

Adopting Microwave Techniques to Analyze Raw Quartz and Clay Materials in Uganda

E.R.R. Mucunguzi-Rugwebe

Kyambogo University, Faculty of Science, P.O. Box, 1, Kyambogo, Uganda

Abstract: An investigation was made into dielectric properties of quartz crystals, brown-clay, grey-clay, whitish-clay and some rocks found in Uganda under temperatures of about 25°C and pressure of approximately 66.0 cm of mercury. This study was conducted at Makerere University in Uganda some time back and revisited in 2009. The analysed materials were provided by the Department of Geology Entebbe, Ministry of Survey and Natural Resources Uganda. Measurements were carried out in the frequency range of 8.2-12.4 GHz, using circular cavities energized in the TM_{01n} family of modes; the signal frequency had no effect on, n , as this was not in the axis of propagation. The relation $D = cx_n / (\pi f_c \sqrt{\epsilon_r})$ was used to calculate the dielectric constant ϵ_r for cut-off frequency f_c , x_n is the n^{th} root of the first kind of the Bessel function $J_1(x) = 0$ and c is the velocity of light. The study revealed that in Uganda there are: (i) crystals of dielectric constant ϵ_r in the range $\epsilon_r = 4.48 - 4.57$ and these values of ϵ_r are close to that of Piezo-electric crystal whose $\epsilon_r = 4.6$, (ii) clays and black rock whose dielectric constants $\epsilon_r = 1$. The study revealed that when a raw material is put in a cavity its dielectric constant can be determined by observing the resonance or (absorption frequency). Material with higher dielectric constants had correspondingly lower frequency than those with lower dielectric constants.

Key words: Clay, microwave, quartz, materials, temperature

INTRODUCTION

The purpose of the study was to determine the dielectric constants of some raw materials in Uganda by applying microwave techniques.

Let us consider electromagnetic excitation along the axis of a cylindrical cavity whose cross-section size and shape remain constant. In order to have easy treatment of the cylindrical geometry we consider the z-spatial variation of sinusoidal time dependent waveform given by the equation, Corson and Lorrain, 2000):

$$E(x, y, z, t) = E(x, y)e^{+i(kz - \omega t)} \quad (1)$$

Assuming that the cavity is empty and applying Maxwell's equations on z-dependent wave we get, Montgomery (2009):

$$\left(\nabla_t^2 + \frac{\partial^2}{\partial z^2} + \frac{\omega}{c^2} \mu \right) |E| = 0 \quad (2)$$

Expressing Δ_t^2 in Curvilinear and Polar co-ordinates we obtain:

$$\nabla^2 = \frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \quad (3)$$

Electric field E and induction B can be represented by (Reddick, 1987):

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\omega^2}{c^2} \mu \right] |B| = 0 \quad (4)$$

Assuming that $(1/\Theta(\theta)) \partial \Theta / \partial \theta^2 = -m^2$ where m^2 must be an integer so that solution to the function $\Theta(\theta)$ is single-valued since the field within the wave guide for $0 < \theta < 2\pi$ is the same as that for $2\pi < \theta < 4\pi$ extra. If m is an integer, then the radial equation can be expressed as (Jackson, 1995):

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \left(k_c^2 \frac{m^2}{r^2} \right) R(r) = 0 \quad (5)$$

where $k_0^2 = \frac{\omega^2}{c^2} \mu \in -k^2$

Equation (4) reduces to:

$$r^2 \frac{d^2 R}{dr^2} + r \frac{dR}{dr} (r^2 - n^2) R = 0 \quad (6)$$

This is the Bessel function of order n where $n^2 = m^2/k_c^2$. The radial function is given by (Reddick, 1987):

$$R(r) = A_n J_n(k_c r) + B_n(k_c r) \quad (7)$$

where A_n , B_n are arbitrary constants $J_n(k_c r)$ and $N_n(k_c r)$ are Bessel functions of first and second kind. The solution for the function $\Theta(\theta)$ is $\Theta = b_m \cos(m\theta + \phi_m)$. Because functions of the second kind are infinite at the origin ($r = 0$), the feasible solution of the Bessel function is that of the first kind.

Electric field E and induction B are related by (Montgomery, 2009):

$$E(z) = k E_o J_n(k_c r) \cos(m\theta) \quad (8)$$

where $E_o = A_n b_m$ and magnetic induction B is given by:

$$B = i B_o J_n(k_c r) \cos(m\theta) \quad (9)$$

E_z must fulfill the boundary condition that $E_z = 0$ at walls of the waveguide i.e., at $r = a$. Hence for $J_n(k_c a) = 0$, $k_c a$ must be a root of the Bessel function and therefore,

$$k_c = \frac{x_n}{a} \quad (10)$$

At resonance or absorption or cut off frequency:

$$f_c, k_c = \omega_c \sqrt{\epsilon \mu} \quad (11)$$

If there is no transmission when the signal is incident on a substance, this implies that maximum power absorption has taken place or one may regard this as a position of cut off frequency. The resonance absorption frequency meant cut off frequency in this case and was related to the dielectric constant, ϵ_r , of the substance in a cavity of radius, a , by (Reich *et al.*, 2010):

$$f_c = \frac{c x_n}{2 \pi a \sqrt{\epsilon_r}} \quad (12)$$

If a substance of different dielectric constant, ϵ'_r , is put in the same cavity, a different resonance frequency, f'_r , is obtained. It can be deduced that resonance frequency, f_c , and f'_r are related by (Montgomery, 2009):

$$\frac{f'_r}{f_c} = \left(\frac{\epsilon_r}{\epsilon'_r} \right)^{1/2} \quad (13)$$

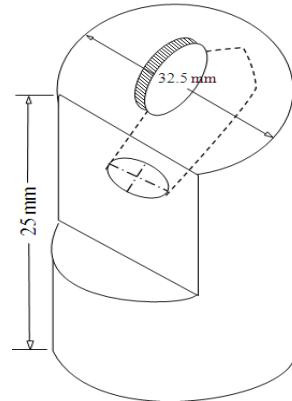


Fig. 1: Cavity used as sample container

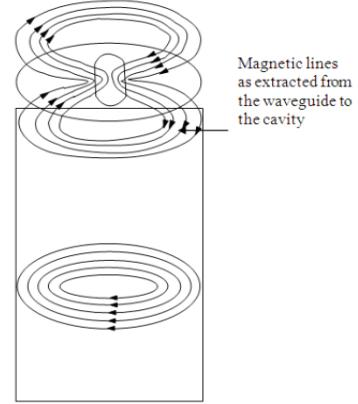


Fig. 2: Showing Magnetic lines extraction

also after considering Poynting's vector, the magnetic and electric energy closed in a volume, dn , is related to dielectric constants by (White, 1990):

$$\left(\frac{\epsilon_r}{\epsilon'_r} \right)^{1/2} = \left(\frac{E}{E'} \right) \quad (14)$$

From (13) above one can derive the relation as shown in (14). From (12) and (13) one can calculate the dielectric constant ϵ_r .

Instrumentation: A cavity was constructed from a cylindrical brass metal external dimensions were about 32.5 mm. In diameter and 25.0 mm in length see Fig. 1. The diameter, D , of the cavity was calculated for cut off frequency TM_{011} using the relation $D = cx_n/\pi f_c \sqrt{\epsilon_r}$ where, c , is the velocity of light in free space; x_n is the n th root of the first kind of the Bessel function $J_{1(n)} = 0$ and $\hat{\epsilon}_r$ is the dielectric constant of the substance assumed to be in the cavity at the absorption frequency. The above mode was

chosen, in order to avoid degeneracy of modes in the cavity. The length of the cavity had no effect on the cut off frequency since it was not in the axis of propagation and therefore, n , could take any value. In order to get maximum energy output, magnetic coupling was used as shown in Fig. 2. The width of the coupling membrane as in Fig. 3 was designed to be about 1.8mm. in order to (i) avoid the cavity from bending at nitrogen temperature and (ii) extract magnetic lines easily from the wave guide to the cavity. A krystron (Panelimatic PM7011X) running from 8.2-12.4 GHz operated using a power supply (Type PM7812) which was internally modulated by 50Hz was used. The optimum anode voltage was 300.0 volts and for the krystron to tune well the reflector voltage was set at 160.0 volts. A wave guide of about 7.5 cm. long was connected between the krystron and power isolator (PM7011X). A wave meter whose meter readings were calibrated on a graph was used to obtain readings that

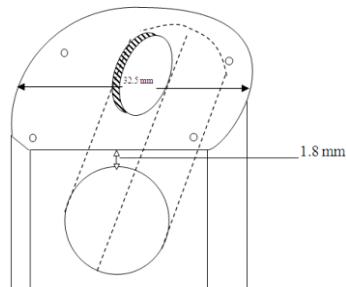


Fig. 3: Showing coupling membrane

were directly converted into corresponding frequencies directly. A lock-in amplifier and C.R.O. (Type 502) were used for amplifying and measuring the output signal from the directional-coupler.

METHODOLOGY

Measurements were taken at Makerere University in Uganda sometime back and revisited in 2009. Quartz crystals, brown-clay, grey-clay, whitish-clay and some rocks were provided by the Department of Geology Entebbe, Ministry Survey and Natural Resources Uganda.

Precautions and preliminary experiments: Special care was taken in selection of wave guides in order to get correct wave guide components with the intention that arcing or damage did not occur to them. In addition to this special selection of components, a suitable variable attenuator was used to permit the control power input to the cavity. The wave guides used in the study, were checked by the spirit level to ensure that they were at same level. If not leveled, one would get side reflections which lead to inaccurate determination of resonance absorption frequency as they cause reduction in coupling. In order to design a cavity, a substance of a dielectric constant $\epsilon_r = 4.0$ was assumed to be in the cavity and that it gave resonance absorption at a frequency, $f_c = 9.46$ GHz. The theoretical calculated dimension of the cavity was 1.21 cm. However, the driller used was 1.191 cm. in

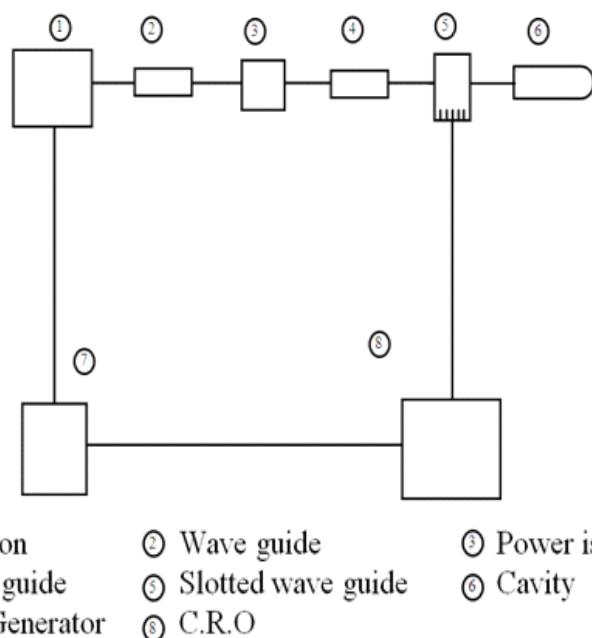
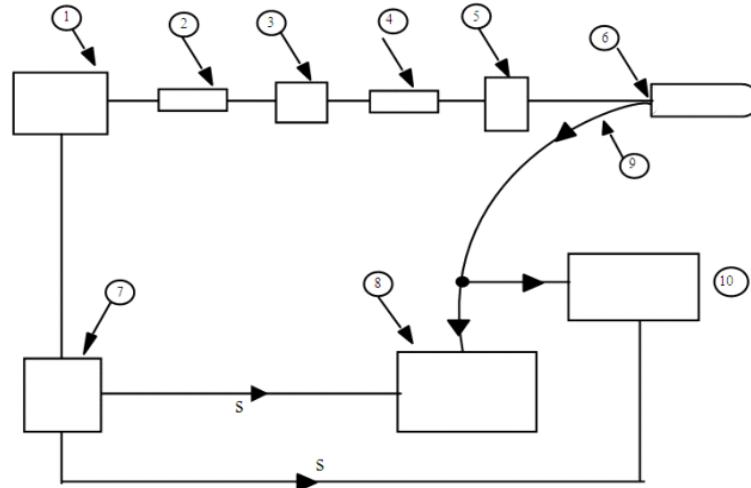


Fig. 4: Showing arrangement of the experimental apparatus for standing wave detection



- | | | |
|------------------------------------------------------|------------------------------|------------------|
| ① Klystron | ② Price of wave guide (4in.) | ③ Power isolator |
| ④ Attenuator | ⑤ Power isolator | ⑥ Cavity |
| ⑦ Klystron power supply | ⑧ C.R.O. | |
| ⑨ Triggering signal 50Hz -from Klystron power supply | | |
| ⑩ Directional Coupler | ⑪ Lock-in-Amplifier | |

Fig. 5: Showing experimental setup

diameter. When the cavity was filled with the standard substance whose $\epsilon_r = 4.0$, and using the layout as shown in Fig. 5 the experimental resonance absorption frequency, $f_{r,c}^l = 9.45\text{GHz}$, was obtained. Precaution was also taken while packing the pieces of the crystal in the cavity because improper packing can bring reduction in coupling and can easily result into false resonance absorption frequency which would yield incorrect value of the dielectric constant. Great attention was given in determining the size of the thickness of the membrane as it attenuates magnetic and electric coupling by different factors. Suppose A_{in} is the voltage amplitude to wave incident in the main line and A_E is the voltage amplitude to the wave coupled into the cavity then (Atwater, 1962):

$$\frac{A_E}{A_{in}} = \frac{\pi j}{\lambda S} (E_1 E_2) f(r) F_E(t) \quad (15)$$

where E_1, E_2 are electric field intensities corresponding to waves of unit amplitude in a wave guide and the cavity respectively, s , is the normalizing factor, $f(r)$ is the function that depends on the dimension of the hole and $F_E(t)$ is the attenuation factor for a hole of thickness t . A large hole to the cavity sets up a reflection in the main transmission line. Here, tight coupling of 10 db or less was considered suitable for the investigation. The length of the long axis of the oval shaped hole that led in to the

cavity was made equal in size as that of the shorter side of the rectangular wave guide with the intention of extracting as much electric field as possible without distortion. The membrane was purposely made 1.8 mm. so that it could not bend at Nitrogen temperature.

Preliminary trials on the experimental arrangement were carried out to detect the following:

- Standing wave profile in the system
- Faulty klystron
- Oscillatory behaviour of klystron through its frequency range
- Degeneracy

In order to detect whether there was a standing wave profile or not, the experimental layout as shown in Fig. 4 was set up. A klystron was used as a microwave generator for a signal between 8.2 - 12.4 GHz. A suitable wave guide was connected between the klystron so that a strong magnet in the power isolator did not affect directly the power output of the klystron. Since a wavelength lying between 2.42-3.7 cm. was to be detected, a wave guide calibrated up to 10.0 cm was considered suitable to accommodate the antinodes and nodes in the standing wave. The calibrated wave guide (slotted wave guide) consisted of a sliding diode for detecting nodes and antinodes. Whenever the antinodes or nodes were not detected this confirmed that the set up had no standing wave pattern.

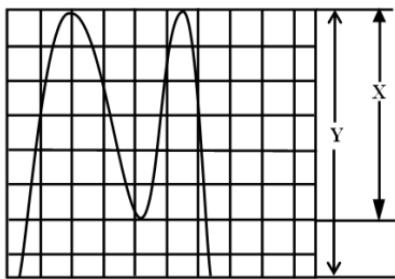


Plate 1: Resonance - Absorption depth X, An appearance of the wave form on C.R.O showing the resonance – absorption depth

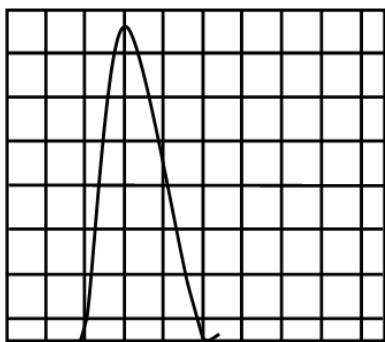


Plate 2: Optimum reflector voltage, the wave form obtained when optimum reflector voltage for the klystron's best performance was reached, with end of the wave guide shorted and the size of the wave-meter cavity at its maximum

The performance of the klystron was observed through its entire frequency range by use of a frequency-meter. The frequency meter was set at a value half-way the spectrum and the klystron was tuned. A coupling was observed indicating resonance absorption due to wave meter cavity. Whenever the wave meter cavity was changed the resonance absorption depth would change. If the absorption depth swept over the whole waveform then it was said that the klystron was operating normally, i.e. it was tuning over the whole frequency range. In normal conditions the resonance absorption depth appeared as shown in Plate 1. However, a coupling due to resonance absorption of the wave meter cavity may be observed at a single value or point and if there is no sweep of this resonance absorption depth over the whole range when the meter cavity is changed, then the klystron is regarded to produce oscillation at only one frequency i.e. it is oscillatory. The reflector voltage was adjusted to a value that allowed the klystron optimum-performance. This was achieved by observing the output waveform when a flat piece of aluminum plate was tightly screwed on to the wave guide in the place of the cavity (a technique known as shorting). With a wave meter cavity

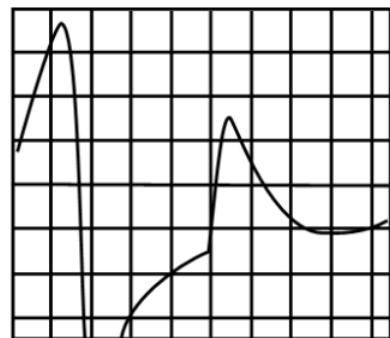


Plate 3: Non-optimum klystron performance, the wave form obtained when using The reflector voltage that does not give the klystron optimum-performance, with the end of the wave guide shorted and the size of the wave-motor cavity its maximum

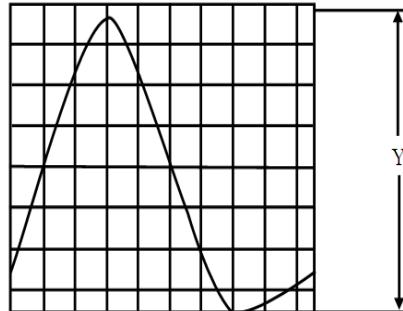


Plate 4: Power Output when shorted, An appearance of the waveform on C.R.O. of the power output when shorted

at its maximum and after shorting at correct reflector voltage, a waveform as shown by the Plate 2, was observed. If the reflector voltage was not at the correct value, after shorting a waveform as shown by the Plate 3 was obtained indicating that the reflector voltage was not giving the klystron optimum performance. In other words at any time the wave guide was shorted, the reflector voltage was adjusted until a waveform with a shape as that one on Plate 2 was obtained.

Measurements of raw quartz and clay materials: A cavity of 1.19 ± 0.01 cm. in diameter was filled fully with pieces of the same crystal. The experimental set up was as shown in Fig. 5. The waveform was displayed on the C.R.O. to observe the resonance absorption depth. Some resonance absorption depths could be detected, and for every absorption depth, the cavity was removed and shorting was done. If the same absorption depth appeared on shorting, it would be concluded that the absorption depth was not due to the substance packed tightly in the cavity. On the other hand, whenever the absorption depth disappeared and there was an increase observed in the waveform output as shown in Plate 4, then that was

considered as true absorption depth (i.e. the depth was due to resonance absorption frequency). In short, if the power output measured as Y in Plate 4 was found greater than Y in Plate 1. Coupling percentage was calculated using the following relation $(X/Y) \times 100$ or (resonance absorption depth: divided by power output) $\times 100$. Coupling percentage was calculated each time whenever true resonance absorption frequency was reckoned, in order to estimate power loss due to (i) the reflection in the wave guide (ii) the improper matching in input impedance and output impedance of the cavity together with the material inside.

Another cavity of diameter 2.43 ± 0.01 cm. was used to study the dielectric constants of brown-clay, grey-clay, whitish-clay and rocks. With $x_n = 2.405$, the dielectric constants of Quartz and brown-clay, grey-clay, whitish-clay and rocks were calculated and presented as shown in Table 1 and 2, respectively.

RESULTS AND DISCUSSION

From experimental results: in Table 1 the average value of the dielectric constant of the crystals, C_1 , C_2 , C_3 and 228B was 4.52. This value of ϵ_r is close to that of Piezo-electric crystal whose $\epsilon_r = 4.6$ (Von Hippel *et al.*, 1995). The dielectric constants of clays and black rock shown in Table 2 fall within $\epsilon_r = 1.0$. This indicates that their ϵ_r is almost equal to that one of air. It further shows that when clays and rocks are in powdered form the degree of porosity increases and this slightly reduces the dielectric constant.

Precautions and error estimation: Before resonance absorption frequencies were obtained, all the cavities in line, (i.e., the attenuator cavity and wave meter cavity) their dimensions were set to minimum. Whenever any resonance absorption depth was observed in the waveform, dimension of any cavity mentioned above, was changed to find out whether the absorption depth was due

to that cavity or not. If the coupling percentage of the absorption depth was below 40%, then reflector voltage was adjusted to obtain a better coupling.

Mode of operation: The cylindrical T_{01n} resonant mode was chosen because (Reich *et al.*, 2010):

- It gives relatively large area of electric field at the membrane of the cavity
- It provides uniform electric field
- It is independent of the lengths of the cavities, so there was no need to calculate their lengths
- One could choose conveniently the value of resonance absorption frequency that would fall halfway the klystron's frequency spectrum to design the cavity size

Transmission:

- At frequencies 9.1-10.2 GHz. Transmission is most easily and conveniently accomplished through standard T_{01n} waveguide sections and at these frequencies low losses are registered; a loss figure being typically 1 db in 76 cm. guide. However, losses were overcome by using wave guides shorter than 76 cm. $P/E_{\max}^2 = 6.63 \times 10^{-4} (\lambda / \lambda_g)$ this is the theoretical limit for peak power transmission of a rectangular wave guide (Kerns, 1967).
- The inside of the wave guides was cleaned with brasso to avoid dirt which causes sharper field gradients that are responsible for power loss.
- Losses can be avoided by minimizing non-zero VSWR.
- Mismatching between the load output of the transmitter and receiving total load including wave guide and cavity with sample could produce: (i) poor coupling (ii) unstable waveform on C.R.O Whenever poor coupling was observed, a longer wave guide

Table 1: Showing dielectric constants of quartz crystals

Diameter of cavity (cm)	Material	Size of sample	Resonance absorption frequency (Ghz)	Coupling (%)	reflector voltage (v)	ϵ_r
1.191	Pieces of quartz C_1	0.969 c.c	9.020	77.5	160.0	4.57
1.191	Pieces of quartz C_2	0.953 c.c	9.110	70.0	160.0	4.48
1.191	Pieces of quartz C_3	0.970 c.c	9.080	70.0	160.0	4.51
1.191	Quartz 228 B	0.968 c.c	9.060	92.0	160.0	4.52

The klystron was internally modulated by 50.0 Hz.

Table 2: Showing dielectric constants of clays and black rock

Diameter of cavity (cm)	Material	Size of sample	Absorption frequency (Ghz)	Coupling (%)	Reflector voltage (v)	ϵ_r
2.43	Brown clay	Filing the cavity	9.235	73.0	60.0	1.06
2.43	Grey clay	Filing the cavity	9.145	62.1	60.0	1.07
2.43	Whitish clay	Filing the cavity	9.150	75.0	60.0	1.07
2.43	Crushed black rock	Small particles	9.390	88.0	60.0	1.02
2.43	Powdered brown clay	Very small particles	9.220	88.2	60.0	1.05
2.43	Powdered whitish clay	Very small particles	9.415	75.0	60.0	1.01
2.43	Powdered rock	Very small particles	9.405	75.5	60.0	1.01
2.43	Power grey clay	Very small particles	9.405	92.9	60.0	1.01

was interchanged for a shorter one and vice-versa so that a better coupling and more stable waveform than the previous ones were obtained and this would also reduce noise.

- Reflector voltage would be adjusted until the klystron optimum-performance was achieved. The klystron would be tuned manually to have absorption depth centred. All these points mentioned above bring systematic and random errors in the experimental results

CONCLUSION AND RECOMMENDATION

This is a versatile method that can be used for determining dielectric constants of raw materials and grade them according to their constants. It can also be utilized in bricks making industry for determining the type of clay to be used in making high quality bricks. It has a high potential in mineral prospecting particularly in assessing the type of mineral being extracted from mines.

ACKNOWLEDGMENT

I am highly indebted to the Uganda Government and Swedish International Development Agency (SIDA) which financed this Project. I am also grateful to members of the Academic Staff of Physics Department who gave pertinent

advice and discussions concerning some aspects of this undertaking. Last but not least Mr. V. Lutalo who drew the diagrams in this paper I say thank you very much.

REFERENCES

- Atwater, H.A., 1962. Introduction to Microwave Theory McGraw-Hill Book Company, New York.
- Corson, D. and P. Lorrain, 2000. Fundamentals of Electromagnetic Phenomena Freeman, New York.
- Jackson, J.D., 1995. Classical Electrodynamics. John Wiley, New York.
- Kerns, D.M. and R.W. Beatty, 1967. Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis. 1st Edn., Oxford, UK.
- Montgomery, C.G., 2009, Techniques of Microwave Measurements. McGraw-Hill Book Company, University of Michigan.
- Reddick, H.W. and F.H. Miller, 1987. Advanced Mathematics for Engineers. John Wiley, New York.
- Reich, H.K., Ordung, Krauss and Skalnik, 2010. Microwave Theory and Techniques. McGraw -Hill Book Company, University of Michigan, Michigan.
- Von Hippel, A.R., S. Alexander and Labounsky, 1995, Dielectrics and Waves. Artech House, London, UK.
- White, F.W.G., 1990, Electromagnetic Waves. John Wiley, New York.