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Grid Generation around a Cylinder by Complex Potential Functions

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Abstract: In this study, orthogonal and structured grid generation around a cylinder is described by using potential functions. In this method, the orthogonality of and φ functions are used for grid generation. First, coordinates of points are given by using the algebraic method on cylinder boundaries and then, according to known potential functions in terms of *x* and *y* values in the external flow around a cylinder, coordinates of other network's points are calculated through solving a system of two nonlinear equations with two unknowns. Iteration methods are used for solving this system of equations. The generated grid besides orthogonal property has small distances on the surface of cylinder and gradually as it goes farther away from the cylinder, the distance between nodes rises. This kind of grid can be useful in solving the flow field around a cylinder.

Key words: Complex potential function, grid generation, orthogonal grid

INTRODUCTION

Computational fluid dynamics is a branch of Mechanical Engineering in which numerical method is used to solve equations governing fluid dynamics and heat transfer. Equations governing fluid motion are usually of partial differential type that partial derivatives should be approximated to solve these kinds of equations. With these approximations, the equations of partial derivatives are converted to finite difference expressions that differential equations are converted in to algebraic equations. Obtained algebraic equations are called finite difference equations that should be solved in the special grid and network. So within the related domain and on its borders, some grid points are determined. Furthermore, the effect of grid's quality on accuracy and convergence of numerical methods has caused particular importance of computational grid generation. Generated grids are classified into two groups of structured and unstructured. Structured grid is generated in a way that grid points can be easily identified on a regular basis compared to grid lines that are defined regularly. In other words, each point of the grid can be defined with a certain ration of *i* and *j*. There are different methods to generate grids. The simplest method is algebraic method generated by using simple algebraic equations with respect to geometry of the appropriate grid domain. Another method is Complex Variable method on two-dimensional spaces developed by Churchill (1948), Moretti (1979), Davis (1979) and Ives (1982). The third common method of generating structured grids is the method of partial differential equations. The idea of using differential equations is based on research works of Crowley (1962) and Winslow (1966) and also based on idea of changing physical domain into a computational domain. For solving differential equations on some specific spaces with respect to physics of the problem, orthogonal grids can have very beneficial and reduce calculation's size. For example, when we investigate flow around an airfoil, we know that pressure gradient in the direction perpendicular to the surface is zero; i.e. pressure in the vertical direction on the surface will not change. According to orthogonal grid generation by methods of differential equations, this method has been used extensively.

In this study by choosing a different method we are looking for generating an orthogonal grid around a cylinder in order to help us be able to analyze the flow around it. In this study, orthogonal property of ψ and φ functions is used for grid generation. In fact, we will generate an orthogonal grid by help of potential functions around a cylinder and its relation with x and y values. It should be noted that this method can be used extensively for all domains that their ψ and φ functions are known.

MATERIALS AND METHODS

Physical domain: It is assumed that we want to generate an orthogonal grid around a cylinder by Complex Potential Functions method. We assume cylinder's the radius as 1 m and define physical space as shown in Fig. 1.

As we know, this method is based on this assumption that ψ and φ functions are orthogonal. Therefore, it is enough just to know the functions property based on x and y, then you can easily generate an orthogonal grid:

Since shape has symmetry, first it is possible to consider one fourth of it as it is shown in Fig. 2. Next, we can use mirroring against the horizontal and orthogonal axis of the grid in order to make the whole shape. Mesh

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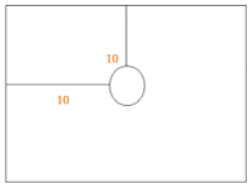

Fig. 1: The Physical domain

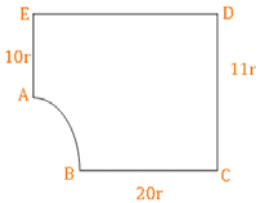

Fig. 2: One forth of the physical domain

should be generated in a way that in areas close to the cylinder is there will be less space between nodes and gradually as it goes farther away from the cylinder, the distance between nodes rises.

Grid generation method: First assume grid for the j = 1, i.e. ABC line. In this area, the shape will be divided into two parts:

AB arc: We divide this arc into N1 parts. Therefore, it gives:

$$\Delta\theta = \frac{\frac{\pi}{2}}{N1-1} \tag{1}$$

$$\theta_i = \frac{\pi}{2} - \frac{\pi}{2}(i-1)\Delta\theta \tag{2}$$

$$x_i = r\cos\theta_i \tag{3}$$

$$y_i = r\sin\theta_i \tag{4}$$

BC line: As it can be observed, the y value on this line is equal with zero and space between nodes should be in a way that as it goes farther away from the cylinder, the distance between nodes rises. Therefore, we should calculate the grid ratio in this stage. Node spaces on the AB arc will be presented with δ and the first node on the BC line will be considered with the same amount. Then gradually we increase the space between nodes according to the ratio of R in order to get the C point. We will show number of nodes on this line with N2.

$$\delta\frac{1-R^{N2-1}}{1-R}BC \tag{5}$$

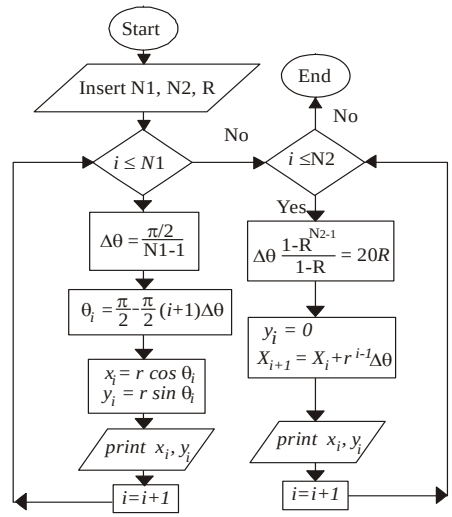

Fig. 3: Grid algorithm generated on j: 1

R or grid ration for every j = 1, i = N1+N2−1 is known. Therefore, the only unknown in this equation is R. This equation is a kind of first-degree non-linear equation that can be solved by iteration method of Newton-Raphson, and R value can be calculated.

With given values of x and y it is possible to use and φ values through relations of complex potential functions for the flow around the cylinder:

$$\psi = A\left(r - \frac{1}{r}\right)\sin\theta = A\left(y - \frac{y}{x^2+y^2}\right) \tag{6}$$

$$\varphi = A\left(r + \frac{1}{r}\right)\cos\theta = A\left(x + \frac{x}{x^2+y^2}\right) \tag{7}$$

Parameter A is a fixed number. It is assumed as1. Therefore, it will be possible to calculate values of x and y on the j = 1 line. You can observe grid algorithm generated on j = 1 line in the Fig. 3.

In the next step, we consider the AE line. Space between nodes 1 and 2 on the AB arc equals with δ that is a known value. First node on AE line will be assumed with the same space and then space between nodes will be increases gradually with R ratio. We want to generate N3 nodes on this line. R ratio in this case can be calculated by following relation:

$$\delta\frac{1-R^{N3-1}}{1-R} = AE \tag{8}$$

It is obvious that x coordinates of the points located on AE line segment equals with zero. Therefore, x and y values can be calculated on this line too. So x(i, j) and y(i, j) for every (i = 1 , j = N3-1) is known.

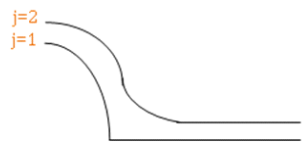

Fig. 4: The fixed ψ line

Now we assume the j = 2 line. Because x and y is known for (I = 1, j = 2) point, then it will be possible to calculate ψ and φ based on given relation. In Fig. 4 you can see the fixed line:
Now we move on the fixed ψ line or j = 2. It is obvious that:

$$\psi(2, 2) = \psi(1, 2) \tag{9}$$

Now we move on the fixed ψ line ψ or move perpendicular to the direction:

$$\varphi(2, 2) = \varphi(2, 1) \tag{10}$$

With known values of and for this point, it will be possible to calculate x and y values for these points through solving two equations and two unknown system (equations). Method of solving nonlinear two equations and two unknown system is given in the next section.
We can continue in the same way:

$$\varphi(3, 2) = \varphi(3, 1) \tag{11}$$

$$\psi(3, 2) = \psi(1, 2) \tag{12}$$

Therefore, on j=constant lines we will have:

$$\varphi(i, j) = \varphi(i, j-1) \tag{13}$$

$$\psi(i, j) = \psi(i-1, j) \tag{14}$$

Therefore, by the above-mentioned relation, and φ values can be easily used for various points. Furthermore, x and y values can be calculated through solving two equations and two unknown system.

Method of solving two equations and two unknown system: Our equation system is as follows in which with known values of and, the value of x and y or nodes coordinates will be calculated (A and B parameters are assumed as 1):

$$f(x,y) = x + \frac{x}{x^2 + y^2} - \varphi = 0 \tag{15}$$

$$g(x,y) = y - \frac{y}{x^2 + y^2} - \psi = 0 \tag{16}$$

To solve the system's equations, we assume that (α, β) is the desired answer of the system and (x_0, y_0) is an approximation of (x, y). So, we can write:

$$\alpha = x_0 + h_0 \tag{17}$$

$$\beta = y_0 + k_0 \tag{18}$$

we use Taylor Expansion method to calculate h_0 and k_0 parameters in Eq. (3) and (4). In the performed expansion, the expressions above second order are ignored:

$$f(\alpha, \beta) = f(x_0 + h_0, y_0 + k_0) \tag{19}$$
$$= f(x_0, y_0) + h_0 \frac{\partial f(x_0, y_0)}{\partial x} + k_0 \frac{\partial f(x_0, y_0)}{\partial y}$$

Since (α, β) is an approximation for answers of the problem, then we will have:

$$0 \approx f(x_0, y_0) + h_0 \frac{\partial f(x_0, y_0)}{\partial x} + k_0 \frac{\partial f(x_0, y_0)}{\partial y} \tag{20}$$

It will be calculated in the same way for g function:

$$0 \approx g(x_0, y_0) + h_0 \frac{\partial g(x_0, y_0)}{\partial x} + k_0 \frac{\partial g(x_0, y_0)}{\partial y} \tag{21}$$

By substitution of f and g values in Eq. (6) and (7), we will have:

$$h_0\left[1 + \frac{y_0^2 - x_0^2}{(x_0^2 + y_0^2)^2}\right] + k_0\left[\frac{-2x_0y_0}{(x_0^2 + y_0^2)^2}\right] = -\left[x_0 + \frac{x_0}{x_0^2 + y_0^2} - \varphi\right] \tag{22}$$

$$h_0\left[\frac{2x_0y_0}{(x_0^2 + y_0^2)^2}\right] + k_0\left[1 - \frac{x_0^2 - y_0^2}{(x_0^2 + y_0^2)^2}\right] = -\left[y_0 - \frac{y_0}{x_0^2 + y_0^2} - \psi\right] \tag{23}$$

The initial assumption of $x_0 = y_0 = 1$ is considered. Therefore, Eq. (8) and (9) can be simplified as follows:

$$h_0 + \frac{1}{2}k_0 = \varphi - \frac{3}{2} \tag{24}$$

$$\frac{1}{2}h_0 + k_0 = \psi - \frac{1}{2} \tag{25}$$

The above mentioned system is a system of linear equations based on h_0 and k_0 that will be solved by Kramer Order:

$$h_0 = \frac{2}{3}\left[2\varphi + \psi - \frac{7}{2}\right] \tag{26}$$

$$k_0 = \frac{2}{3}\left[2\psi - \varphi + \frac{5}{2}\right] \tag{27}$$

By getting h_0 and k_0 values, then $x_0 + h_0$ value will be a better approximation of x_0 for α and also $y_0 + k_0$ will be a better approximation of y_0 for β.

Therefore,

$$x_1 = x_0 + h_0 \tag{28}$$

$$y_0 = y_0 + k_0 \tag{29}$$

Now we repeat the same operation for x_1 and y_1 and calculate x_2 and y_2 approximations.

Generally, if x_n and y_n to be calculated, then by solving the following system:

$$h_n\left[1 + \frac{y_n^2 - x_n^2}{(x_n^2 + y_n^2)^2}\right] + k_0\left[\frac{-2x_n y_n}{(x_n^2 + y_n^2)^2}\right] = -\left[x_n + \frac{x_n}{x_n^2 + y_n^2} - \varphi\right] \tag{30}$$

$$h_n\left[\frac{2x_n y_n}{(x_n^2 + y_n^2)^2}\right] + k_n\left[1 - \frac{x_n^2 - y_n^2}{(x_n^2 + y_n^2)^2}\right] = -\left[y_n - \frac{y_n}{x_n^2 + y_n^2} - \psi\right] \tag{31}$$

The h_n and k_n values will be calculated and we will have:

$$x_{n+1} = x_n + h_n \tag{32}$$

$$y_{n+1} = y_n + k_n \tag{33}$$

We repeat the iteration operation to the point that:

$$|x_{n+1} - x_n| < 0.0001 \tag{34}$$

$$|y_{n+1} - y_n| < 0.0001 \tag{35}$$

RESULTS AND DISCUSSION

In this study by using complex potential function method we are looking for generating an orthogonal grid around a cylinder in order to help us be able to analyze the flow around it. The generated mesh is defined in a way that in areas close to the cylinder is there will be less space between nodes and gradually as it goes farther away from the cylinder, the distance between nodes increases. According to symmetry of the physical space, first grid is produced for one fourth of it and then is generalized to the whole problem according to the symmetry. In Fig. 5, you can see the generated grid for the one fourth and one-half of the shape related to the N1 = 50, N2 = 50, N3 = 50 case.

To represent orthogonal grids more clearly, you can observe magnification of some parts of Fig. 6 in the Fig. 7. As you can observe in these figures, as we go farther away from the cylinder surface, the distance between nodes increases because our calculation's size decreases. On the other hand, the generated mesh is orthogonal that can be used efficiently to solve the flow field in the external flows. In the Fig. 8, you can see generated grid for the whole shape related to the N1 = 50, N2 = 50, N3 = 50 case.

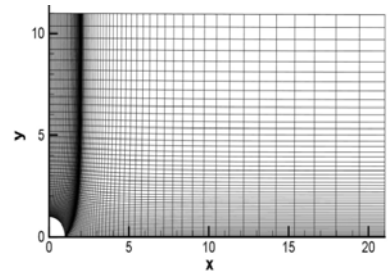

Fig. 5: The generated grid for the one-fourth of the shape related to the N1: 50; N2: 50; N3: 50 case

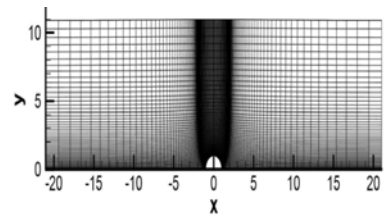

Fig. 6: The generated grid for the one-half of the shape related to the N1:50; N2:50; N3:50 case

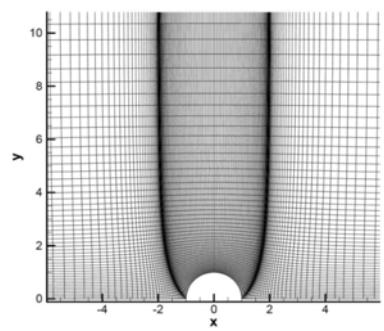

Fig. 7: Magnification of some parts of Fig. 6

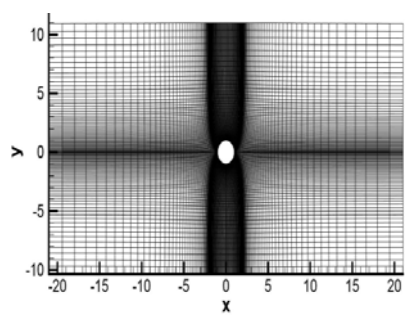

Fig. 8: The generated grid for the whole shape related to the N1 = 50, N2 = 50, N3 = 50 case

CONCLUSION

In this study, an orthogonal mesh is generated around a cylinder by using orthogonal property of ψ and φ functions. The generated grid besides orthogonal property has small distances on the surface of cylinder and gradually as it goes farther away from the cylinder, the distance between nodes increases. This kind of grid can be useful in solving the flow field around a cylinder. This kind of mesh besides its orthogonal property is generated in a way that there are many little distances on the cylinder surface, and as we go farther away from the cylinder surface, the distances between increases, because the closer distances exist between nodes, the more accurate answers will generate for solving mesh equations. Since responses on the surface of the cylinder and in the closer points are of more importance, then fewer distances are considered between nodes.

Most important advantage of this method is orthogonal mesh generation with a low amount of calculations in comparison with other methods of grid generation like partial derivatives method. However, disadvantage of this method is that in special spaces we consider ψ and φ functions based on r and θ or x and y, or in other words it is possible to calculate the Function Complex Potential.

RECOMMENDATIONS

In this study only the grid generated around a cylinder is described by using potential functions method. It is suggested that this issue to be considered in other spaces with known functions like wedge and obtained results to be analyzed. On the other hand, it is suggested to solve a flow equation around a cylinder with generated mesh for investigation of validity of the method. Next, obtained results will be compared with results of other methods.

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