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Robust Sliding Mode Control of Electromagnetic Suspension System with Parameter Uncertainty

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Abstract: Due to the nonlinearities inherent in electromagnetic suspension systems, it is difficult to design a linear controller which gives satisfactory performance and stability over a wide range of operating points. Besides, uncertainties in modeling of the system make it difficult to control the system robustly. The parameter uncertainties such as mass and electric resistance variations of the system and external disturbances affect the performance of the system. In this study a sliding mode controller is designed which is robust to bounded mass and electric resistance changes and reject the external disturbances. Besides the robustness of the mentioned controller, its simplicity makes it interesting to apply to Electromagnetic Suspension System. The system and controller are simulated in Matlab/simulink environment. The results of the simulations confirm the satisfactory performance and robustness of the designed controller against uncertainties and disturbances.

Key words: Electromagnetic suspension system, robustness, sliding control, uncertainty

INTRODUCTION

Electromagnetic Suspension systems (EMS) are widely used in high speed ground transportation and magnetic bearings because of their ability to eliminate friction between rail and magnetic body. Due to inherent nonlinearities associated with the electromechanical dynamics, the control problem is usually quite challenging to engineers, since a linear controller is valid only about a small region around a nominal operating point. In recent years a lot of works have been reported in literature for controlling magnetic suspension systems. Linear control strategies have been developed by researchers, are based on a Taylor series expansion of the actual nonlinear force distribution at the nominal operating point (Sinha, 1987). Despite of their simplicity, the tracking performance of the linear control strategies deteriorates rapidly with increasing deviations from the nominal operating point. Two approaches to the problem of ensuring consistent performance to be independent of operating point have been reported in literature (Kim and Kim, 1994; Trumper *et al*., 1997; Joo and Seo, 1996). One method is that of gain scheduling where nonlinear force- current- air gap relationship of the magnetic suspension system is successively linearized at various operating points with a suitable controller designed for each of these operating points (Kim and Kim, 1994). Another approach to deal with the magnetic suspension problem is the feedback linearization technique. Feedback linearization utilizes the complete nonlinear description of the system and yields consistent performance largely independent of operating point of the plant. Many investigations have been done by researchers around controlling magnetic suspension systems by feedback linearizing technique (Joo and Seo, 1996; Trumper *et al*., 1997). This technique besides having mentioned advantages is sensitive to parameter variations and other disturbances. Usually, feedback linearizing control does not guarantee robustness in the presence of model uncertainty and disturbances (Joo and Seo, 1996). In the other works the cascade design method (Yu *et al*., 2002) and nonlinear H4 approach (Bittar and Sales, 1997; Sinha and Pechev, 2004) reported that suffer from complication.

Modeling inaccuracies can have strong effects on nonlinear control systems. Therefore, any practical design must address them explicitly. A simple and effective approach to deal with model uncertainty is sliding mode control methodology.

For the class of systems to which it is applied, sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision. Sliding control has been successfully applied to robot manipulators (Efe *et al*., 2000), underwater vehicles, high performance electric motors (Cupertino *et al*., 2000), power systems and power electronics (Biel *et al*., 2004). In this study we apply the robust sliding mode controller to the magnetic suspension system to study the effect of the mass and electric resistance variations and other disturbances on the output of the system.

This approach directly addresses the nonlinear and uncertain nature of the magnetic suspension dynamics and is capable of achieving not only robust stability but also robust performance against parameter changes and disturbances.

SYSTEM MODELING

To synthesize a feedback control system, a relatively precise model of the plant is required. It is known that a mathematical model cannot fully express the behavior of the real physical plant. An ideal mathematical model has various uncertainties such as parameter identification errors, unmodeled dynamics and neglected nonlinearities. The controller is required to have robust stability and performance in presence of uncertainties in the model and also due to bounded disturbances.

The mechanical dynamic of the suspension system (Fig. 1) is given by (Sinha, 1987):

$$mg+fd \,!\, fem(I, x) = m\frac{d^2x}{dt^2} \tag{1}$$

where m is the suspended object mass, *fd* is disturbance force, $g = 9.81m/s^2$ is the gravity acceleration and *fem(i,x)* is the electromagnetic force acted on suspended object and is equal to:

$$fem(i,x) = \frac{C}{2}\left(\frac{i}{x}\right)^2 \tag{2}$$

In the above equation, C is a constant and is equal to:

$$C = \frac{1}{2}N^2\mu_0 A \tag{3}$$

where N is the number of turns of the magnetic winding, $:_0$ is the vacuum permeability and A is the pole face area. Electrical dynamic of the system governed by the following equation:

$$v(t) = Ri(t) + i(t)\frac{dL(x)}{dt} + L(x)\frac{di(t)}{dt} \tag{4}$$

where, R is the total resistance of the electric circuit. Supposing the permeability of the iron to be infinity (: >> 1) and ignoring the linkage flux, the inductance of the system, L will be:

$$L = \frac{C}{x} \tag{5}$$

Replacing (5) in (4) and doing some manipulations we can get:

$$u(t) = Ri(t) - \frac{C}{x^2}i(t)\frac{dx}{dt} + \frac{C}{x}\frac{di(t)}{dt} \tag{6}$$

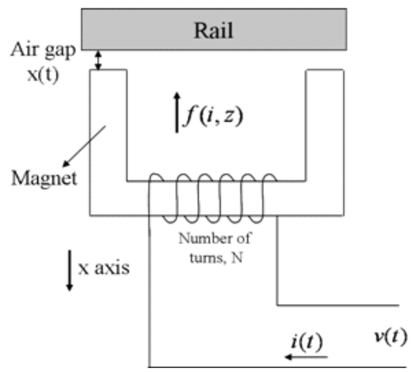

Fig. 1: The EMS system configuration

where, u(t) is the control input voltage. Choosing the state variables as: x_1 = x, vertical air gap, $x_2 = \dot{x}$, relative velocity and x_3 = i, the coil current, the state equations of the magnetic suspension system will derived as follow:

$$\dot{x}_1 = x_2 \tag{7}$$

$$\dot{x}_2 = -\frac{C}{2m}\left(\frac{x_3}{x_1}\right)^2 + g + \frac{1}{m}fd \tag{8}$$

$$\dot{x}_3 = \frac{x_2x_3}{x_1} - \frac{R}{C}x_1x_3 + \frac{x_1}{C}u(t) \tag{9}$$

It is obvious from the above equations that if $|fd| > mg$ the system is uncontrollable; this is because the electromagnetic can only exert attractive force. This is one of the drawbacks of the suspension systems. It should be mentioned that the state equations of the EMS are derived under the unsaturated magnetic materials conditions.

Control system design: Consider the magnetic suspension system which is derived from (7), (8) and (9) as:

$$x_1^{(3)} = f(x) + g(x).u \tag{10}$$

where *fd*, the disturbance force supposed to be zero. *f* (x) and g(x) are functions of state variables and are equal to:

$$f(x) = \frac{R}{m}\frac{x_3^2}{x_1} \tag{11}$$

$$g(x) = -\frac{1}{m}\frac{x_3}{x_1} \tag{12}$$

The parameters R and m can change due to the system operating conditions. The resistance of the electrical part of the system is a function of temperature and so change with operation of the system and ambient temperature. When an EMS system is used in practical applications, e.g. maglev transportation, there are various factors to change mass, such as taking passengers or loading freight, etc. Furthermore, there are many causes to act as disturbances into the system such as wind, irregularities of the rail, etc. Therefore, to derive sliding control which can be used in practical applications, we must design a controller to stabilize not only the nominal system but also the perturbed system. To design the controller we should determine the bounds of the system uncertainties. Regarding the practical situations, it is supposed the variation of the system resistance to be as follow:

$$0.8\hat{R} \le R \le 1.2\hat{R} \tag{13}$$

and the mass of the system change in the following bound:

$$0.5\hat{m} \le m \le 1.5\hat{m} \tag{14}$$

In the above equations $\hat{R}\, and\, \hat{m}$ are nominal resistance and mass of the system, respectively.

According to variation of the system parameters, the dynamics of f(x) is not exactly known, but estimated as $\hat{f}(x)$. The estimation error on f(x) can be bounded as:

$$\left|\hat{f} - f\right| \le F(x,\dot{x},\ddot{x}) \tag{15}$$

According to (13) and (14), we can get the following equations for $\hat{f}(x)$ and *F*:

$$\hat{f}(x) = 0.5\frac{x_3^2}{x_1} \tag{16}$$

$$F = 0.7\frac{x_3^2}{x_1} \tag{17}$$

To satisfy the sliding mode control with uncertainty on its dynamic the control law will be (Slotine and Li, 1991):

$$u = \hat{g}^{-1}[\hat{u} - k.\mathrm{sgn}(s)] \tag{18}$$

where $\hat{g}, \hat{u}$ and the gain k are given by the following equations:

$$\hat{g} = \left(g_{\max}g_{\min}\right)1/2 \tag{19}$$

Table 1: The nominal parameters of the system

Description	Value
Nominal air gap, x_0 (mm)	5
Nominal mass, $\hat{m}$ (Kg)	10
Number of turns, N	600
Pole face area, A (mm2)	900
Nominal resistance, (**S**)	5

$$\hat{u} = -\hat{f} + \ddot{x}d_1 - 2\lambda\ddot{\tilde{x}}(t) - \lambda^2\dot{\tilde{x}}(t) \tag{20}$$

$$k \ge \beta(F + \eta) + \left|\beta - 1\right|\hat{u} \tag{21}$$

In the above equations, x_d (t) is desired value of the x(t),**8** is a positive constant, $\tilde{x}(t)$ is the tracking error in the variable x, **0** is positive constant and **$** is the gain margin of the design and is equal to:

$ = $(g_{max}/g_{min})^{1/2}$ (22)

To eliminate the chattering phenomena, we can use continuous approximation of the discontinuous sliding mode controller (Khalil, 1996). To do this the sign function in (18) is replaced by saturation function and we can get the final control law as:

$$u = \hat{g}^{-1}[\hat{u} - k.sat(s/\phi)] \tag{23}$$

Where, **N** is a positive constant and determine the sliding bound and steady state error of the output. This control law prohibits the excitation of unmodelled high frequency dynamics and results smooth switching control.

RESULTS AND DISCUSSION

To confirm the robustness of the designed controller, the magnetic suspension system and the controller have been simulated in simulink/matlab environment. The nominal parameters of the system are presented in Table 1. The control input voltage is limited within 0 volt to 50 volt. Figure 2 illustrates the transient response of the system with changing the reference air gap from 5 mm to 8 mm. To make the output follow its reference, the control signal decreases immediately. With increasing the output, the control signal increases slightly to compensate the variations. When the output reaches the reference value, the control signal takes the constant value of 39 volts. It is clear from the figure that there is no chattering phenomenon in the control signal. The chattering problem is solved by selecting the appropriate value for **N** in (23). In Fig. 3 we change the reference value of the air gap

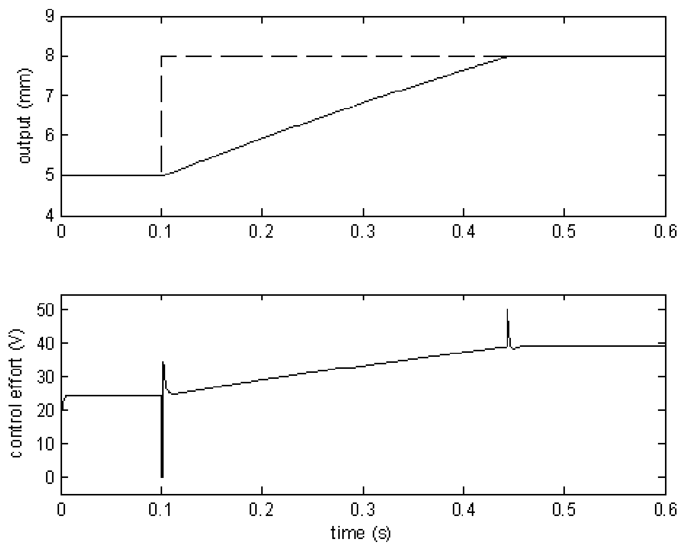

Fig. 2: Transient response of the system to air gap reference change

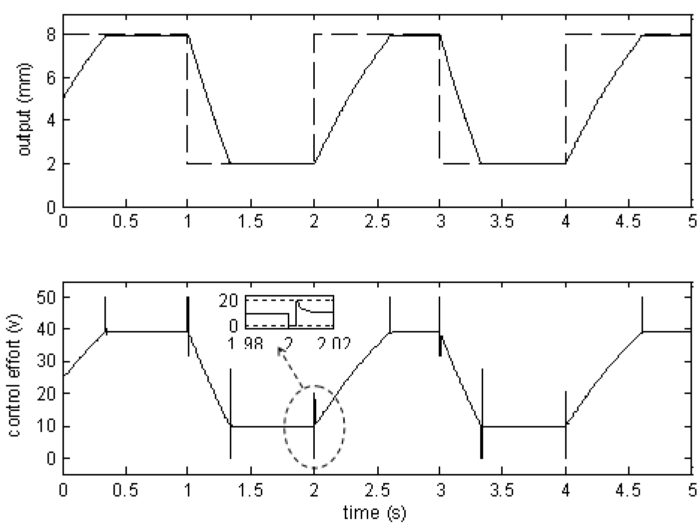

Fig. 3: Response to the pulse form air gap reference

between 2 mm and 8 mm (the nominal value of the air gap is 5 mm). As it can be seen from this figure, the output tracks smoothly the reference signal without any oscillations. In Fig. 3 the control signal is enlarged around 2 s. It is observed that the variation of the control signal is not very sharp and it can be implemented in practice. Also it is not seen any chattering phenomenon in the control effort signal.

Mass change: One of the main sources of perturbation in EMS systems, especially in transportation trains is changing the mass of the system. In this sub-section we investigate the effect of the mass variations of the system on its performance. Figure 4 shows the effect of the changing the mass of the system on output and control signal. In this simulation the mass of the system increased by 50%, i.e., from 10 Kg (nominal value) to 15 Kg. As it can be seen from the figure, this change in the mass of the system has negligible effect on the output. The suspension system falls 0.02 mm but maintain its stability after increasing the mass. In Fig. 5 we change the mass from 10 Kg to 5 Kg. With decreasing the mass, the control voltage decreases and cause the system move upward slightly and finely settle at nominal air gap.

The Fig. 4 and 5 confirm the robustness of the controller against variations of the system mass.

Resistance change: To study the effect of the resistance variations on the performance of the designed controller, we change the electrical resistance of the system in the

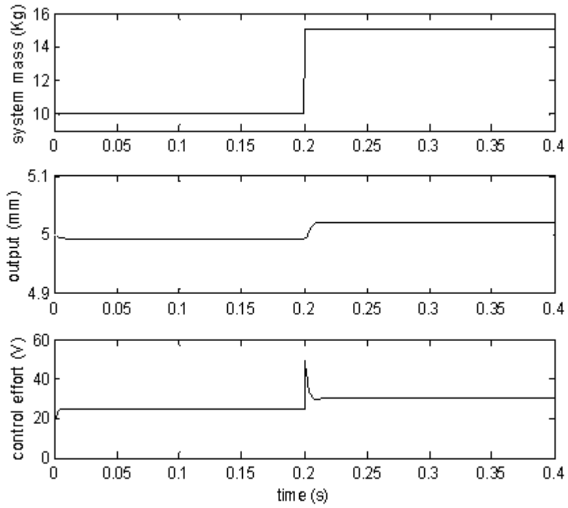

Fig. 4: Response to mass change of the system (increasing of the mass)

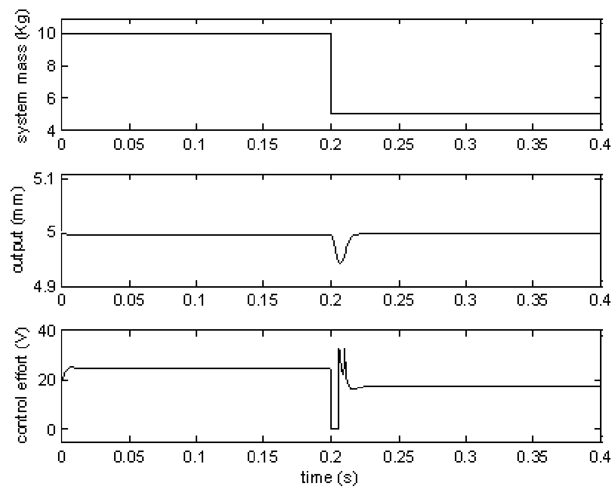

Fig. 5: Response to mass change of the system (decreasing of the mass)

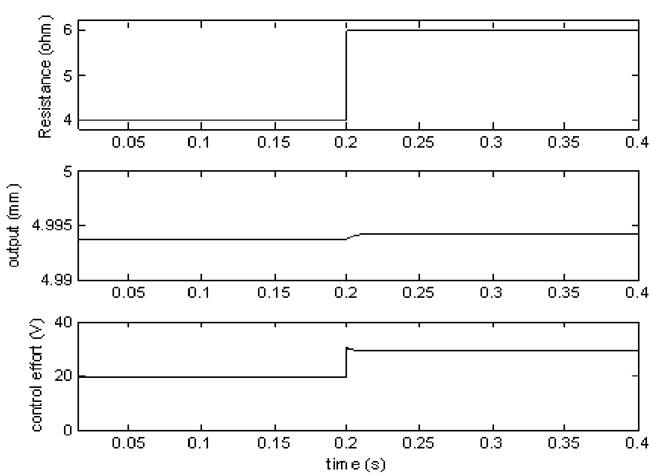

Fig. 6: Response to resistance change of the system (step change)

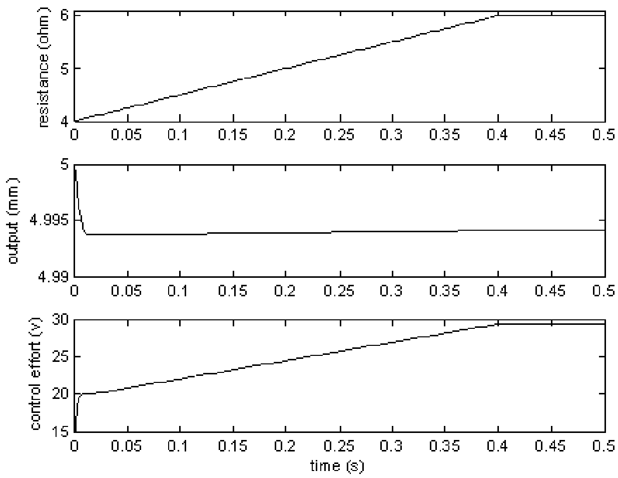

Fig. 7: Response to resistance change of the system (ramp change)

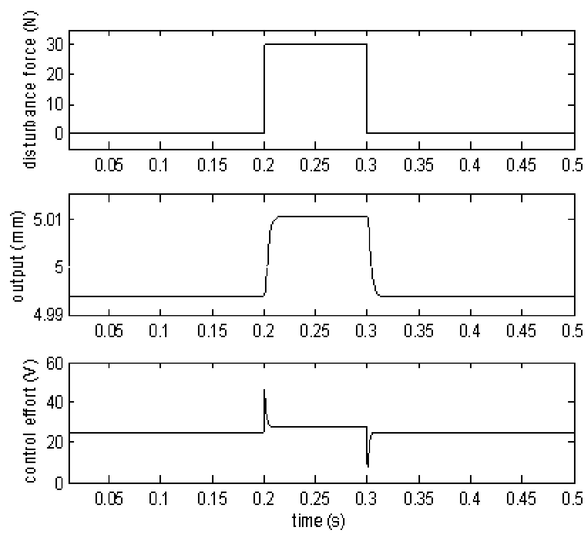

Fig. 8: Response to the disturbance force input

limited bounds discussed in the section III. In Fig. 6 the step change applied to resistance and in the Fig. 7 the change of the resistance is modeled as a ramp signal. In both cases variation of the system resistance has a negligible effect on the output of the system; but the control signal is changed about 10 volts to compensate the resistance variations. Both of the figures show the robustness of the controller against electric resistance.

Disturbance rejection: To conform the robustness of the controller against the disturbance force input, f_d in this study we apply an impulse as a disturbance force in time 0.2 sec with duration of 0.1 sec and amplitude of 30 N. The results are illustrated in the Fig. 8. This figure shows that despite of the entrance of high value of disturbance force to the system, the controller maintains its stability and shows good performance. The output of the system change only 0.017 mm during the disturbance and return to its initial value after removing the disturbance.

CONCLUSION

A robust sliding mode controller is designed to control an electromagnetic suspension system. This control is robust against parameter uncertainty and external disturbances. In EMS the variation of the system mass (for example in maglev trains) and resistance deteriorate the performance of the system. Matlab/Simulink based simulations are carried out to investigate the performance and robustness of the controller. The results of the simulations show that the bounded mass variations have a negligible effect on the system dynamic. Also the resistance change effects have been studied in this study. Although the resistance variation, in comparison with the mass, has not a noticeable effect on the output, it changes the control effort signal to compensate the variation of the resistance. Besides with appropriate chosen of control law, the chattering phenomenon has been cancelled from the control signal.

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