A Cost Model of Partial Postponement Strategy of the Single-Period Product under Stochastic Demand

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Abstract: The target of our study is to set a new cost model to solve the partial postponement problem by adding penalty cost parameter of shortage under stochastic demand. By doing this, we hope the new model can be better applied to real conditions. Postponement is an important strategy to achieve mass customization and it has been adopted by many companies to improve production operation, inventory and logistics management and supply chain operation, but the postponed activity will cause additional costs at the same time. There have been many literatures trading off between the relative cost and the postponement benefits of product manufacture, and our paper is to solve the similar problem.

Key words: Inventory, mass customization, partial postponement

INTRODUCTION

Postponement strategy is an effective strategy to achieve mass customization and it has been perceived as one of the major supply chain management practices, because it can better deal with the product proliferation without incurring large operating costs caused by postponement activities (Li et al., 2007). By holding some or all common inventory in the early stage of product with uncertainty demands and delaying the customizing final product until the demand information is certain, the postponement strategy can save inventory levels, reduce delivery lead times, and various product portfolio to satisfy various customer demand, by doing this, the postponement strategy can lessen the mismatch between the forecast-driven production and the actual demand, reduce the effect of bullwhip in the supply chain and improve customer satisfaction obviously (Wong et al., 2010, 2009). But how many inventories can be held to reduce cost and response to customer quickly. If the inventory is all hold as final products, it will derive the scale economy of mass production with high production efficiency, but it will not meet the various customer requirements or it will cause very high inventory level caused by the various product portfolios (Chod et al., 2006). In the contrary, if the inventory is all hold as common inventory of modules or basic products, maybe it will reduce the inventory level fully for the risk pooling effect and achieve the scope economy, but the lost sale will occur when the delivery time exceeds the expect time of customer waiting for the final customization (Ronald and Bassok, 2004). So there is a dilemma to the common inventory level and final product inventory (Anupindi and Jiang, 2008). There have been many researches on the postponement problem, typically, as in Graman and Magazine (2002), denoted G&M, the authors studied the impact of postponement capacity on the benefits of inventory savings, given a defined customer service level, denoted as fill rate and which is decided by managers in the paper, and then a model of single-period and two products capacitated-postponement inventory is analyzed, where the non-postponed finished final product inventory and the generic product inventory (i.e., postponed inventory), which will be customized until the real customer demand is known are both held by the manufacture. The two final different products can be made from the generic products (i.e., postponed products) by packaging, additional parts or other customized service item etc. to meet different demand. The finished product inventory will be used to meet the real demand first, but once the demand are excess of finished product inventory, some or all of the generic product will be completed to meet the excess demand as much as possible within the specified delivery time. The author obtained some important conclusions, such as when the fill rate, the coefficient of variation, the number of products being postponed increase, the demands are more negatively correlated and the demand distribution of different products are approaching each other, and then inventory saving will increase and most importantly, the author observed an important phenomenon that a relatively small amount of postponement capacity (about 40% of total expected demand) can be achieve all of the benefits of completely postponing all demands, i.e., keeping all the inventory as generic product and different customized parts rather than any final finished products, and the customized parts can be assembled to the generic product to form the final customized products quickly until the
real demands is certain. This important phenomenon will inspire many firms to adopt the partial postponement strategy, and especially, postponement strategy is relative to some additional postponement cost, such as the investment cost, processing cost, handling cost of common inventory. But there is a potential preliminary behind the numerical analysis and observation, i.e. the fill rates of different products are the same.

So in the later research of Graman (2010), denoted G, the author further set a similar model to the model in Graman and Magazine (2002), but there are some distinct differences. The first difference is Graman (2010) focused on solving the minimum-cost objective function by considering some additional costs caused by postponement, where the postponed manufacturing or assembly will cause the more frequent setups of production line to process smaller lot size, additional handling, packaging to facilitated the handling and maintaining integration of generic products, denoted as assemble labor and material cost. Secondly, Graman (2010) showed some additional different conclusions based on the research of Graman and Magazine (2002), such that when the value of generic products and relative postponement cost decrease (including packaging postponed inventory to maintain product integrity, additional operation, waiting caused by inability to delivery within the specified lead time), the holding cost increase, etc., the total inventory and expected total cost will decrease. The third difference between the two continuous researches is: in Graman and Magazine (2002), the assumption that each fill rate or the expected stock-out number for each customized final product equal are the basis for comparison among different level of partial or capacitated postponement strategy and other sensitivity analysis of different parameters change on the inventory level of generic products and final products. But in the version of Graman (2010), the fill rate or the expected stock-out number is set as an constraint in solving the minimal cost non-linear programming problem, and in the process of reasoning and analysis, Graman (2010) didn’t discriminate the regions Anupindi and Jiang (2008), Ronald and Bassok (2004) and Li et al. (2007) in Fig. 1. That is to say, Graman (2010) treat the condition happened in region Anupindi and Jiang (2008) and Ronald and Bassok (2004) as the same in region Li et al. (2007) in computing the expected stock-out number, as shown in the Eq. (C.4) and (C.5) in the Appendix C in Graman (2010), which will cause error reasoning in solving the non-programming problem. Importantly, the fill rates for different customer are often different according to the profits obtained from the sales for different customer, the important degree of the orders, or the penalty cost caused by the demand unmet. The more important of the order, the more profit obtained from the order or the bigger penalty cost, the responding order should be met in the more anterior sequence, which is a universal phenomenon in many enterprises, so there should be some difference between the fill rates for different customers or products, and we will demonstrate that the problem formulation and computation will be simplified and direct by introducing the penalty cost parameter substituting for the fill rate.

So the main targets of our paper include two aspects:

- Why the expression of expected stock-out product should consider the region Li et al. (2007), Anupindi and Jiang (2008) and Ronald and Bassok (2004) individually and discriminate the integration expression of expected stock-out number happened in region Li et al. (2007), Anupindi and Jiang (2008) and Ronald and Bassok (2004)
- When the introduce of the different penalty cost parameter rather than the equal fill rate parameter for different customized product, and when the shortage-cost items set as an item in the objective total cost function, but not as a constraint included in Graman (2010), the reasoning and computation can be simplified, and more importantly, it is more fit the decision condition as many companies meet.

For convenience, the following notation and definitions used are same in Graman (2010) except the new parameter of penalty cost $t_1$ and inventory number of left ove.

This study will compute the optimal inventory level of both common product and final product by trading off between the relative cost and the postponement benefits of product manufacture involving penalty cost of shortage under stochastic demand.

Assumption and notation: Of course, there may be some potential assumptions in the Graman and Magazine (2002) and Graman (2010) and in this study as following:

- The two products completed from the same generic inventory can be substituted for the other one, so the inventory of product 1 can be used to meet demand of product 2, vice versa.
Either of the finished products can be further revised or customized to meet the demand of the other product.

Customization lead times for both products are zero.

Each product has a linear customization cost, i.e. the additional expense of using postponed manufacturing mode over the cost of non-postponement mode. The unit customization cost for both products is same. Besides, the customized production capacity is assumed to be unlimited, so the fixed cost associated with postponement is assumed to be minimal and can be set equal to zero.

When the postponed inventory is available, once there is a shortage of either or both of products, customizing the products takes place.

Each final product contains one unit of the generic product and the difference between final products is cosmetic, so all quantities are in terms of the generic product.

If we further assume that the order with more unit penalty cost is met first by the postponed capacity, the problem will be simplified when the demand exceed the finished product, and there is no need to discriminate the demand happened in region Li et al. (2007) and Ronald and Bassok (2004), i.e., we can adopt the Fig. 2 directly to solve the capacitated postponement problem.

Expressions for the expected stock-out inventory: In model of Graman (2010), the objective function was to minimize the cost made up of assembly cost, postponement cost, packaging cost and holding cost of combined finished goods with postponed inventory. The constraints include fill rate constraints, boundary condition constraints, postponement capacity allocation constraints and non-negativity constraints. The expression of the expected stock-out products $E[SO_i]$ ($i = 1, 2$) in the region Chod, et al., (2006), Graman and Magazine (2002), Graman (2010) Wong et al. (2009) and Wong et al. (2010) is same to Graman and Magazine (2002) and Graman (2010), when the real demand of both products can’t be met by all finished product inventory and generic or postponed inventory which is happened in region Li et al. (2007) and Ronald and Bassok (2004) of Fig. 1 or in region Li et al. (2007) of Fig. 2, then the computation of expected stock-out number for each product is based on the decision rule: equalize the fill rates of each product Graman (2010), i.e.

$$\frac{S_i + P_1}{x_1} = \frac{S_2 + P_2}{x_2}$$

and $P_1 + P_2 \leq C$, which is same to $E[SO_1] = E[SO_2]$, but this constraint was not included or reflected in the model of Graman (2010). Besides, Graman (2010) didn’t
discriminate the regions Anupindi and Jiang (2008), Ronald and Bassok (2004) and Li et al. (2007), in Fig. 1. That is to say, Graman (2010) treat the condition happened in region Anupindi and Jiang (2008) and Ronald and Bassok (2004) as the same in region Li et al. (2007), in the computation the expected stock-out number, as shown in the Eq. (C.4) and (C.5) in the appendix C, which will cause error reasoning in solving the non-programming problem. The reason for this is as following:

When the demand happened in the region Anupindi and Jiang (2008) and Ronald and Bassok (2004) all finished and postponed inventory will all be exerted where $P_1 + P_2 = C$, and this is basis to compute $E[SO_i]$. At the same time, the equation will decide how the postponed capacity (or generic products) is allocated to each type of final products, but as shown in Graman and Magazine (2002), when the demand happened in region Anupindi and Jiang (2008) or Ronald and Bassok (2004) one of the product demand is much more larger than the other one, so even all the postponement capacity is allocated to complete the product of large demand, the fill rate can’t be raised to equal that of lower product demand. For example, in region Anupindi and Jiang (2008) demand for product 2 is much more larger than product 1, so all the postponed capacity is allocated to product 2 to attempt to equalize the fill rate, as a result, the boundary function between Li et al. (2007) and Anupindi and Jiang (2008), a is:

$$\frac{S_1 - S_2}{x_1}$$

similarly, the boundary function between Li et al. (2007) and Ronald and Bassok (2004) is

$$\frac{S_1 + C}{x_1} - \frac{S_2}{x_2}$$

so we can get the expected stock-out number for product 1 in region Li et al. (2007), and Ronald and Bassok (2004):

$$E[SO_1(7)] = \int_{x_2 = S_2}^{S_1} \int_{x_1 = S_1 + S_2 + C-x_2}^{(S_1 + C)x_2/S_2} (x_1 - S_1 - P) \times f(x_1, x_2) dx_1 dx_2$$

$$E[SO_1(8)] = \int_{x_2 = S_2}^{S_1} \int_{x_1 = S_1 + S_2 + C-x_2}^{(S_1 + C)x_2/S_2} (x_1 - S_1 - C) f(x_1, x_2) dx_1 dx_2$$

$$E[SO_1(9)] = \int_{x_2 = S_2}^{S_1} \int_{x_1 = (S_1 + C)x_2/S_2}^{(S_1 + C) - C} (x_1 - S_1 - C) f(x_1, x_2) dx_1 dx_2$$

The equation for product 2 $E[SO_2(7)] E[SO_2(8)]$ and $E[SO_2(9)]$ can be denoted by the same reason. But not the general expression (C.4) and (C.5) in the Appendix C in Graman (2010) which will ignore the condition in region Anupindi and Jiang (2008) and Ronald and Bassok (2004) in the process of computation which will influence the result of program solving.

The new model including penalty cost parameter: In this section, we will introduce the unit penalty cost parameter $t$, denoting the penalty cost of unit shortage of product/ (which is also often used to reflect the customer satisfaction level from the viewpoint of manufacture), then there is no need to discriminate the condition in Li et al. (2007), and Ronald and Bassok (2004), as shown in Fig. 2. For the demand for the products of larger unit penalty cost must be always satisfied before the demand of smaller one, or the total cost can’t be minimized at all.

After the penalty cost parameter is introduced to substitute for the fill rate, the fill rate constraints can be relaxed, the expression and computation of $E[SO_i]$ can be simplified.

- When none of products stock out, the generic inventory will not be used, and some finished inventory and generic inventory will be left over in the end of period, as the demands in region Chod et al. (2006) in Fig. 2.
- When only one of finished products stock outs, i.e. the demands in region Graman and Magazine (2002), Graman (2010) Wong et al. (2009), Wong et al. (2010), some or all generic inventory will be allocated to the product of shortage.
- When both demands can’t be met from the finished inventory directly in region Lee et al. (1997) but $x_1 + x_2 \leq S_1 + S_2 + C$, so the postponed inventory can be allocated as: $x_1 - S_1$ for product 1 and $x_2 - S_2$ for product 2. Obviously, there is none shortage of product in region Chod et al. (2006), Graman and Magazine (2002), Graman (2010) Wong et al. (2009), Wong et al. (2010) and Lee et al. (1997), so there is no penalty cost when demands happened in these regions.
- When all the postponed inventory are exhausted, both of the demands can’t be met at all, as the condition in region Li et al. (2007), and Ronald and Bassok (2004) in Fig. 1 or the region Li et al. (2007), in Fig. 2. The allocation principle is that the demand of larger penalty cost will be all satisfied in the first place attempting to minimize the total cost, so there is no need to discriminate the expression of $E[SO_i]$ which is different from Graman and Magazine (2002) and Graman (2010). The reason behind this is simple, as long as the product can reduce the cost to more extent, the postponed inventory will be allocated to it, even all the generic inventory will be allocated to the product of max {$t_1, t_2$}:
so the objective function is:

\[
E[SO_i(t)] = \int_{t_3-i}^{t_3} \int_{S_i}^{S_i+i} [x_i - \bar{S}_i - C \times \max(0, \text{sig}(t_i - t_{3-i})]) \times f(x_i, x_{3-i}) \, dx_i \, dx_{3-i} - \int_{t_3-i}^{t_3} \int_{S_i}^{S_i+i} C \times \max(0, \text{sig}(t_i - t_{3-i})]) \times f(x_i, x_{3-i}) \, dx_i \, dx_{3-i} 
\]

The inventory level of final product 1, 2 and common product C, can be illustrated in Table 1 (obviously, \( E[LO_i] + E[P_i + P_2] = C \)), based on the depiction in Fig. 2.

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>( 7(t_{1}&gt;t_3) )</th>
<th>( 7(t_{1}&gt;t_3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO1</td>
<td>S1-x1, S1-x2, 0</td>
<td>S1-x1, 0</td>
<td>S1-x2, 0</td>
<td>S1-x2, 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LO2</td>
<td>S2-x1, 0</td>
<td>S2-x1, 0</td>
<td>0</td>
<td>S2-x2, 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P1</td>
<td>0</td>
<td>0</td>
<td>x1-S1, 0</td>
<td>C</td>
<td>x1-S2, C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P2</td>
<td>0</td>
<td>x2-S1, 0</td>
<td>0</td>
<td>x2-S2, C</td>
<td>0</td>
<td>C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SO1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>x2-S1-C</td>
<td>0</td>
<td>x2-S2-C</td>
<td>0</td>
<td>x2-S2-C</td>
</tr>
<tr>
<td>SO2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>x2-S1-C</td>
<td>0</td>
<td>x2-S2-C</td>
<td>0</td>
<td>x2-S2-C</td>
</tr>
</tbody>
</table>

The total cost includes assembly labor and material cost, postponement cost, packaging cost, holding cost of finished final product but none generic product, and shortage cost, so the objective function is:

\[
E[TC_{PP}] = m \times (S_1 + S_2) + m \times C + w \times C + d \times (S_1 + S_2) + d \times E[LO_i] + E[P_i] + E[SO_i] + E[LO_i] + E[P_i] \times h_p \times E[SO_i] + t_i \times E[SO_i] + t_2 \times E[SO_i] 
\]

subject to \( C, S_i \geq 0, \forall i \).

when \( C = 0 \), it means that the non-postponement strategy is adopted, i.e. in the initial period, the inventory only includes the finished final product but none generic product, and there is none customization activity. So the best choice would be the one with the lowest total cost. In the equation:

\[
E[P_i + P_2] = \int_{S_1}^{S_2} \int_{S_1}^{S_2} \left( S_1 + S_2 \right) f(x_1, x_2) \, dx_1 \, dx_2 
\]

There are three decision variables \( S_1, S_2, C \) and only some non-negativity constraints compared with the five decision variables and more additional constraints in Graman (2010), so the computation process will be simplified and it will be easy to observe the effects of variable \( C \) on the value of \( S_i \) and the total cost, when the postponed capacity is set to different value.

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Case study: The corresponding parameters are denoted as \( m = 0.5, d = 0.5, r, h_p = 0.5, h_F = 0.5, h_F = 0.5, w = 0.5u_i = 1000, u_2 = 1000, \sigma_1 = 200, \sigma_2 = 200 \).

The Genetic Algorithm is adopted to solve the problem, the solution is:

\( S_1 = 750.64 \), \( S_2 = 670.32 \)
\( C = 710.30 \), TC = 2806, TC = 2806

The impact of common inventory on the number of each final product inventory and total cost is computed by
MatLab7.0, as illustrated in Table 2. From the Table 2, we can see that the partial postponement strategy can save inventory level and reduce the total manufacture cost. Besides, the inventory level of product 1 is higher than product 2, the reason behind this is the penalty cost of product 1 is higher, so the inventory level of final product 1 and the number of common inventory allocated to product 1 is more than product 2 to reduce the cost to maximal extent.

**CONCLUSION**

It is difficult to figure out the condition where more than two products customized from the generic inventory, but computation process will be simplified for the allocation rule of the postponement capacity proposed in this paper by introducing the different penalty cost parameter. In future research, we can still consider two final products which are still assembled or customized from a common inventory or product platform, but the two products can be partial substituted for each other, such that one of the product with better characteristics can be used to substitute for the other one to meet demand, but the reverse substitution can’t be accepted or price of the product with better characteristics is general higher than the other one and the substitution is relative to tradeoff between product price and fill rate of customer demand. Besides, some important factors should be considered such as product characteristics (electronics, automotive, clothing, and so on), special delivery time window, customization time from generic inventory, the optimal numbers of parts in the generic product, or the ratio of the volume of generic product to final customized product, the difficulty degree of customized assembly and packaging, etc. which will influence the postponement cost, delivery time and customer satisfaction.

**REFERENCES**


