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The Numerical Analysis of the Schemes of 1-Order Ordinary Differencial Equations

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Abstract: In this study, we will consider the numerical methods of 1-order differential equations, such as forward euler scheme, predict-correct scheme and the 4-order Runge-kutta scheme. The truncation errors of these schemes are analyzed theoretically and numerically in there. The error tables and the numerical experiments are also presented at the last.

Key words: Euler scheme, numerical analysis, ordinary differential equations, runge-kutta scheme

INTRODUCTION

In this study, the initial value problem of 1-ordinary differential Eq. (1) is considered (Hairer and wannerin,1993; Hu and Tang, 2006).

$$\begin{cases} u = f\left(t,u\right), t_0 \leq t \leq T \\ u\left(t_0\right) = u_0 \end{cases} \tag{1}$$

We are going to analyze the truncation error of the forward-euler scheme, predict-correct scheme and the 4-order Runge-kutta scheme theoretically or in numerically. The study is organized as following: The numerical schemes for the 1-ordered ordinary equations; numerical experiments; summary.

THE NUMERICAL SCHEMES OF THE 1-ORDERED ORDINARY EQUATIONS

In this section we will introduce the 3 schemes of the 1-ordered differential equations and analyze the truncation error of each scheme (Li *et al*., 1999).

We divide the closed interval [t_o, T] to nth subinterval and get some mesh points: t_o, t_1, t_2, t_n.

Then $h = \frac{T - t_o}{n}$, Let u_n to approximate the exact solution of (1) $u^{(t)}$ at t_n.

Forward-Euler scheme:

$$U_{n+1} = u_n + hf(t_n, u_n) \tag{2}$$

The truncation error of forward-euler scheme: replacing the numerical solution u_n in (2) by the exact solution $u(t_n)$ then the difference of each side of Eq (2) is the truncation error of the scheme.

$$R = u(t_{n+1}) - u(t_n) - hf(t_n, (t_n)) \tag{3}$$

The Taloy series of $u(t_{n+1})$ for t_n is written as:

$$u\left(t_{n+1}\right) = u\left(t_n\right) + hu\left(t_n\right) + \frac{h^2}{2}u''\left(t_n\right) + o\left(h^3\right) \tag{4}$$

Substituted (4) to (3) then we get:

$$\begin{aligned} R &= u\left(t_n\right) + hu\left(t_n\right) + \frac{h^2}{2}u''\left(t_n\right) + o\left(h^3\right) - u\left(t_n\right) \\ &- hf\left(t_n, u\left(t_n\right)\right) = \frac{h^2}{2}u''\left(t_n\right) + 0\left(h^3\right) = o\left(h^2\right) \end{aligned} \tag{5}$$

So the forward-euler scheme is a 1-order,s method.

Predict-correct scheme:

$$\begin{aligned} u_{n+1} &= u_n + \frac{h}{2}\left[f\left(t_n, u_n\right) + f\left(t_{n+1}, \tilde{u}_{n+1}\right)\right] \\ \tilde{u}_{n+1} &= u_n + hf\left(t_n, u_n\right) \end{aligned} \tag{6}$$

We can also analyze the truncation error of predict-correct scheme as that for the forward euler scheme.

$$R = -\frac{h^3}{12}u'\left(t_n\right) + o\left(h^4\right) = o\left(h^3\right) \tag{7}$$

So the predict-correct scheme is a 2-order,s method.

Order Runge-Kutta scheme:

$$\begin{cases} u_{n+1} = u_n + \dfrac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \\ k_1 = f\left(t_n, u_n\right) \\ k_2 = f\left(t_n + \dfrac{h}{2}, u_n + \dfrac{h}{2}k_1\right) \\ k_3 = f\left(t_n + \dfrac{h}{2}, u_n + \dfrac{h}{2}k_2\right) \\ k_4 = f\left(t_n + h, u_n + hk_3\right) \end{cases} \tag{8}$$

This scheme is also called Simposon method. The truncation-error of this scheme is:

$$R = O(h^5) \tag{9}$$

So the scheme is 4-order method.

NUMERICAL EXPERIMENTS

In this section, we will show the differences of each scheme by numerical experiments.
Example (Hu and Tang, 2006):

$$\begin{cases} u' = u, 0 \le t \le 1 \\ u_0 = 1 \end{cases} \tag{10}$$

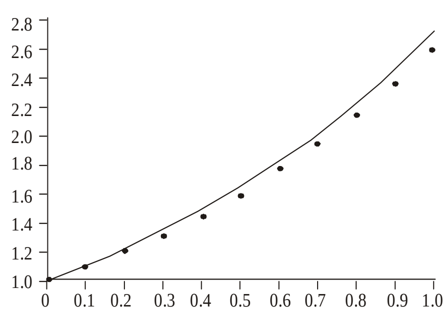

Fig. 1: Forward scheme where h = 0.1

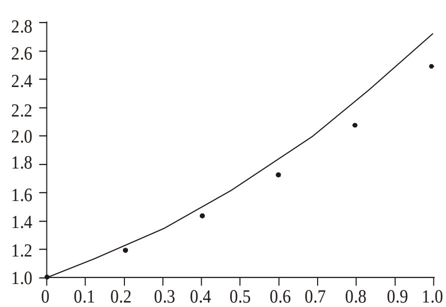

Fig. 2: Forward scheme where h = 0.2

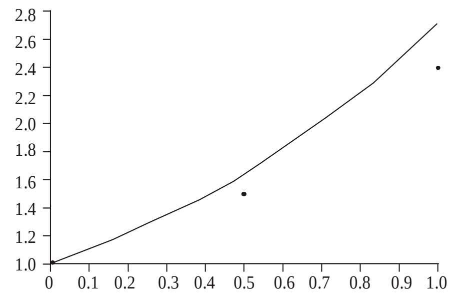

Fig. 3: Forward scheme where h = 0.5

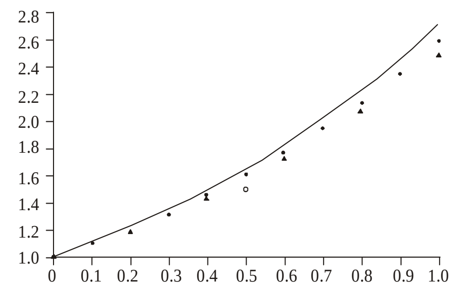

Fig. 4: Forward scheme all together

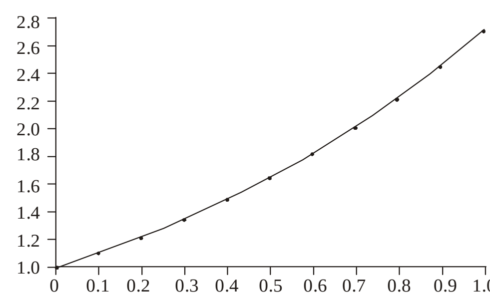

Fig. 5: Predict-correct scheme where h = 0.1

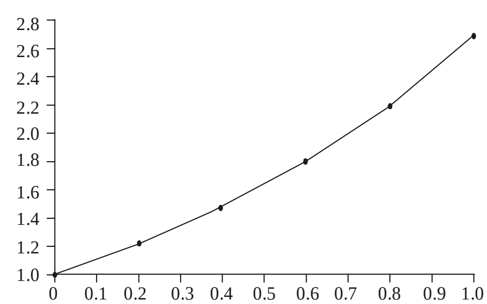

Fig.6: Predict-correct scheme where h = 0.2

Case 1: The numerical solutions of forward scheme. We will show the figures of forward scheme with different values of h.

Case 2: The numerical solutions of predict-correct scheme.

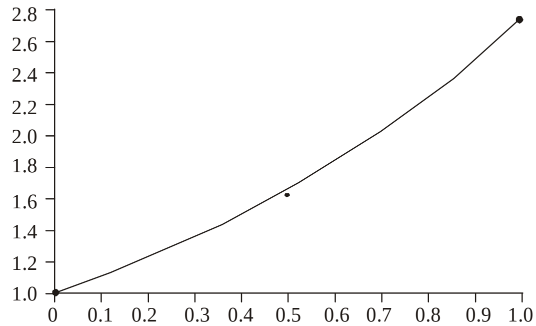

Fig. 7: Predict-correct scheme where h = 0.5

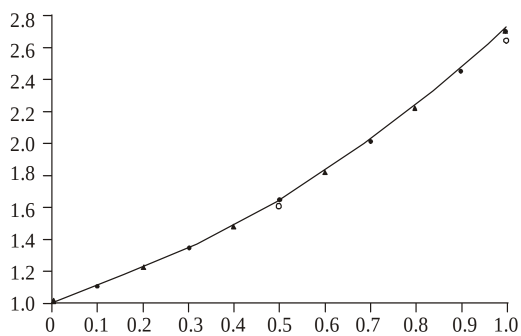

Fig. 8: Predict-correct scheme all together

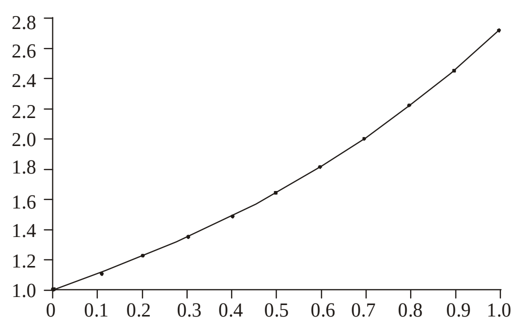

Fig. 9: 4-order Runge-kutta scheme where h = 0.1

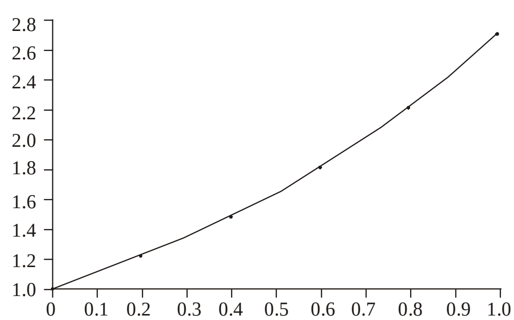

Fig. 10: 4-order Runge-kutta scheme where h = 0.2

We will show the figures of predict-correct scheme with different values of h.

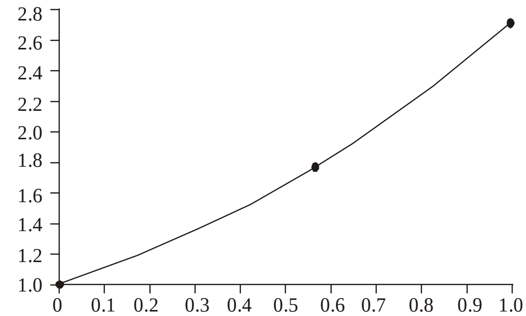

Fig. 11: 4-order Runge-kutta scheme where h = 0.5

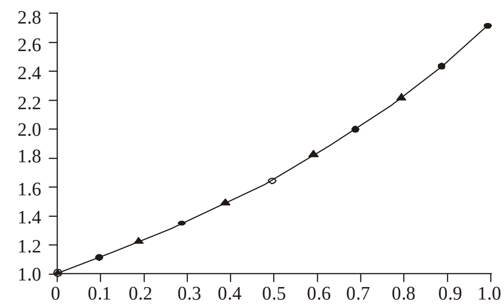

Fig. 12: 4-order Runge-kutta scheme all together

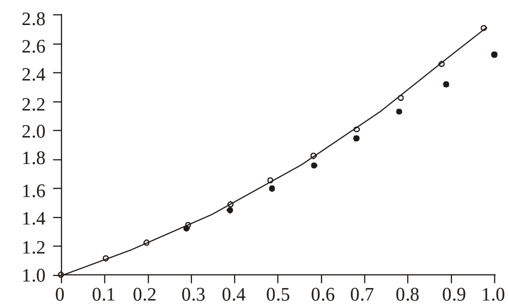

Fig. 13: Forward-eulerl predict-correct; Runge-Kutta for h = 0.1

Table1: The truncation error of all schemes for h = 0.1

	Foward-euler	Predict-correct	Runge-Kutta
0	0	0	0
0.2	0.02139	0.001403	2.76E-06
0.4	0.05183	0.003425	6.74E-06
0.6	0.06912	0.006271	1.23E-05
0.8	0.15194	0.010206	2.01E-05
1	0.22996	0.015574	3.07E-05

Table 2: The truncation error of forward scheme for h = 0.1; 0.2; 0.5

h	h = 0.1	h = 0.2	h = 0.5
X = 1.0	0.12454	0.22996	0.46828

Case 3: The numerical solutions of 4-order Runge-Kuttascheme. We will show the figures of 4-order Runge-Kutta scheme with different values of h.

Case 4: The numerical solutions of all schemes for h = 0.1

We can see from Table 2 and Fig. 1-12 that the samller h is the more exact numerical solution is for the same scheme.

We can also so from Table 1 and Fig. 13 the 4-order Runger-kutta scheme is most exact; the predict-correct scheme is more exact and the accuracy of the forward scheme is low.

CONCLUSION

In this study, we have analyzed the error of the numerical schemes for 1-ordered ordifferential equations. We also prensented the numerial approximations of some experiments to illustrate the difference of the scheme. At last, we get the conclusion that 4-orderd Runge-Kutta scheme is the highest resolution scheme, and the forward-euler scheme is easy to implement.

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REFERENCES

Hairer, E. and G. Wannerin, 1993. Solving Ordinary Differential Equations I, Springer-Verlag Beilin. Heideberg.

Hu, J. and H. Tang, 2006. Numerical Solution of Differential Equationss. (In Chinese): The Scientific Press.

Li, L., C. Yu and Z. Zhu, 1999. Numerical Solution of Differential Equationss. (In Chinese): The Press of Fu Dan University.