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Optimizing the Natural Frequencies of Beams via Notch Stamping

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Abstract: Natural frequency optimization is important to avoid the coincidence of excitation frequency and natural frequency which causes resonance phenomenon. In this study, the natural frequencies of a beam, with different boundary conditions, are enhanced by stamping V-notches on its surface. These notches alter the local stiffness in the beam while keeping the mass the same. This method is cost-effective in comparison with other Structural Dynamics Modification methods (SDM) because it is a one-step manufacturing method and because it enhances the dynamic behavior of beam structures without additional weight or additional joints. The natural frequencies of notched beam are calculated by finite element method. In particular, ANSYS package is used in building the notched beam models for modal analysis. The effect of notch location and size on the beam fundamental frequency is investigated. The simulation results indicated that creating notches on free-free beam decreases its fundamental frequency, while creating notches on clamped beam may increase its fundamental frequency. The optimal designs of notched beams are presented. The proposed method couples a finite element method for the modal analysis with an optimization technique based on Genetic Algorithm (GA). Three examples are presented to show the optimal design of free-free and clamped notched beams. The optimization results show that V-notch stamping technique is an effective technique to optimize the natural frequencies.

Key words: Finite element method, genetic algorithms, modal analysis, natural frequency, optimization, V-notch

INTRODUCTION

Beams are widely used in engineering applications such as bridge construction and aerospace and automobile industries. Reduction of vibration and noise of such structures is a fundamental requirement in engineering design. The vibration and the sound radiation can be reduced in the design process by different approaches, such as; using the nodalizing approach (i.e., by locating the excitation forces at vibration nodes or by moving the nodes to force locations), shifting the natural frequencies away from the excitation force frequencies to avoid resonance phenomenon, etc. In this research, we are interested in shifting and optimizing the natural frequencies of thin-beam structures.

In most low-frequency vibration problems, excessive response resulted when the excitation frequency is near the fundamental frequency, so maximizing or minimizing the structure fundamental frequency is important to solve such problems. In designing most space vehicles, it is important to shift lower frequencies outside a certain frequency band to avoid coupling with the control system (Grandhi, 1993). Also, the vibration response can be reduced by achieving certain spacing between two adjacent natural frequencies, i.e., when the excitation frequency lies between two natural frequencies, the suitable design is to maximize the distance between the excitation frequency and the nearest higher and the nearest lower frequency (Bendsøe and Olhof, 1985).

The natural frequencies of beam structures can be modified by different methods such as changing the boundary conditions and adding internal supports, adding auxiliary structures, such as masses and springs and changing the structural geometry. The effect of changing the boundary conditions of beams on their natural frequencies are well known and may be found in many books (Meirovitch, 1967). One of the earlier studies on adding internal supports to a beam in order to optimize its natural frequencies is the study by Szelag and Mroz (1979), in which the support positions and stiffnesses were changed. A good example of designing a beam with an internal support to maximize its natural frequency has been provided by Wang *et al*. (2006). The authors investigated the minimum stiffness of an intermediate point support that raises the beam natural frequency to its upper limit before mode switching. Adding concentrated masses to modify the natural frequencies of a cantilever beam was studied by Wu and Lin (1990). They found that

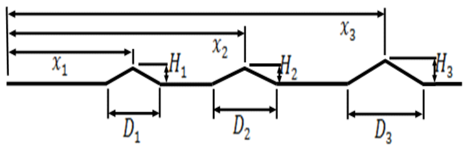


Fig. 1: Schematic representation of notched beam

the fundamental frequency of a cantilever beam which is carrying a single mass increased when the mass was moved from the free end to the fixed end. An example of using distributed masses to modify the natural frequencies of beam structure is a study of Wang and Cheng (2005). The authors used structural patches to shift the natural frequencies of a beam to designated values. They shifted the natural frequencies by tuning the patch thickness and the patch location on the host beam. Modifying the natural frequencies of beam structures by changing their geometry (cross section) focused mainly in designing tapered beams with different profiles and in designing stepped beams. A good example of designing tapered beams has been provided by Ece *et al*. (2007). The authors studied the natural frequencies of a tapered beam in which its width was varied exponentially along its length. They found that this design increased the natural frequencies of clamped beams. Another example of this group is a study by Jaworski and Dowell (2008) which presented an investigation of the free vibration of a stepped cantilevered beam. They determined the first three beam natural frequencies by using the Rayleigh-Ritz method, the component modal analysis and the finite element method. A more recent method is changing the natural frequencies of a beam using dimpling technique. Cheng *et al*. (2008) studied shifting the natural frequencies of a beam to designated values by forming a series of cylindrical dimples on its surface.

The optimal design of beams with respect to natural frequencies can be found in literature. Karihaloo and Niordson (1973) maximized the fundamental frequency of a cantilever beam by tapering its cross section while keeping its mass constant. Gupta and Murthy (1978) investigated the optimal design of uniform non-homogeneous beams. They varied the modulus of elasticity distribution through the beams, assuming constant density, to maximize its fundamental frequency. Olhoff and Parbery (1984) maximized the gap between two adjacent natural frequencies by using the beam cross-sectional area function as a design variable.

In this study, the natural frequencies of different types of beams will be optimized by modifying its geometry. In particular, the natural frequencies are altered by creating a V-notch or a series of V-notches on beam surface. The main advantage of this technique over other structural dynamic modification techniques is that the dynamic modification can be achieved without any weight increase, without additional joints and with only a one-step manufacturing process. The presented design strategy is as follows. First, finite element modal analysis of the notched beam is performed. In particular, ANSYS® is used in modeling and finite element analysis of the modified beam. Second, an optimizer based on the method of Genetic Algorithm (GA) is used to optimally locate and size the notches on the beam.

NOTCHED BEAM MODELING

The notched beam is modeled as shown in Fig. 1, where *x* represents the distance between the notch center and the left beam boundary, represents the notch height and *D* represents the notch width. The v-notch extends through the width of the beam. The mass of the beam is held constant before and after creating the notch; thus the beam is thinner at notch location than the flat parts of the beam. Assuming the notch has a uniform thickness; its thickness can be found using:

$$h_n = \frac{1}{\sqrt{4\left(\frac{H}{D}\right)^2 + 1}} h \tag{1}$$

where, h and h_n represent the original thickness of the beam and the notch thickness, respectively. The notched beam is modeled by ANSYS Parametric Design Language (APDL). In this scripting language the model can be built in terms of parameters, so the locations and geometries of the notches can be defined as variables which can be modified systematically to enable the optimization process to take place. A beam element, with two nodes and three degrees of freedom at each node, is used for meshing the solid model. The degrees of freedom of each node are two translations in the *x*- and *y*-directions and one rotation about the *z*-axis.

Influences of notches on natural frequencies of beams: While the notch does not change the mass of the beam, it does affect the stiffness of the beam. To design the notched beam, three independent geometrical parameters, for each notch, need to be determined; x, D and H. Consider a beam with length L = 0.3 m, width b = 0.025 m, thickness h = 0.00116 m, modulus of elasticity E = 190 GPa and density ρ = 7700 kg/m^3. Two sets of boundary conditions will be taken on consideration during this study; free-free and clamped (fixed-fixed) boundary conditions. In the fixed boundary condition, the motion of the end is fully constrained, while in the free boundary condition, the end can translate in both *x*- and *y*-directions and can rotate around *z*-axis. In this study, to study the effect of notch position and size on the beam natural frequencies, the notch width D is held constant and set equal to a tenth of the beam length and the notch position and height are changed. The notch location is changed all the way from the left end to the right one, i.e., from x/L≈

to x/L ≈ 1. The notch height to width ratio varies from 0 to 0.5. The results for the first three natural frequencies are shown in Fig 2 and 3 where $f_{1,WN}$, $f_{2,WN}$ and $f_{3,WN}$ are the fundamental, the second and the third natural frequencies of the beam respectively before notch stamping.

The changes of the first three natural frequencies of a free-free beam by varying the notch position and height are shown in Fig. 2. The natural frequencies of the free-free beam can be decreased by forming a single notch on it. In general, increasing the notch height decreases the natural frequencies. For the notch with high-height to width ratio, the natural frequencies are very sensitive to the change of its location, i.e., the first natural frequency is minimum when the notch lies in the middle of the beam (x/L = 50%), i.e., near the higher modal strain area. Similarly, the second and third natural frequencies are minima when the notch lies near the higher modal strain areas.

As shown in Fig. 3, the natural frequencies of the clamped beam may be increased or decreased by creating a single notch on it. In general, the maximum increase on the beam natural frequencies occurs when the notch height to width ratio is moderate (approximately between 0.15 and 0.2). While the maximum decrease on the beam natural frequencies occurs with notches have high-height to width ratio (approximately greater than 0.4). It is also noted that creating the notches near the boundary conditions and at the higher modal strain areas has a significant effect on clamped beam natural frequencies. This is different from the results which are obtained by creating the notches on the surface of free-free beam. To emphasize this difference, the effect of creating the notch on the surface of the beam should be understood. The effect of creating a notch on beam natural frequencies can be summarized by two points:

- The thinning effect
- The V-shape effect

Since the notch is thinner than the other parts of the beam, the bending stiffness of the notched beam is smaller than that of the original beam, as a result of this thinning the natural frequencies decrease. On the other hand, the V-shape increases the bending stiffness of the beam, i.e., creating a V-shape in a beam couples its transverse and axial vibrations. If the boundary conditions prevent the beam from extension axially during lateral deflection, then the bending stiffness will increase. In free-free beam considered before, the boundary conditions allow for the longitudinal motion of the beam. Thus, the V-shape of the notch does not increase the bending stiffness of these beams. However, the thinning effect reduces the bending stiffness and as a result reduces the natural frequencies. On the other hand, the boundary conditions in the clamped beams prevent any longitudinal motion. Thus, the V-shape effect increases the bending stiffness of the beam. Therefore, two factors influence the natural frequencies of clamped beam; the thinning effect which reduces the beam natural frequencies and the V-shape effect which increases the beam natural frequencies.

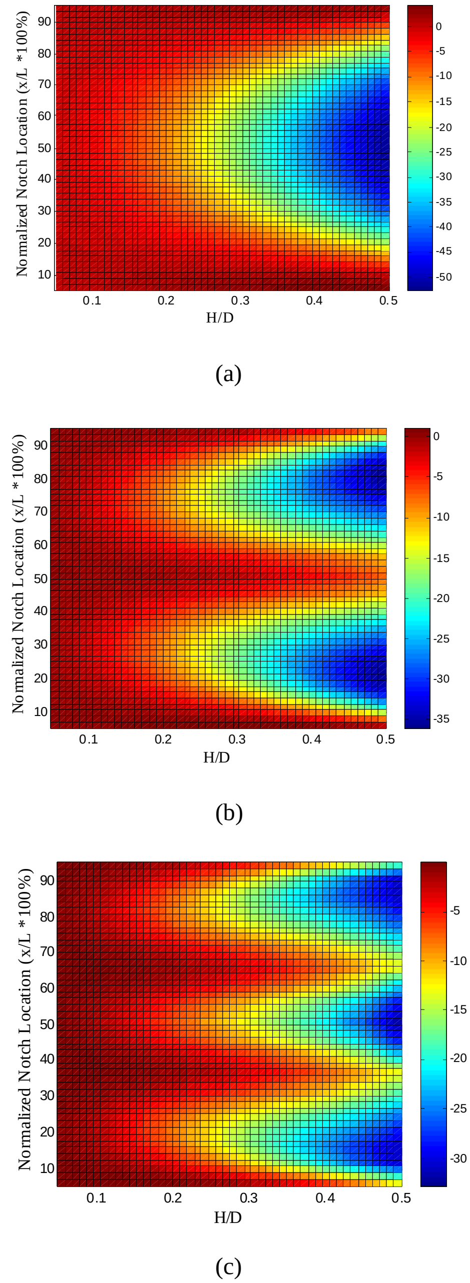


Fig. 2: Percentage change of natural frequencies of free-free beam by varying the notch location: a) fundamental frequency ($f_{1,WN}$ = 65.8Hz), b) second natural frequency ($f_{2,WN}$ = 181.4Hz) and third natural frequency ($f_{3,WN}$ = 355.6Hz)

OPTIMIZATION

As shown in the previous section, the beam natural frequencies are non-linear functions of notch position and size. Also, they have multilocal optimal points. As a result, there is a need for a stochastic search approach, such as the Genetic Algorithm (GA), to determine the optimal design. Genetic Algorithm (GA) is a stochastic and heuristic optimization method that is based on the concept of biological evolution developed by Holland (1975). It is started with a set of candidate points (called population) which cover the entire search space. Then the objective function (fitness function) is evaluated for each point. The points with relatively good objective functions are used to generate a new set of candidate points. The new candidate points (population) are generated by three methods: reproduction, crossover and mutation. In the reproduction method, the candidate point in the current generation with the best objective function is automatically passed to the next generation. The crossover method creates a new point by combining the vectors of two candidate points in the current generation. The mutation method creates a new point by randomly changing some components of a candidate point in the current generation. The algorithm continues iteration (generation) until reaching the stopping criteria, such as finding the optimum, reaching the maximum number of generation, constraints tolerance, etc. In comparison with traditional optimization methods, GA has the following advantages:

- It studies without any need to differentiate the objective function or constraints, which is useful when the objective function or constraints are non-differentiable and when it is difficult to write the objective function in a closed form
- It studies with a population of points at each iteration instead of one point, which provides the ability to find multiple optimal solutions in the research space. Searching in many different areas of the search space increases the probability of finding the global optimum

In the optimization examples, four general constraints are considered. The first constraint is the distances between the notches have to be large enough to prevent notches from overlapping onto each other. The second constraint is that the notches must retain a certain minimum distance from the beam ends. The third constraint is that each notch height to width ratio has to be less than a certain value in order to ensure it meets manufacturing limitations. Finally, the width of the notch has to be bounded. In this study, a Genetic Algorithm (GA) is used to optimize the natural frequencies. During optimization, ANSYS® is integrated into MATLAB® and executed via batch-mode to determine the natural frequencies.

OPTIMIZATION EXAMPLES

To show the efficiency of using notches in optimizing the natural frequencies of beam-structure, three examples are presented. The first example shows the case of minimizing the fundamental natural frequency of a free-free beam. In the second example, the fundamental frequency of a clamped beam is maximizing, while in the third example the gap between the fourth and fifth natural frequencies of a clamped beam is maximized.

Minimizing the fundamental frequency of a free-free beam: As shown in the parametric study, creating a single notch on a free-free beam decreases its fundamental frequency. In this example, the minimization is applied by using three notches. The GA is used to find the positions x_i, diameters D_i and height to width ratios $(H/D)_i$ of notches which give the minimum fundamental frequency. The population size in each generation is 60 and the stopping criterion is chosen to be the maximum number of generations not to exceed 100 generations or achieving a tolerance of less than 10^{-3} for the objective function.

The beam under consideration is a free-free beam of 0.3 m length, 0.025 m width and 0.00116 m thickness. It is made of steel with modulus of elasticity E = 190 GPa and density of ρ = 7700 kg/m^3. The fundamental frequency of this beam without any notches is 65.8 Hz. In general, the optimization problem can be stated as:

Minimize f_1,

Subject to: $x_{i+1} - x_i - \frac{D_i}{2} - \frac{D_{i+1}}{2} c, >$ i = 1, 2,..., n-1 (2)

Design variables $x_{lb} \le x_i \le x_{ub}$,

$D_{lb} \le D_i \le D_{ub}$,

$$\left(\frac{H}{D}\right)_{lb} \le \left(\frac{H}{D}\right)_i \le \left(\frac{H}{D}\right)_{ub}, \quad i = 1, 2, ..., n$$

where, c is the minimum tolerance between any two notches or between the notch and beam edge, lb and ub are the lower and upper bounds of design variables respectively, i is notch number and *n* is the total number of notches. In this example, c = 0.003 m, x_{lb} = 0 m, x_{ub} = 0.3 m, D_{lb} = 0.01 m, D_{ub} = 0.03 m, $\left(\frac{H}{D}\right)_{lb}$ = 0.05 and $\left(\frac{H}{D}\right)_{ub}$ = 0.3. The optimal design variables which minimize the fundamental frequency are shown in Table 1. By creating only three optimal notches, the fundamental frequency of the free-free beam can be

Table 1: The optimum design variables of notched free-free beam which minimize the fundamental frequency

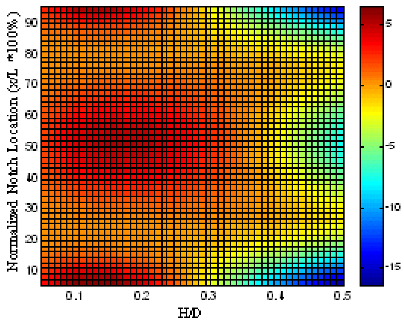
f_1(Hz)	x_1(m)	x_2(m)	x_3(m)	D_1(m)	D_2(m)	D_3(m)	$(H/D)_1$	$(H/D)_2$	$(H/D)_2$
38.3	0.1169	0.1502	0.1832	0.0300	0.0299	0.0299	0.2989	0.3000	0.2993

Table 2: The optimum design variables of notched clamped beam which maximize the fundamental frequency

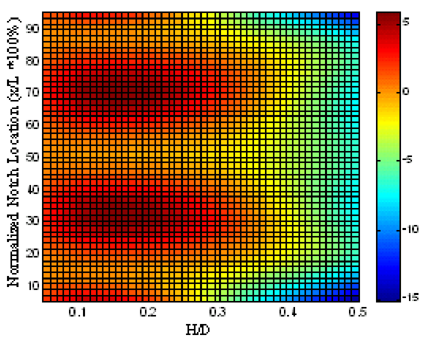
f_1(Hz)	x_1(m)	x_2(m)	x_3(m)	D_1(m)	D_2(m)	D_3(m)	$(H/D)_1$	$(H/D)_2$	$(H/D)_2$
78.5	0.0180	0.0511	0.2828	0.0296	0.0300	0.0300	0.1117	0.0528	0.1019

Table 3: The optimum design variables of notched clamped beam which maximize the gap between the fourth and fifth natural frequencies

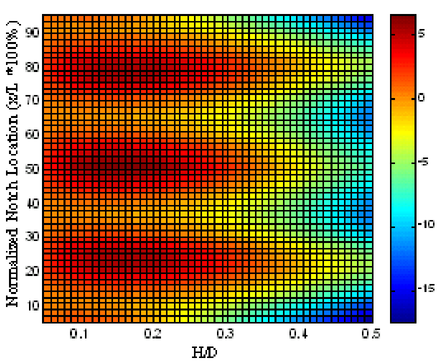
f_4(Hz)	f_5(Hz)	x_1(m)	x_2(m)	x_3(m)	D_1(m)	D_2(m)	D_3(m)	$(H/D)_1$	$(H/D)_2$	$(H/D)_2$
517.3	1068.2	0.0467	0.1500	0.2535	0.0300	0.0300	0.0299	0.1960	0.1701	0.1959



(a)



(b)



(c)

Fig. 3: Percentage change of natural frequencies of clamped beam by varying the notch location: a) fundamental frequency ($f_{1,WN}$ = 65.8Hz), b) second natural frequency ($f_{2,WN}$ = 181.4Hz) and c) third natural frequency (Hz)

Fig. 4: The optimum design of notched free-free beam which minimize the fundamental frequency

Fig. 5: The optimum design of notched clamped beam which maximize the fundamental frequency

Fig. 6: The optimum design of notched clamped beam which maximize the gap between the fourth and fifth natural frequencies

decreased from 65.8 to 38.3 Hz, which is reduced by 41.8% compared to the funda mental frequency of the beam without notches. As shown in Fig. 4, the optimal positions of the notches are near the middle of the free-free beam and the optimal widths and heights are near the upper bounds of the feasible region.

Maximizing the fundamental frequency of a clamped beam: In this example, the goal is to maximize the fundamental frequency of a clamped beam by using a series of notches. The maximization is applied by using three notches. The GA is used to find the positions x_i, diameters D_i and height to width ratios $(H/D)_i$ of notches which give the maximum fundamental frequency. Similar to the previous example, the population size in each generation. The stopping criterion is chosen to be the maximum number of generations not to exceed 100 generations or achieving a tolerance of less than 10^{-3} for the objective function.

The beam has the same dimensions and material properties as in the previous example. The optimization problem is the same as the problem that illustrated in Eq. (2), but with maximizing f_1 instead of minimizing f_1. The optimal design variables that maximize the fundamental frequency of a notched clamped beam are listed in Table 2. By creating only three optimal notches, the fundamental frequency of the clamped beam can be increased from 65.8 to 78.5 Hz, which is 19.3 % greater than the fundamental frequency of the beam without notches. The optimal notched clamped beam is shown in Fig. 5. It can be seen from Fig. 5 that the optimal positions of the notches are near the ends of the clamped beam and the optimal widths are near the upper bound of the feasible region.

Maximizing the gap between two adjacent frequencies of a clamped beam: When the beam is subjected to excitation frequencies within a range defined by an upperand a lower frequency bound, the suitable design of the beam may be obtained by maximizing the gap between adjacent beam natural frequencies close to the excitation range.

In this example, the distance between the fourth and fifth natural frequencies of a clamped beam is maximized by creating three notches on it. The beam has the same dimensions and material properties as in the previous two examples. The fourth and fifth natural frequencies of this beam before creating the notch are 587.8 and 878.1 Hz respectively. So the difference between the frequencies is 290.3 Hz. The optimization problem is the same as the problem which illustrated in Eq. (2), but with maximizing (f_5-f_4) instead of maximizing f_1.By using GA, the optimization problem is solved and the optimal design variables which give the maximum gap between the fourth and fifth natural frequencies are summarized in Table 3. The optimal clamped beam which has the maximum gap between the fourth and fifth natural frequencies is shown in Fig. 6. As shown in Table 3, by creating only three notches, the gap between the fourth and fifth natural frequencies of the clamped beam can be increased from 290.4 to 550.9 Hz, which is 89.7 % greater than the gap between the frequencies without notches.

CONCLUSION

A method for optimizing the natural frequencies of beam structures using V-notches was demonstrated. The natural frequencies were determined numerically using the finite element method. The parametric study shows the efficiency of V-notch stamping technique to reduce the fundamental frequency of free-free cantilever beams, while it shows the efficiency of notching technique to increase the fundamental frequency of clamped beams.

The GA can be used to determine the best design of notched beam. The finite element method was coupled with GA to minimize the fundamental frequency of free-free beam, to maximize the fundamental frequency of clamped beam and to maximize the gap between two adjacent natural frequencies of clamped beam.

It should be noted that creating V-notches in a beam is subject to manufacturing limitations which must be taken in consideration during design.The main limitation is the material capacity to deform plastically without cracking. This effectively places an upper constraint on the notch height. The methodology presented in this study can take these manufacturing limitations into account. Future work will focus on experimental verification of the results presented in this study as well as an expansion of the techniques presented here to include an optimization of radiated sound from baffled beams.

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