus has been widely used in high stability requirements systems, such as the system design (Doltsinis and Kang, 2004; Park *et al*., 2006), mathematical programming (Bertsimas *et al*., 2004), discrete optimization (Bertsimas, 2003) and economic optimization (Kang *et al*., 2004).

Robust optimization considering all the parameters, it requires to minimize the maximum of the gap between the values and optimal value under all parameter the minimum. It has two disadvantages: The first is the decision which satisfies all changes of the parameters may not exist, even if there is, its value may be very poor. The second is decision given by robust optimization may be not reasonable choice in special circumstances. Supposes there have two decisions, to the first decision, its value under only one parameter has large gap with the best value, and under the other parameters the gap is very small, but its regret value is very big. The other decision's regret value is smaller than the first, but the gaps between its values and the best value are relatively large under all

most the parameters. According as robust optimization, it should select the second decision. But if the probability of singular parameter is very small to the first decision, then, select the first decision more reasonable.

To solve two disadvantages of robust optimization, we consider new method to improve the optimality by reducing the requirements of stable range.

STABILITY DECISION

The influence of parameters on decision may be in the constraints or in the objective function. In order to facilitate description of the problem, the parameter can be unified only in constraint or objective function. Robust optimization generally transforms the parameters of objective function to the constraints' parameters. The parameters of constraints also can be transformed to the parameters of objective function by increase the penalties of infeasible constraints on the objective function, this paper mainly use parameters of the objective function to describe the problem.

General optimization method considers the optimal decision of given parameters, the optimal solutions under parameters are different, optimal value is a function of parameters, called the optimal value function. Optimal solutions of different parameters are also different, the optimal solution under one parameter may be very bad to other parameters, and the gap between its value and the optimal value of same parameter may be very large. So solution would be changes when the parameter changes. In order to obtain one decision which can be acceptable in a greater range, optimality requirement should be reduced. Search acceptable decision in as large as possible parameter range, we call as stability decision.

The stability decision is to balance between the stable region and the optimality of decision. There have two ideas, one is maximize the stability region, consider the stability of all parameters, called as global stability decision. In general, the optimality of global stability decision is not good. The other is to guarantee optimality within an acceptable range, to maximize the stability region, because of it can not meet requirements of optimality in all parameter range, called it as local stability decision.

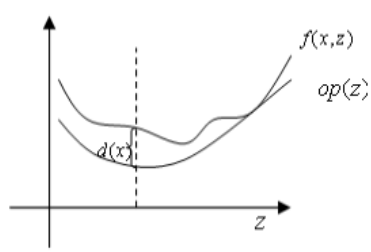

Fig. 1: The regret value of decision

Global stability decision: The key to consider the global stability decision is how to evaluate the global stability. For a given decision variable, its objection function is a function of parameters. The gap between one decision's objective function and the optimal function value at same parameter is power to change the decision, the gap greater the change power greater. The average or the maximum of these gaps under all parameters can be used to represent the global stability of the decision, called it as regretted value of the decision. When the regret value is the maximum value of the gaps under all parameters, the global stability is a robust decision making.

Lets parameters $z \in R^m$ are, their ranges are interval or set $H \subset R^m$, decision variables are x, their feasibel set $S \subset R^n$, the corresponding gain function is f(x,z). Optimal value function is op(z). The regret value is (z):

$$d(x) = \max_{z \in H}\left(f(x,z) - op(z) \right) \tag{1}$$

If only one parameter, its range is a interval, then the regret value of decision x is shown as Fig. 1.

The decision with minimum regret value is global stability decision. If the number of feasible decision is small, then their gap values can be calculated directly and find out the minimum from them. Otherwise, it needs to build a mathematical model to find the global stability decision. Global stability decision model can be written as:

$$\min_{x \in S} \max_{z \in H}\left(f(x,z) - op(z) \right) \tag{2}$$

The optimal value function $op(z)$ should be given first in the model, and then to solve the mathematical programming. Assumes decision problem's objective is to minimize, then has the following conclusion:

For every feasible decision x and parameter z, because:

$$op(z) = \min_{y \in S} f(y,z)$$

so it has:

$$f(x,z) - op(z) = \max_{y \in S}\left(f(x,z) - f(y,z) \right)$$

$$\max_{z \in H}\left(f(x,z) - op(x,z) \right) = \max_{z \in H, y \in S}\left(f(x,z) - f(y,z) \right)$$

So global stability decision's model (2) can be transformed into the following plan:

$$\min_{x\in S}\ \max_{z\in H, y\in S}\left(f(x,z)-f(y,z)\right) \tag{3}$$

Programming (3) does not need to seek $op(z)$ firstly, but some variables are added in the programming.

Local stability decision: The basic idea of local stability is to make decisions acceptable range as large as possible, the acceptable decision is feasible, and its regret value is in reasonable limits. To determine the local stability decision, it should be first determined the requirements of optimality. If the decision is feasible and its objective value does not exceed a times of the optimal value, then the decision is acceptable under the corresponding parameters. Call a as acceptable threshold. Every decision x has an acceptable parameter region, called this parameter region as the acceptable region of decision x, denoted as R(x). If lets the objective function of infeasible decision is infinity, or put it on the objective function as a penalty, then for minimize problems there has:

$$R(x) = \{z \in H \mid f(x,z) \le aop(z)\} \tag{4}$$

The acceptable region R(x) is a subset of set $H \subset R^m$, The local stability decision is a decision which acceptable region is maximum.

There are three comparison methods of the sets' size. One is to compare measures of sets, the number of their elements or their areas are used. The decision with maximum acceptable region is called as absolute stability decision. Another is to compare the inclusion relationship of sets, for a decision, if there is no decision's acceptable region really contains its acceptable region, called the decision as relative stability decision; Also one is to compare the probability of sets, the decision which acceptable region's probability is maximum is called as expectation stability decision.

As a measure of the set is not less than its subset's measure, so absolutely stability decision must be relatively stability decision, similarly, when the probability or density function is positive, expectation stability decision is relative stability decision.

For example, there has only one parameter z in the interval $[z_1, z_4]$, there are three decisions x_1, x_2, andx_3. All decisions' objective functions are shown as Fig. 2.

The function of optimal value with the parameters is the under envelope of the three curves. For a given gap threshold, acceptable regions of three decisions are $[z_1, z_3]$, $[z_1, z_2]$ and $[z_2, z_4]$. The acceptable region of decision x_1 contains the acceptable region of decision x_2, so decision x_2 is not relative stability local decision, but x_1 and x_3 are $[z_2, z_4]$ is greater than $[z_1, z_3]$, so x_3 is the absolute stability decision.

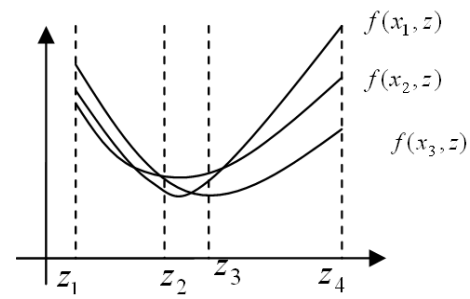

Fig. 2: Decision's accep region

THE MODEL OF LOCAL STABILITY DECISION

The model of local stability decision is more complex than global stability decision, because it needs to compare decisions' acceptable regions. So here only consider the models of three special cases:

The absolute stability decision's model with limited parameters: Consider the parameter value is limited, assuming that there are k kinds of parameter values, denoted as z^1, z^2, ..., zk $\in R^m$. The optimal values under these parameters are op_l, l = 1,2, ..., k. And the acceptable region of every decision is a discrete set, it can be measured by the number of its elements. Lets variable v_l, l = 1, 2, ..., k denotes whether the lth parameter values is in decisions acceptable regions, namely:

$$v_l = \begin{cases} 1, & if\ \ f(x,z_l) \le aop_l \\ o, & else, l = 1,2,\ldots,k \end{cases}$$

It is equivalent to:

$$f(x,z_l) - M(1-v_l) \le aop_l,\quad l = 1,2,\ldots,k$$

where, M is large enough positive number.And the elements' number of acceptable regions is $\sum_{l=1}^{k} vl$ so absolute stability decision's model is

$$\max \sum_{l=1}^{k} vl \quad s.t. \begin{cases} f(x,z_l) - M(1-v_l) \le \alpha op_l, l = 1,2,\ldots,k \\ x \in S, v_l, \qquad l = 1,2,\ldots,k \end{cases} \tag{5}$$

The determine model of the relative stability decision: Searching for the relative stability decision needs to find

maximum subset by comparing every two decisions' acceptable regions. If the number of feasible decisions is few, pair wise comparisons is possible, else, it is difficult to compare every two decisions' acceptable regions. We give a method to judge whether the feasible decisions are relative stability decision.

Lemma: Assumes the acceptable region of decision $\bar{x}$ is $R(\bar{x})$, if the optimal value of mathematical programming (6) is large than zero, then decision $\bar{x}$ is relative stability decision.

$$\min_{\substack{x\in S\\ x\neq x}} \max_{z\in R(\bar{x})} \left(f(x,z) - \alpha op(z) \right) \tag{6}$$

Proof: For decision $\bar{x}$, its acceptable region is $R(\bar{x})$. According to the definition of relatively stability decision, the decision is relative stability decision if and only if it is no decision's acceptable region contains its acceptable region. In other words, if for every decision $x \neq \bar{x}$, there has a point $z \in R(\bar{x})$ and f(x,z) > αop(z), the decision $\bar{x}$ is relatively stability decision.

If the optimal value of the programming (6) is large than zero, to every decision $x \neq \bar{x}$. The maximum of f(x,z) > αop(z) in the region is large than zero. This shows that for every decision $x \neq \bar{x}$, there has a point $z \in R(\bar{x})$ and , the decision $\bar{x}$ is relatively stability decision.

So, to judge whether decision $\bar{x}$ is relative stability decision needs to solve the corresponding programming (6), if its optimal value is large than zero, then decision $\bar{x}$ is relative stability decision. Else, it needs to calculate all optimal solution of programming (6) and their acceptable regions, if one of these acceptable regions is really contains $R(\bar{x})$, decision $\bar{x}$ is not relative stability decision.

The model of expected stability decision under discrete parameters: When the probability of parameters is known, the probability of each decision's acceptable region can be calculated, called as acceptability probability of the decision.

Assuming that the probability of every parameter value is p_l, l = 1, 2, ..., k, then probability of acceptable regions is:

$$\sum_{l=1}^{k} p_l v_l$$

and expected stability decision's model is:

$$\max \sum_{l=1}^{k} p_l v_l \quad s.t. \begin{cases} f(x,z_l) - M(1-v_l) \le \alpha opl, \ l = 1,2,\ldots,k \\ x \in S \end{cases} \tag{7}$$

where, M is large enough positive number.

THE STABILITY MODEL OF SUPPLIER SELECTION PROBLEM

It needs to establish long-term strategic business partnerships between supply chain's nodes. When the core enterprise chooses suppliers, it needs to consider the stability of supplier (Bertsimas *et al*., 2004).

Consider the core enterprise of a supply chain selects a product supplier from m candidate enterprises. Each enterprise's maximum production capacity is s_l^+, l = 1, 2, ..., m, the minimum production capacity s_l^-, l = 1, 2, ..., m is ,and demand of the core enterprise is *d*. If suppliers can not meet the need, it needs to buy from the market, the cost higher than the supplier is r_1. If the demand is less than the minimum production capacity, the amount of actual production is equal to the minimum production capacity. The excess production sold into the market, increasing cost of sales is r_2. Now need to select k enterprises as a supplier from m candidate enterprises to minimize the cost of additional expenditure.

This problem is decide whether each candidate enterprise is selected as supplier, lets variable is x_l = 0, 1, l = 1, 2, ..., m, if the lth candidate enterprise is selected x_l = 1, otherwise x_l = 0. The number of the selected enterprises is $\sum_{l=1}^{m} x_l$.

The additional expenditure cost is determined by the demand of the core enterprise, let y is the part that the demand exceeds the largest supply and z is the part that the demand is lower than the minimum production. Apparently y and z can not be greater than 0 simultaneously, the actual production of the suppliers is d-y+z. The maximum production capacity of selected suppliers is:

$$\sum_{l=1}^{m} s_l^+ x_l$$

and the minimum production capacity is:

$$\sum_{l=1}^{m} s_l^- x_l$$

so there are constraints:

$$\sum_{l=1}^{m} s_l^+ x_l + y - z \geq d$$
$$\sum_{l=1}^{m} s_l^- x_l + y - z \leq d$$
$$yz = 0$$

The additional expenditure cost is $r_1y + r_2z$. According to the principles of goal programming, when the additional expenditure cost is the minimum, y and z should not simultaneously be greater than zero, so the mathematical programming is:

$$\min r_1 y + r_2 z$$
$$\begin{cases} \sum_{l=1}^{m} s_l^+ x_l + y - z \geq d \\ \sum_{l=1}^{m} s_l^- x_l + y - z \geq d \\ \sum_{l=1}^{m} x_l = k \\ x_l = 0,1, l = 1,2,\ldots,m, y, z \geq 0 \end{cases}$$

The demand d is its parameters and assumed it is discrete, its value may be d_1, d_2, ..., d_n. For each demand value, the programming (8)'s optimal value is u_1, u_2, ..., u_n.

Global stability decision model: Let y_i is the part that the demand d_i exceeds the largest supply and z_i is the part that the demand d_i is lower than the minimum production. For given variable value $x_l = 0, 1$, $l = 1, 2, ..., m$, its regret value is:

$$\max_{i=1,2,\ldots,n} r_1 y_i + r_2 z_i - v_i$$

As programming (3), its global stability decision model is:

$$\min_x \max_{i=1,2,\ldots,n} r_1 y_i + r_2 z_i - u_i$$
$$s.t. \begin{cases} \sum_{i=1}^{m} s_l^+ x_l + y_i - z \geq d_i, \quad i = 1,2,\ldots,n \\ \sum_{i=1}^{m} s_l^- + x_l + y_i - z_i \leq d, \quad i = 1,2,\ldots,n \\ \sum_{i=1}^{m} y_i z_i = 0 \\ \sum_{i=1}^{m} x_l = k \\ x_{l=0,1,l=1,2,\ldots,m,} \\ y_{i,} z_i \geq 0, i = 1,2,\ldots,n \end{cases} \quad (9)$$

Table 1: Production capacity of enterprises

Enterprise	1	2	3	4	5	6	7	8
Maximum Capacity	4	5	4	3	6	5	6	4
Minimum Capacity	2	3	3	2	5	2	4	3

Table 2: Occurrence probability of each demand

Demand	8	9	10	11	12	13	14	15
Probability (%)	6	3	7	2	26	20	8	10

The model of absolute stability decision: Similarly with the programming (5) then its absolute stable model is:

$$\max \sum_{i=l}^{x} v_i$$
$$s.t. \begin{cases} \sum_{l=1}^{m} s_l^+ x_l + y_i - z_i \geq d_{i,} i = 1,2,\ldots,n \\ \sum_{l=1}^{m} s_l^- x_l + y_i - z_i \leq d_{i,} i = 1,2,\ldots,n \\ r_1 y_i + r_2 z_i - a u_i - M(1 - v_i) \\ \qquad \leq 0, i = 1,2,\ldots,n \\ \sum_{l-1}^{m} x_l = k \\ x_{l=0,1,l=1,2,\ldots,m} \\ y_i, z_i \geq 0, v_i = 0,1. i = 1,2,\ldots,n \end{cases} \quad (10)$$

The model of expected stability decision: If p_i is probability of demand d_i, i = 1, 2, ..., n, similarly with the programming (7), its model of expected stability decision is:

$$\max \sum_{i=1}^{n} p_i v_i$$
$$s.t. \begin{cases} \sum_{l=1}^{m} s_l^+ x_{l+y_i} - zi \geq di, i = 1,2,\ldots,n \\ \sum_{l=1}^{m} s_l^- x_{l+y_i} - zi \leq di, i = 1,2,\ldots,n \\ r_{1 y_i + r_2 z_i - a u_i - M(1 - v_i) \leq 0, i=1,2,\ldots,n} \\ \sum_{l=1}^{m} x_l = k \\ x_l = 0,1, l = 1,2,\ldots,m, \\ y_i, z_i \geq 0, v_i = 0,1, i = 1,2,\ldots,n \end{cases} \quad (11)$$

Example: Consider there have 8 candidate enterprises, to choose 3 suppliers, the maximum production capacity and minimum production capacity of each alternative

enterprise as Table 1. $r_1 = 2$, $r_2 = 3$ and its the optimization model is:

$$\min 2y + 3z$$
$$s.t.\begin{cases} 4x_1 + 5x_2 + 4x_3 + 3x_4 + 6x_5 + 5x_6 \\ + 6x_7 + 4x_8 + y - z \geq d \\ 2x_1 + 3x_2 + 3x_3 + 2x_4 + 5x_5 + 2x_6 \\ + 4x_7 + 3x_8 + y - z \leq d \\ \sum_{l=1}^{8} xl = 3 \\ x_l = 0,1, l = 1,2,...,8, y, z \geq 0 \end{cases} \quad (12)$$

The demand d is a parameter, its value may be integer from 8 to 15. For all demand values, the programming (12)'s optimal values are 0. Obviously its global stability decision model is:

$$\min u$$
$$s.t.\begin{cases} \sum_{l=1}^{8} x_l = 3 \\ 4x_1 + 5x_2 + 4x_3 + 3x_4 + 6x_5 + 5x_6 + 6x_7 \\ + 4x_8 + y_i - z_i \geq d_i, & i = 1,2,...,8 \\ 2x_1 + 3x_2 + 2x_4 + 5x_5 + 2x_6 + 4x_7 \\ + 3x_8 + y_i - z_i \leq d_i, & i = 1,2,...,8 \\ 2y_i + 3z_i - u \leq 0, & i = 1,2,...,8 \\ x_l = 0,1, & l = 1,2,...,8, \\ y_i, z_i \geq 0, & i = 1,2,...,8 \end{cases} \quad (13)$$

The optimal solution of the program is to select Enterprise 3, 4, and 7 for suppliers, and its regretted value is 4.

If its is less than 2, the decision is stable. Lets M = 30, the absolutely stable model is:

$$\min \sum_{i=1}^{8} p_i v_i$$
$$s.t.\begin{cases} \sum_{i=1}^{8} x_l = 3 \\ 4x_1 + 5x_2 + 4x_3 + 3x_4 + 6x_5 + 5x_6 + x_7 \\ + 4x_8 + y_i - z_i \geq d_i, & i = 1,2,...,8 \\ 2x_1 + 3x_2 + 3x_3 + 2x_4 + 5x_5 + 2x_6 + 4x_7 \\ + 3x_8 + y_i - z_i \leq d_i, & i = 1,2,...,8 \\ 2y_i + 3z_i + 30v_i \leq 32, & i = 1,2,...,8 \\ x_l = 0,1, & l = 1,2,...,8 \\ y_i, z_i \geq 0, v_i = 0,1, & i = 1,2,...,8 \end{cases} \quad (14)$$

The optimal solution of the program is to select Enterprise 3, 4 and 7 for suppliers, and it is stable on 6 cases of all potential demands.

Supposed the occurrence probability of each demand is shown as Table 2. Then the model of expected stability decision is:

$$\min \sum_{i=1}^{8} vi$$
$$s.t.\begin{cases} \sum_{i=1}^{8} x_l = 3 \\ 4x_1 + 5x_2 + 4x_3 + 3x_4 + 6x_5 + 5x_6 + 6x_7 \\ + 4x_8 + y_i - z_i \geq d_i, & i = 1,2,...,8 \\ 2x_1 + 3x_2 + 3x_3 + 2x_4 + 5x_5 + 2x_6 + 4x_7 \\ + 3x_8 + y_i - z_i \leq d_i, & i = 1,2,...,8 \\ 2y_i + 3z_i + 30v_i \leq 32, & i = 1,2,...,8 \\ x_l = 0,1, & l = 1,2,...,8 \\ y_i, z_i \geq 0, v_i = 0,1, & i = 1,2,...,8 \end{cases} \quad (15)$$

The optimal solution of the program is to select Enterprise1, 7 and 8 for suppliers, and its stability probability is 91%.

CONCLUSION

We considered stability of decision under uncertainty environment, and put forward the concepts of global stability decision, absolute stability decision, relative stability decision and expected stability decision. Global stability decision is a robust decision, and local stability decision is different, in essence, is to pursuit stability by reduced optimality requirements.

The models of global stability decision, absolute stability decision and expectations stability decision were given, and determine condition of relative stability was given. Then the supplier selection problem was solved by using these models.

The parameters of these stability decision models are discrete parameters. For continuous parameter, it can be solved by discretization, but the parameter value too much. It needs to consider solution method of the stability decision model with continuous parameter values.

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