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Chaotic Self-motion of a Spatial Redundant Robotic Manipulator

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Abstract: In this study, a floating spatial robotic manipulator model and kinematic equations are established, Chaos by numerical simulation and analysis of direct observation, time history method, the self-motion state of a spatial 3R redundant robotic manipulator's links has been studied which its end-effector tracking the a plane path of its work space repeatedly for PD controlling by numerical simulation and chaos analysis such as direct observation, Time course approach, phase diagrams and poincare mapping method. The results show that the self-motion is chaotic when solving the robotic manipulator's inverse kinematics based on pseudo-inverse Jacobian matrix.

Key words: Chaos, inverse kinematics, self-motion, spatial redundant robot

INTRODUCTION

Redundant robotic manipulator is a robot whose number of degrees of freedom (df) is more than the at least number of df need to which to complete a specific job tasks (Lu, 2007). Redundant robotic manipulator is a strongly coupled, highly nonlinear dynamic system, with zero space, ensuring the end-effector working, and at the same time can complete auxiliary tasks such as avoiding obstacles, overcoming singularity, minimizing joint torque, avoiding vibration caused by flexible joints (Zhang and Li, 2004), so redundant robotic manipulator often used as the executing agency for intelligent robots. For redundant robotic manipulator, self-motion will produce in the course of joint movement for zero space. many problems has been brought in redundant robotic manipulator controlling for existence of self-motion, and therefore recognizing self-motion of redundant robotic manipulator, controlling it and making use of it that can open up new avenues for improving the performance of redundant robotic manipulator and can lay the theoretical foundation for redundant robotic manipulator's application.

Previous studies have shown (Klein and Huang, 1983), there exist joint movement non-repetitive problem of control algorithm based on pseudo-inverse Jacobian matrix, namely end-effector is performing a repeated work of a closed path in work space, the corresponding joint trajectory is not repeated, and appeared the "drift" phenomenon of joint trajectory. The "drift" phenomenon of joint trajectory can be well explained by using chaos theory. Drift" phenomenon is essentially corresponded to the conclusion that chaos is exist in self-motion of redundant robotic manipulators. the study of chaos in self-motion of redundant robotic manipulator raised the interest of some scholars at home and abroad, Shrinivas and Ghosal (1996) and Ravishallkar and Ghosal (1999) studied chaotic motion of two df's and three DOFs planar robotic manipulator with revolute and prismatic joints respectively without considering the flexible, friction, and etc., Li *et al*. (2002) made further research and found parameters conditions of a plane 2R robotic manipulator in periodic and chaotic motion; Matthew and Fuehs (1990, 1991) studied the chaotic self-motion of the redundant robotic manipulator; Liu *et al*. (2004, 2005) studied chaotic phenomena of redundant robotic manipulator and proposed a decomposition motion algorithm to control the chaotic motion of planar redundant robotic manipulator; Li *et al*. (2003) and Li (2005) studied chaotic phenomena of the spatial 4R redundant robotic manipulator and proposed a delayed feedback control algorithm to control chaotic motion of spatial redundant robotic manipulator; Vakakis (1990, 1991) studied chaotic motion phenomenon of Hopping robot.

But the study to the redundant robotic manipulator's chaotic self-motion above which is to the fixed base of redundant robotic manipulator, the study to the redundant robotic manipulator's chaotic self-motion to the free-floating spatial robotic manipulator is not involved, the free-floating spatial redundant robotic manipulator's chaotic self-motion is studied in this study.

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MATERIALS AND METHODS

Kinematics analysis of the spatial redundant robotic manipulator:

The spatial redundant robotic manipulator's model: Because of the spatial robotic body is not fixed, reaction and torque to the robotic body will be generated by motion of the robotic manipulator, and caused position and posture of the robotic body change, the position and posture changes of the robotic body in turn affect positioning of the robotic manipulator, thus affecting the end-effector's operational planning, so the free-floating spatial redundant robotic manipulator's kinetic characteristics are very different from the fixed base robotic manipulator's (Hong *et al*., 2000). Establish inertial coordinate system $\sum_i$ which fixed on center of mass of system, joint coordinate system $\sum_0$ which fixed on the i_{th} joint, D-H parameters of joint coordinate system according to D-H parameters method (Danevit and Hartenberg, 1995), body coordinate system r_0 which fixed on center of mass of the body, the position and posture which body coordinate system relative to inertial coordinate system are expressed by the position vector r_0 and Z-Y-X Euler angles α, β, γ.

Kinematics analysis of the spatial redundant robotic manipulator: According to the definition of center of mass system, there are:

$$m_0 r_0 + \sum_{i=1}^{n} m_i r_c^i = (m_0 + \sum_{i=1}^{n} m_i) r_G \tag{1}$$

where, m_o is the body mass, m_1 is the mass of the ith joint, r_0 is the position vector which is the origin i^{th} of inertial coordinate system relative to the origin of body coordinate system, r_c^i is the position vector which is the origin of inertial coordinate system relative to the center of mass of the link, r_G is the position vector of robotic system center of mass in inertial coordinate system. The geometric relationship of each link shown in Fig. 1:

$$\begin{cases} r_c^i - r_0 = {}_i^{i-1}l \quad i = 1 \\ r_c^i - r_c^{i-1} = {}_i^{i-1}l \quad i = 2, L, n \end{cases} \tag{2}$$

where, ${}_i^{i-1}l$ is the position vector which is the origin of link $(i - 1)^{th}$ coordinate system relative to the origin of link i^{th} coordinate system, r_0, r_c^i can be obtained by Eq. (1) and (2). According to the recurrence relations of the robotic manipulator's each link (Craig, 1986):

$$\begin{cases} \omega i = {}_{i-1}^{i}R\omega_{i-1} + q^{\&}{}_i Z_i \\ Vi = {}_{i-1}^{i}R(v_{i-1} + \omega_{i-1} \times {}_i^{i-1}l) \end{cases} \tag{3}$$

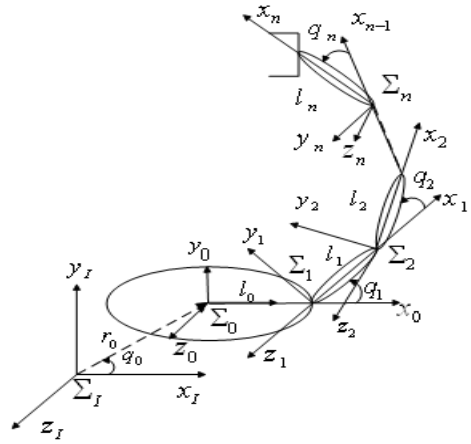

Fig. 1: The free-floating spatial redundant robotic manipulator model of n joints

where, $q^{\&}{}_1$ is angular velocity vector of the ith joint, ω_i is angular velocity of the ith joint that expressed in coordinate system $\sum_i$, v_i is linear velocity of the ith joint that expressed in coordinate system $(i - 1)^{th}$ is rotational transformation matrix which is the ith link coordinate system relative to the Z_i link coordinate system, Z is unit vector of Z axis in coordinate system $\sum_i$. Assuming $V_i = [v_i^T, \omega_i^T]^T$ is generalized velocity of the ith link that expressed in coordinate system $\sum_I$, by Eq. (3), we have:

$$V_i = \phi(i.i-1)V_{i-1} + n_i q^{\&}{}_i \tag{4}$$

where,

$$\phi(i,i-1) = \begin{bmatrix} {}_{i-1}^{i}R & -{}_{i-1}^{i}R_i^{i-1}l^v \\ 0 & {}_{i-1}^{i}R \end{bmatrix} \in R^{6\times 6},$$

$$n_i = [0,0,0,0,0,1]^T$$

The end-effector's velocity can be expressed as follows in its joint coordinate system by Eq. (4):

$$\begin{aligned} V_{n+1} &= \phi(n+1,n)V_n = \\ &\phi(n+1,0)V_0 + \phi(n+1,n)GN^{q\&} \end{aligned} \tag{5}$$

where, v_0 is the generalized velocity of the body,

$$\begin{aligned} V_0 &= [v_0^T, \omega_0^T]^T, q\& = [q_1^{\&} L\, q^{\&}{}_n], G = \\ &[\phi(n,1) L\, \phi(n,n)], N = diag(diag(n_1 L\, n_n) \end{aligned}$$

The end-effector's velocity in the body coordinate system can be expressed as $X^{\&}{}_e$, form Eq. (5), there have:

$$X\&^e = J_b V_0 + Jq^{q\&} \tag{6}$$

where,

$$J_b = R_e \phi(n+1,0)$$
$$J_q = R_e \phi(n+1,n) GN$$
$$R_e = {}^0_{n+1}R,\ {}^0_{n+1}R$$

is rotational transformation matrix which is the end-effector's coordinate system relative to the body coordinate system. From Eq. (6) can be seen that the velocity of the robotic manipulator's end-effector has two parts: the body velocity and joint velocity. But the velocity of the fixed-base robotic manipulator's end-effector is only with the joint velocity. J_q is Jacobian matrix of the fixed-base robotic manipulator.

There are the following relationships between the velocity of the ith link and its center of mass velocity:

$$\begin{bmatrix} V_c^i \\ \omega_c^i \end{bmatrix} = \begin{bmatrix} E & -P_i^v \\ 0 & E \end{bmatrix} \begin{bmatrix} V_i \\ \omega_i \end{bmatrix} \tag{7}$$

where, ω_c^i is angular velocity of the ith joint center of mass that expressed in coordinate system $\sum_i$, V_c^i is linear velocity of the ith joint center of mass that expressed in coordinate system $\sum_i$, P_i^v is the ant- symmetric matrix of position vector which is the origin of the ith joint coordinate system relative to its center of mass, assuming ith is generalized velocity of the ith link that expressed in body coordinate system, so:

$$V_c^{-i} = S_i V_i \tag{8}$$

where,

$$S_i = \begin{bmatrix} {}^0_i R & -{}^0_i RP_i^{\times} \\ 0 & {}^0_i R \end{bmatrix}, {}^0_i R$$

is rotational transformation matrix which is the Ith link coordinate system relative to the body coordinate system, From Eq. (4), (8) and the nature of ϕ(i, k)),the following is available:

$$V = \Phi N_q^{\&} + DV_0 \tag{9}$$

$$V_c = Sv \tag{10}$$

where,

$$V = [V_1^T\ LV_1^T]^T, V_c = [V_c^{1T}\ LV_c^{nT}]^T,$$
$$q^{\&} = [q^{\&}{}_1 L\ q^{\&}{}_n]^T, S = diag(S_1 LS_n),$$
$$D = [\phi^T(1,0)\ L\ \phi^T(n.0)]^T,$$
$$\Phi = \begin{bmatrix} \phi(1,1) & 0 & L & 0 \\ \phi(2,1) & \phi(2,2) & L & 0 \\ M & M & O & M \\ \phi(n,1) & \phi(n,2) & L & \phi(n,n) \end{bmatrix}$$

free-floating redundant spatial robotic manipulator is in a free floating state, so the momentum is conserved, assuming the initial momentum of the robotic system is zero, so the momentum of the robotic system remains zero in the whole course of the motion, namely:

$$\left(\hat{M}_0 + M_m SD\right) V_0 + M_m S\Phi N q^{\&} = 0 \tag{11}$$

The relationship between the body posture and the body center of mass position and joint angular velocity can be obtained form Eq. (11), that is:

$$I_s \omega_0 + I_m{}^{q\&} = 0 \tag{12}$$

$$V_0 = Jv^{q\&} \tag{13}$$

where, $I_s \in R^{3\times 3}$, $I_m \in R^{3\times n}$ are the inertia matrices of the robotic body and the robotic manipulator respectively, J_v is the interference matrix that the robotic manipulator movement to the center of mass position and have to do with the mass of the robotic system's each part. The following can be obtained from Eq. (6), (12) and (13):

$$X^{\&}{}_e = J_s \omega_0 + J_m{}^{q\&} \tag{14}$$

where, $J_5 \in R^{6\times 3}$, $J_m \in R^{6\times n}$ are matrices that are concerned with the robotic geometric structure.

The relationship between body velocity and joint velocity is shown by Eq. (13), which is:

$$V_0 = J_v{}^{q\&} \tag{15}$$

where, $J_v = -(\hat{M}_0 + M_m SD)^{-1} M_m S\Phi N$. Therefore, the body velocity is determined by Eq. (15), which reflects

the influence that the robotic system end-effector and manipulator motion applied to the body, when the end-effector in operation. The following can be obtained from Eq. (6):

$$X^{\&}{}_e = J_b V_0 + Jq^{q\&} \tag{16}$$

Substitute Eq. (15) into (16), we can obtain:

$$X^{\&}{}_e = J_b j_v q^{\&} + Jq^{q\&} \tag{17}$$

Assuming $J_e = J_b J_v + J_q$, Eq. (17) make into the following:

$$X^{\&}{}_e = J_e q^{\&} \tag{18}$$

where,

$$X^{\&}{}_e \in R^m,\ q^{\&}{}_e \in R^n,\ m \le n$$

J_q is Jacobian matrix of the fixed-base robotic manipulator, J_e is the generalized Jacobian matrix. As can be seen from the above process of derivation: Jacobian matrix J_e of the fixed base robotic manipulator is only with the geometric parameters, while the generalized Jacobian matrix J_q of the spatial robotic manipulator is not only with the geometric parameters, but also with dynamic parameters of the body and robotic manipulator.

Inverse kinmatics of a spatial redundant robotic manipulator: Kinematics manipulability W of the spatial redundant robotic manipulator is select as optimal objective function of avoiding singular, PD controller is designed to make the end-effector track the closed curve of the robotic work space repeatedly, joint acceleration expression can be obtained based on the pseudo-inverse of Jacobian matrix:

$$\begin{aligned} q^{\&} = {} & J + (X^{\&}{}_{ed} + K_p e_p + K_v \&_p - J^{\&\&}{}_q) \\ & + \alpha(I - J_e^{+} J_e)\omega \end{aligned} \tag{19}$$

where,

$$\omega = \nabla W(q) = \nabla \sqrt{\det(J_e J_e^T)}$$

is the gradient of manipulability function, $K_v = k_v.I$, $K_p = k_p.I$, k_p, k_v are the magnification times of the displacement error and velocity error respectively, I is unit matrix, $X^{\&}{}_{ed}$ is the desired acceleration of the end-effector,

$$e_p = p_d - X_e,\ e^{\&}{}_p = X^{\&}{}_e$$

are the position deviation and velocity deviation of the end-effector respectively. Scalar magnification coefficient $\alpha = 1$. In order to facilitate research, the state variables defined as follows:

$$X = [X_1 \ \ X_2] \tag{20}$$

where,

$$X_1 = q = [q_0, q_1, q_2, L, q_n]^T,$$
$$X_2 = q^{\&} = [q^{\&}, q^{\&}{}_1, q\&2, L, q^{\&}{}_n]_T$$

The original system equation can be expressed as:

$$X^{\&} = \begin{bmatrix} X_2 \\ J_e^{+}(X_1)\left(P_d^{\&} + K_p e_p + K_v e_p^{\&} - J^{\&}(X_1)X_2\right) \\ + \alpha\left(I - J_e^{+}(X_1)J_e(X_1)\omega\right) \end{bmatrix} \tag{21}$$

Chaotic self-motion of a spatial redundant robotic manipulator: Three joints free floating redundant spatial robotic manipulator operated plane motion which is taken as an example and computer numerical simulation is made in this paper. Simulation requires the end-effector moving by desired trajectory. System parameters are as follows: the body mass is, $m_0 = 200$ kg each link mass are $m_1 = m_2 = m_3 = 10$ kg, the body rotary inertia is $I_0 = 200$ kgm^2, each link rotary inertia are $I_1 = 0.3$ kgm^2, $I_2 = 0.2$ kgm^2, $I_3 = 0.1$ kgm^2, the length of links are $l_1 = l_2 = l_3 = 1$ m, the distance from the body center of mass to the first joint is $l_o = l$m , position vector of the body center of mass in inertial coordinate system is $r_1 = (0, 0.2)$, the desired trajectory, the desired velocity and the desired acceleration of the end-effector are respectively:

$$\begin{aligned} Pd &= [1 + 0.5\cos(2\pi t)\ 2 = 0.5\sin(2\pi t)] \\ P^{\&}{}_d &= [-\pi g \sin(2\pi t)\ \pi g \cos(2\pi t)] \\ P^{\&}{}_d &= [-2\pi^2 g \cos(2\pi t) - 2\pi^2 g \cos(2\pi t)] \end{aligned} \tag{22}$$

By simulation we can see that different initial shape of the free-floating spatial redundant robotic manipulator and coefficient k_v, k_p which make system closed-loop stability can lead to chaotic motion of the spatial redundant robotic system. Limited space, here only given a set of values simulation results.

$$K_v = \begin{bmatrix} 40 & 0 \\ 0 & 40 \end{bmatrix}, K_p = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix} \tag{23}$$

Initial shape:

$$q = [\pi, -\pi/2, \pi/2]^T, q^{\&}[0,0,0] \tag{24}$$

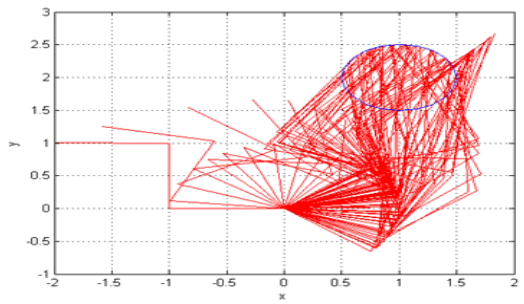

Fig. 2: Motion simulation figure of the spatial 3R robotic system

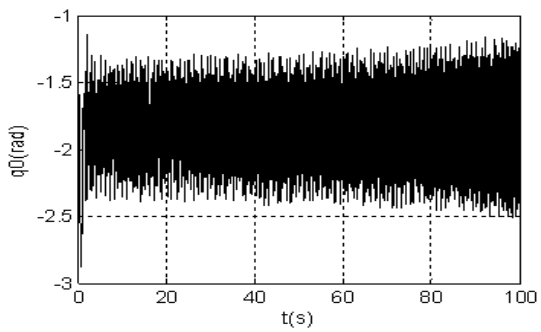

Fig. 3: The time course of q_0

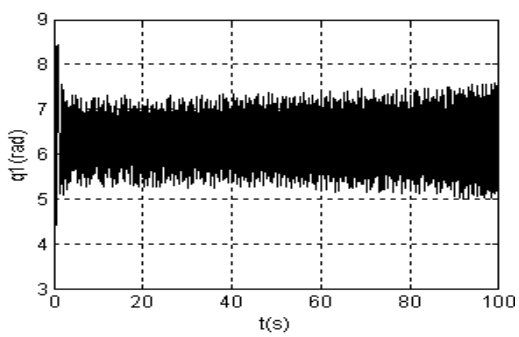

Fig. 4: The time course of q_1

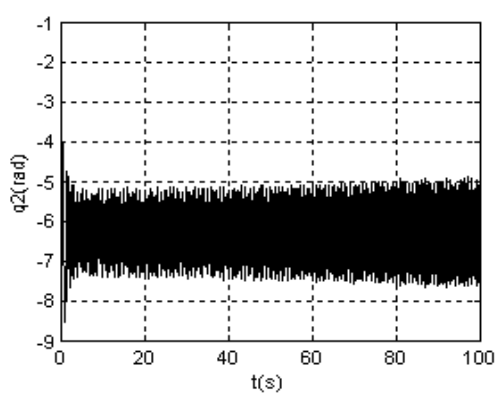

Fig. 5: The time course of q_2

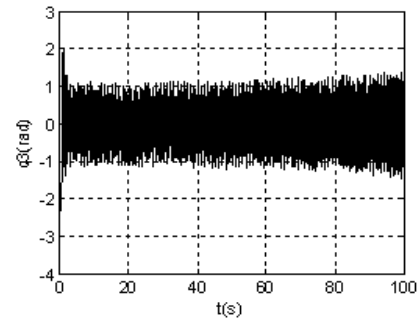

Fig. 6: The time course of q_3

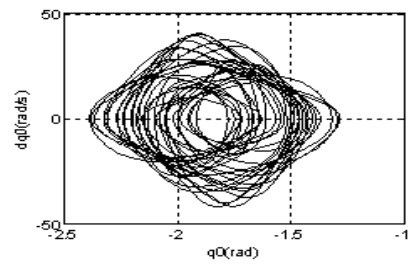

Fig. 7: Phase figure of q_0

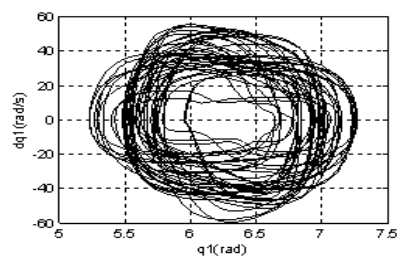

Fig. 8: Phase figure of q_1

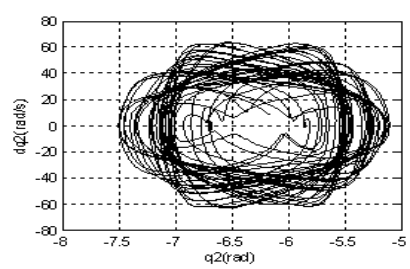

Fig. 9: Phase figure of q_1

According to the numerical computation results of nonlinear differential equations of the robotic system, the robotic motion simulation figure can be drawn. Redundant robotic system's motion simulation is shown in Fig. 2 when solving the inverse kinematics based on the pseudo-inverse Jacobian matrix under the conditions of the above parameters. The self-motion between links is irregular at

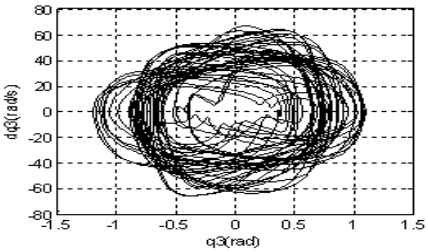

Fig. 10: Phase figure of q_0

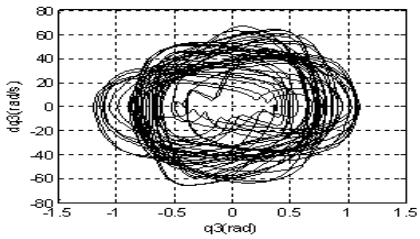

Fig. 11: Poincare section of q_0

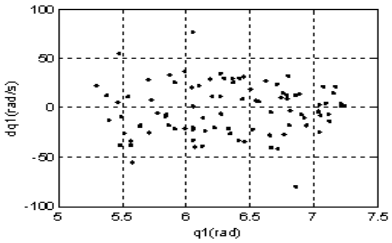

Fig. 12: Poincare section of q_0

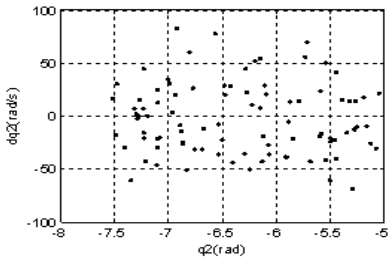

Fig. 13: Poincare section of q_2

preliminary observation. To further observe the system's self-motion state, the time course of the joint figure, phase figure and Pioncare section are drawn, and as shown in Fig. 3 to 14. From the time course can be seen that the base angle and joint angle changes over time showed the characteristics of chaotic motion. During the experiment, modify the initial value of the robotic system, the time course of the robotic system has greatly changed, which indicates the initial sensitivity of the robotic system.

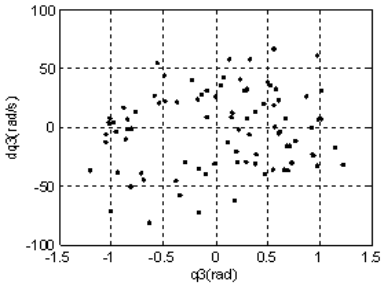

Fig. 14: Poincare section of q_1

From Fig. 7-10 can be seen that the phase trajectory is randomly distributed, neither overlap nor intersect, repeated folding in a certain area. There is chaotic motion in redundant spatial robotic system at initial judgments.

From Fig. 11-14 can be seen that Poincare section composed by an infinite number of discrete points. The self-motion of robotic system has characteristics of chaotic motion for the nature of the Poincare section.

CONCLUSION

The spatial free-floating 3R redundant robotic manipulator is taken as an object,and self-motion state of the free-floating spatial redundant robotic manipulator is studied in detail. A large number of numerical simulation is exercised by chaos analysis method such as direct observation, time-course method, phase figure and Poincare section method. The results show that: the free-floating redundant spatial robotic manipulator's self-motion is chaotic when solving the robotic manipulator's inverse kinematics based on pseudo-inverse Jacobian matrix, designing PD controller to make the end-effector track the closed curve of the robotic work space repeatedly. This discovery laid the foundation to which study the chaos control and use of the spatial redundant robotic manipulator.

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