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Hybrid Systems Modeling in Non Standard Queue and Optimization with the Simulation Approach in CNG Stations

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Abstract: Modeling line in non standard way occurs when layout constraints and inappropriate placing customer is limited for taking customer service by the servant. The aim of this study is providing a mixed model for analyzing the system of non-standard line with Considering the limitations of the layout with Using the concepts and principles of queuing theory So that the main parameters of the model for this type of system can be calculated and The basis of queuing systems with non-standard parameters may be considered. In these nonstandard systems, because of special arrangement of servants, there are some delay times for giving services and exit. The use of simulation tools to demonstrate the relatively low efficiency of CNG (Compressed Natural Gas) stations in Iran, To provide an optimum combination of servers (Fuel nozzle) Also more efficient layout for the CNG stations has Studied. Manufacturing firms and service managers can use this model and evaluate and analysis their own system and get a better recognition of their system. One of the most widely used queuing systems in the country are CNG stations, in consideration high investment cost and land value in large cities, so we decided to studied on this area as one of the servicing activities.

Key words: Constraints deployment, fuel stations (CNG), non-standard system, queuing theory

INTRODUCTION

Although waiting in line is very unpleasant, but it is unavoidable fact of our life. People in their daily life face with different lines that leads them to losing time, energy and funds. Reduction of waiting time is often needs additional investment. Therefore, knowing the influence of investment amount on the waiting time is very important. It is expected that the elimination of unfavorable outcomes of in line without knowing the characteristics of this phenomenon is not possible. Queuing theory is to study queuing systems with the mathematical point of view and study Reasonable ways to reduce waiting times and queues forming factor. The art of queuing is that construct a simple model firstly and then by using mathematical analysis the results will be compared with actual results after that add necessary details to the model which will be coordinated actual system with built model (Ivo and Resing, 1999).

In 1940s, queuing models was used for solving different types of machines/equipments problems. Responding to the questions same as how many telephone operators are needed for answering the customers, is counted as such issues (Bhaskar and Lallement, 2010). But nowadays manufacturing and servicing organizations for better making decision of their customers waiting time reduction, should apply queuing theory, which not only define resources needs level for investment, but meet the customer satisfaction as it possible. Although we cannot omit the queue at all but we can reduce the losses of it, this matter is very important for sustaining growth of the companies in the competitive situation. Thus it is very necessary to describe the performance of queuing theory in different environmental condition. (Gupta and Khanna, 2006).

The most important use of queuing models is in production and engineering systems, computer science, transportation, warehousing systems and information processing. Applying queuing theory models is very useful especially for designing such systems based on layout, size of capacity or the controlling method of them. (Donald and Harris, 1985). Nowadays, management is one of the fundamental basis for manage the developed societies. Optimized mixing of facilities and right cooperation between them with considering the limited sources for desired goal achieving is the main criteria for increasing the organizations efficiency in the mentioned areas.

The equipments layout in queuing systems has the key role in the productivity, waiting time and servicing. Meanwhile, some of these systems do not have enough freedom because of location limitation for taking service and also exiting from the system and this matter cause the increasing waiting time in system and servicing. In

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current study, the modeling and analyzing the non-standard systems was studied. CNG fueling stations is one of the most useful service queuing systems which are studied.

Inappropriate location in fueling station, type of petrol pumps arrangement, low space between two rows of fueling pumps, lack of enough space for cars in optimized use of all pumps and at last the type of internal pumps has caused long lines in fueling stations in country. In this situation people have increasing waiting time for receiving the services by long queues. Studying this issue will help to increase the quality of urban fueling station services and will lead to increasing customer satisfaction for using from these centers. So it is necessary to study and pay more attention for measuring and optimizing on this system.

Current research is done base on the preparing right principals for managing the urban societies to increasing and developing the life quality. In next part of this article, some basic issues of queues are reviewed and in third part is about problems, limitation and assumptions definition. In fourth part is about explaining the model in every row and calculating the entrance mode and real servicing and other parameters for each row and for the whole system. At the ends with presenting a case study, the model reviewed and proving the low performance of CNG stations and presenting a solution for increasing the stations productivity with simulation approach.

PRINCIPLES AND MEANINGS OF QUEUING THEORY

Literature review: In 1966, Hermida and his colleague in Ecuador searched about problems causes' related long waiting time in one of the medical clinics. Researchers have focused on reducing patients' dissatisfaction with improving waiting time and proposing a good solution for reduces great waiting time and their dissatisfaction (Hermida and pina, 1996). In 2004, Kinan Wang has suggested a model with considering the risk which a patient to be tolerated, this is a function has entrance rate, servicing capacity and disease acuteness. With considering this matter that passing of time affect on each patient in line, the model was developed. Also when the patients' condition is different, a model of prioritized patient's queue is considered for minimizing the risk. With considering the risk for different patients group, features and affective sizes of prioritized queue is calculated, also the Management problems that may occur due to the priority queue to be resolved (Wang, 2004). Behskar and Lallement (2010) has used from queuing system for evaluating the supply chain. In this study one supply chain is consist 3 levels of queue network and two entrances. An order is suggested for supply chain by two probable variables, one for time and the other for the goods volume that should be delivered in each order.

In this study our aim is, calculating the lowest responding time for delivering the goods through a 3 levels of queue network Bhaskar and Lallement (2010). The average number of the goods that can be delivered in the lowest time is considered as the optimized capacity of queuing network.

Canonaco and his colleagues, has studied for managing the optimized loading and offloading the containers in every spot of the quay with purpose of the highest productivity from the expensive resources such as rail crane with the aim of reducing the waiting time of ship equipment and its coordination with the suitable rate for completing the server. For solving the problem, one model of queue network has been suggested and given its complexity, simulation of discrete-event, was selected the best approach for analyzing. For showing the real limitations, resource allocation policies and schedule of activities, Event Graph (EG) in designing has been used. Canonaco *et al*. (2008) and Maglaras and Mieghem (2005) have searched how the multi production queuing systems should be controlled that service providing time be less than preparation time. For analyzing this problem, they suggested an approach based on an analysis of floating and flexible model which convert the specifications and characteristics of time preparation to certain limitations on the queue length. The main advantage of this approach is that timing possibility of arranging the time and providing the necessary entrance policies and multi product's acceptance for controlling the preparation time will be provided. (Maglaras and Mieghem, 2005).

Wang and Tai (2000) considered that the number of servers in a M/M/3 system in stable mode will change with the change of the length of queue. And the server's ability is different in every service station. Whenever length of queue in the first servers reach to J, the second server will start and when the length of the customers of second server reach to K as that (K>J) the third server will start to work. The aim of this research in addition to identify the parameters such as average number of the waiting customers in stable mode, is consider the cost for determining the optimized length of K, J queue for getting the highest profit.

Before there were some searches on fueling stations. One of them was Shahmoradi's thesis in 1999 that calculate the server (number of pumps) in fueling stations with using queuing theory and its comparing with the current condition to find the best way to improve the fueling stations. The suggested scientific solutions in this research enable to calculate the best number of server in fueling stations in a way that the sum of customers 'cost and unemployment cost of unused servers reach to the lowest level. Considering M/M/C hypothesis, flow of giving service is one of the most important factors in this thesis with taking advantage of queuing theory which techniques has analyzed the mentioned system (Shahmoradi, 1998). In 2005, a research that has been done by Mirzaee (2005) the suitable situation for establishing petrol stations, with using a mathematic program has been evaluated in a way that minimum numbers of petrol pump has covered the maximum within the scope of his studying. Finally in considering the output of mathematic programming model priority of some area for building of fuel stations base on highest level of servers has been chosen (Mirzaee, 2005) in 2010, Abedi and colleague has studied on the fueling system in Iran and modeling according to queuing theory. In this article despite referring to the limitations of these systems, the result and modeling is based on being M/M/C Of this system. We understand this consideration, ignoring the limitations of the system.

In these research two main limitations has been explained completely. In both ways the exact rate of servicing with the systems' limitations has been calculated and the expected model has been shown regarding to balanced functions. With considering the weight factor for each of the cases, the hybrid model is presented (Abedi *et al*., 2009).Teimoury *et al*. (2011) also has been developed a queuing model for non standard queuing systems with special modes for using in fueling station in Iran that such kind of non standard queuing systems are used for assembly and production line.

Evaluating criteria and basis specifications for queuing systems: With measuring the queuing models, we can do optimizing system on both profiting and reduced customers' waiting time with considering cost's functions. We can use these effective sizes in making decisions and system analyzing for increasing the number of server. Thus, evaluating the queuing system performance, we can take advantage of the below criteria Ivo and Resing, 1999):

- Arrival time distribution of the queue system and customer's waiting time in the system. n^{th} customers' waiting time in system is equals the waiting time for receiving service plus servicing time. In other word:

$$T_S^{(n)} = T_q^{(n)} + S_n \tag{1}$$

- Average number of available customers' in the system
- Estimating distribution of service time: that is included the sum of servicing time when the customers are receiving the service and the remained servicing time of other customers in waiting queue. Finally with considering the mentioned parameters and with using the mean waiting time and mean sojourn time we can measure the system's performance. As random variable L(t) shows the number of available customers in *t* time of system's and S_n shows waiting time of *n* customer in system, the number of available customers in a range of time [0, t] in system and mean sojourn time in long term is equals E (L), E(S) (Ivo and Resing, 1999):

$$E(S) = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} S_k \tag{2}$$

$$E(L) = \lim_{t\to\infty} \frac{1}{t}\int_{X=0}^{t} L(x)dx \tag{3}$$

At the end for the real recognition of a queuing system it is needed to specify its fundamental processes. Six basic characteristic of queuing process is:

- Arrival model of customers
- Servers' model
- Order queue
- System capacity
- Number of counter's service
- Number of service process stages

In most cases, these six characteristics are sufficient for determining the queuing system (Cooper, 1980).

Markov chains with continuous time and characteristic of Markov's models: Markov chains normally are used for modeling in many systems like queuing, manufacturing and inventory systems (Ching *et al*., 2008). As we know, for studying and analyzing a queuing system, we should formulate it in Markov chains frame work. Therefore in this sector we have review on Markov chains and Markov models. A random process has a set of series of random variable X (t) which provide that for each $t \in T$ we have a random variable, then we can take the X (t) as the status of a random process in t time. Considering random process with continues time $t \geq 0$ that takes no negative and integer values.

The process of $[N(t), t \geq 0]$ is called Markov chain with continues time. If we have $0 \leq u \leq s$, X(u), I, j for all s, $t \geq 0$ and non negative value (Gupta and Khanna, 2006):

$$P\{X(t+s) = j|X(s) = I, X(u) = x(u), 0 \leq u \leq s\} = P\{X(t+s)|X(s) = i\} \tag{4}$$

In other word, Markov chain with continues time is a random process that has Markov characteristic that its conditional probability of a future state X(t+s) with

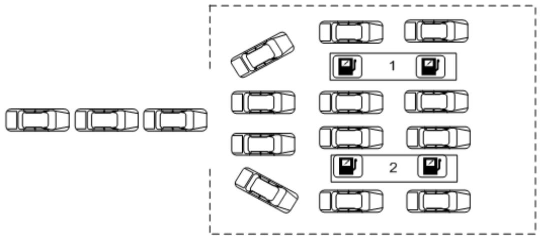

Fig. 1: A view of fueling station model (Abedi *et al*., 2009)

knowing the current situation, namely X(t) and Knowing the past situation, namely X(u) and 0 < = u < s, depends on the present condition and haven't any dependency to the past process (Ross, 2007).

Special features of the exponential distribution cause the simple analyzing of queuing matters. So normally in making random queuing models, it is assumed that intervals of arrival and servicing time follow exponential distribution. As the exponential distribution often a good approximation of reality and the process moves on to Markov-chain bonds (Lipschutz, 1968).

EXPLAINING THE PROBLEM

Queuing model in non standard mode happens due to limitations of the particular layout, the customer for getting the service from servers is faced with restrictions. For analyzing these systems some parameters like waiting time and average length of queuing in such non standard systems, should be calculated. In such non-standard systems, time delay can exist in the service. These models can be seen in fueling stations or assembly and production line, so fueling stations are one of the biggest and the most efficient queuing line in the country and with considering to great amount of investment for settling the station has the special importance that will be analyzed.

Petrol pumping layout in the existing stations in country has been designed, which the cars cannot exit the station when they have finished fueling (except cars that start fueling in the first front pump and also cars which waiting for replacing). Available model of queuing system consists of a main queue that is often outside the space of station (because of small space of station). Cars enter the sub branches after passing sometime in the main queue, after being in the sub branches , these cars enter a smaller queuing system in a way that cannot receive service from other servers therefore, we can consider each sub branches as an in dependent system and entering the cars to the sub branches are like their entering to a smaller queuing system, because if a nozzle in other platform was released, the car that is in another branch, cannot take service. In this case waiting time for cars will be exceeded and we will have no-optimal and non standard queuing system with long queues and high waiting time (Abedi *et al*., 2009), this model is shown in Fig. 1.

Hypothesis and system's limitations: Available parameters, variables and hypothesis are defined as below:

- Customer's arrival is done according to POISSON process with λ
- x_1, x_2, …, x_k in order the service time of the first person (The person in front of each platform) the service time of the second person and … The service time of the k^{th} person that has exponential distribution with μ parameter.
- Selection of branches with equal probability is done
- After choosing the branch, we cannot change it.
- Two servers are serving in series and in certain condition.
- when both servers are free, the customer refer to the first server and when the first server is busy, that customer refer to the second server and if that is busy, he will refer to the next server and this trend will continue to the K^{th} server.
- Every available K customer in each queue (except the cars that receive fueling from the first pump of platform) that is receiving its service, cannot leave the system till all the fronting customers finish their services (because of layout limitation).We call this mode "exit limitation" (first mode).
- In this system if second or third or K^{th} server is busy and first or second or $(k-1)^{th}$ server be free, next customer in queue cannot refer to first or second or $(k-1)^{th}$ server and should wait to finish services of the K^{th} customer. We call this mode "replacement limit" (second mode).

Explaining the real model of non standard systems in fueling stations in every branch: As before mentioned, in this type of system, the waiting car in queue for receiving the service has to wait till k^{th} car leave the system. (Lack of replacement ability in exiting every fronting car from K^{th} car as the result of layout limitation), on the other hand K^{th} car can leave the system when all the fronting customers have finished their servicing. Its mean leaving the fronting cars that are in front of the car with longest servicing time, do not help to the reduction of the car waiting time in queue. This assumption is a reasonable hypothesis, which car's arrival is one by one and their exit in a group at the same time.

In Fig. 2 we can consider this condition equivalent to a system that has a server in every queue and at the same time can provide service for k customer. In other word

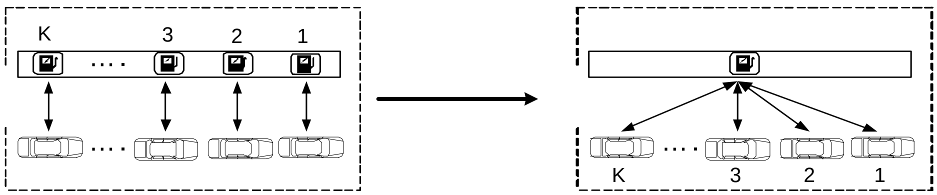

Fig. 2: View of actual system and equivalent system

because of limitations in this system, existing K server that each of them can provide service to one customer is equivalent to the mode that we have one server that can provide service to k customer at the same time.

With the above assumption, the queuing model of these systems in every queue (Branch) is consistent to M/M/1 model with group servicing that is the first type which can serve a maximum capacity of each service can be considered equal to k and servicing time has exponential distribution and customer's entrance is one by one. Service to a group of customers instead of one customer are served simultaneously, also, because server can give service to any number of customers and don't wait till the customers' number reach to K (maximum capacity of the server). This system is the first type mode. The important point is, that due to the limitations mentioned, the servicing mode no longer equal to μ. In continue we will have the calculation of server's real rate in every row.

Calculation of server's rate (μ′) in every branch: Always for analyzing a system, reviewing the arrival rate to the system and servicing rate of each server's is necessary. As was explained, regarding two main limitations that is happened in this system, servicing time in both modes because of layout limitation, is more than the ordinary mode. In other word, if every one of the customers has finished their receiving fuel, they should be waiting up to finishing the receiving fuel of fronting car. In point of view of these waiting customers, they should wait to the maximum servicing time of K customer and not t_{ave} in queuing line, so the servicing time equals to $Max(x_1, x_2,\ldots, x_k)$ and is not the average time. Thus the mean waiting time of queue in every service receiving from a customer perspective equals to:

$$E(Max(x_1, x_2, x_k)),\ x_i \sim exp(\mu),\ i = 1, 2, 3, ...,$$

So in this case we have M/M/1 system with group servicing, that real servicing rate (μ′) for every group servicing equals to:

$$\mu' = \frac{1}{E(Max(x_1, x_2, ..., x_k))} \quad (5)$$

As we know lack of memory is the special characteristic of exponential distribution that makes the simple analyzing of queuing problems. So $E(Max(x_1, x_2, ..., x_k))$ will be calculated by using lack of memory characteristics of exponential distribution, as below (Sheldon Ross 2002):

$$E\left(Max(x_1, x_2, ..., x_k)\right) = \frac{1}{k\mu} + \frac{1}{(k-1)\mu} + \frac{1}{(k-2)\mu} + \ldots + \frac{1}{2\mu} + \frac{1}{\mu} \quad (6)$$

We can express the above relation in this way that the time takes for leaving the station of all cars equal to the sum of time that take the first car finish its service receiving. It equals to $Min(x_1, x_2,\ldots, x_k)$, (it is an exponential random variable with kμ parameter) and extra time that takes next car finish its service receiving equals to $Min(x_1, x_2,\ldots, x_{k-1})$ which is an exponential random variable with (k-1)μ parameter and etc.

Developing system to i branch (row) of K and calculating the entrance flow to every row by Jackson's network approach: We have proved that the studying system has one junction (row), one system M/M/1 with first type of group servicing, servicing flow is μ' and entrance flow is λ, but when we develop this system from one junction to i branches, entrance flow to each branch is an amount like λ′. Its calculation by Jackson's approach network is A discipline within the mathematical theory of probability is a theorem by James R. Jackson offered with little changes.

At first, we review all queuing network. Generally in queuing net, when a customer enters the system for the first time, it is possible to refer to every service station. After receiving service from this station, it can refer to any of the other stations. So, the customers of each station enter from outside directly or come from other stations. A specific mode of queuing net work called Jackson's net work. The assumption of Jackson's queuing network is as below (Proportional changes in the model are listed in parentheses) Sheldon (2002):

- Customers enter the system from outside to the servicing station number i, according to Poisson process with λ_i' flow (i = 1, 2, .., k). (λ_i' = entrance flow to every junction).

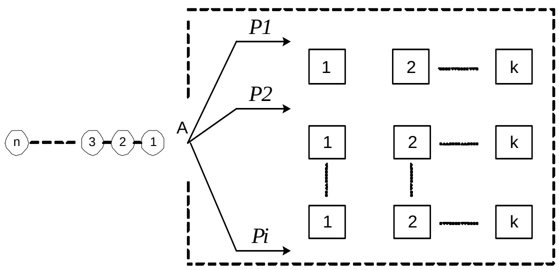

Fig. 3: Schematic view of the entire queuing system with i row and k server

- Service time at station number i assume as an exponential random variable by μ_i parameter and independent of the length of service in the other stations. (In this system $\mu_i = \mu$ due to equality of servicing time of providers).
- Queuing capacity is infinite in all stations.
- A customer who is out of the station i, by p_{ij} probability will refer to station j for receiving the services. The probability of customer's exit from station i after receiving the service, is shown by p_{io}. (In our system $p_{ij} = 0$ because the customer receive its service only from one station and after that exit the system, then we will have $p_{io} = 1$).

Figure 3 shows, after customer arrival to the start point of main queue (point A) unlike the M/M/C model, has not the choice option from one of the C servers, in this style, customer after choosing and entering to sub-queue cannot enter to the other sub queues. If we show the choice probability of branch i from customer with p_i, real entrance flow of station l to station k in branch i with Jackson's net work approach is calculating as below (λ = general entrance flow to system):

$$\lambda_i' = \lambda p_i \tag{7}$$

Considering that the probability of branch i selection in long term reach to equal amount ($P_1 = P_2 = ... = P_i$) so we can convert customers entrance rate of the whole system λ to the amended entrance rate λ'. According to the below relation, the results of Jackson combined model and M/M/1 with the first type of group servicing generalize to the whole system. Amended entrance flow of every branch can be expressed as following:

$$\lambda_1' = \lambda_2' = \cdots = \lambda_i' = \lambda' = \frac{\lambda}{i} \tag{8}$$

FUELING STATION'S SYSTEMS MODELING

After calculation the entrance flow and real service providing, other effective measures will be calculated. So first of all we do the calculation in every branch same as previous part then develop it to the whole system.

Formulating and calculating the effective measures in each row independently: We can consider the stations as queuing system that contains i independent queuing system (i platform's row) that has k server (k nozzle), therefore considering that the performance of each row is independent and also similar to the other rows and everyone has M/M/1 model by first type of group servicing with the maximum capacity k in every servicing, firstly we explain the calculating method of effective measures in one row by Markov chain meanings and then get the final model with developing the suggested model in a queuing system which has i rows by k server. So in this part, system's mode is equivalent to one row's mode.

In this research, for formulating the problem in Markov chain's mode, we consider n customer in the system. Therefore the system status will change for two reasons. One is the new customer's entrance with λ' flow that adds one to the system. The other one is that server's finishing with μ' flow which the number of customer in system will be reduced. In this case system's condition change depends on the present situation and if the system's condition be k or more than that, the number of customers who exit the system will be k people; but if the system's condition be less than k people then server has done the servicing to all the customers in system and system's condition is zero. Regarding to these explanation, system's transferring flow diagram and balanced equations are shown in Fig. 4.

Considering the transferring flow diagram of system, balanced equation's systems are as below:

$$\begin{cases} \lambda'\pi_0 = \mu'(\pi_k + \pi_{k-1} + \cdots + \pi_1) \\ (\lambda' + \mu')\pi_n = \mu'\pi_{n+k} + \lambda'\pi_{n-1}; n \ge 1 \end{cases}$$

For solving the above equation, we write second equation base on operators, as below:

$$\left[\mu' x^{k+1} - (\lambda' + \mu')x + \lambda'\right]\pi_n = 0 : n \ge 0 \tag{9}$$

We know that if ($x_1, x_2,..., x_n$) is the equation's roots, the answer of mentioned deferential equation is as below:

$$\pi_n = \sum_{i=1}^{k+1} c_i x_i^n : n \ge 0 \tag{10}$$

As $\sum_{n=0}^{\infty} \pi_n = 1$, therefore every x_i should be less than one or $c_i = 0$. It is shown that the mentioned specific equation has one root (x_0) between zero and one. With considering this point, we have:

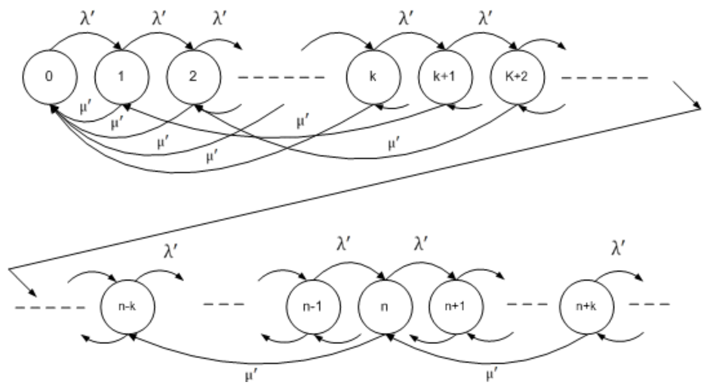

Fig. 4: Transferring flow diagram for mentioned non standard model for each branch with M/M/1 model and group servicing

Table 1: Calculating effective sizes in non standard queuing systems, with having i servicing row and k server in each branch

Calculation method	Comments	Effective sizes
Its value is determined by the sampling	Arrival flow to whole system	λ
$\lambda_i^{'} = \lambda^{'} = \frac{\lambda}{i}$	Arrival flow to a branch	λ´
Its value is determined by the sampling.	Servicing flow for each server, regardless limitation	μ
$\mu^{'} = \frac{1}{E\left(Max(x_1, x_2, ...x_k)\right)}$	Real servicing flow in a branch	μ´
$w^{'} = \frac{x_0}{\lambda^{'}(1-x_0)}$	Waiting time in system for a branch	W´
$W_q^{'} = w^{'} - \frac{1}{\mu^{'}}$	Waiting time in queue for a branch	$W´_q$
W = w	Waiting time in system for whole system	W
$W_q = w´_q$	Waiting time in queue for whole system	W_q
$L^{'} = \frac{x_0}{1-x_0}$	Number of people in system and in a branch	L´
$L_q^{'} = \lambda^{'} W_q^{'}$	Number of waiting people in queue and in a branch	$L´_q$
$L_q = L´_q \times i$	Number of waiting people in queue for whole system	L_q
L = L´× i	Number of exist people in system for whole system	L

$$\pi_n = cx_0^{\,n} : n \geq 0,\ 0 < x_0 < 1 \tag{11}$$

For calculating C, we can use $\sum_{n=0}^{\infty} \pi_n = 1$:

$$\sum_{n=0}^{\infty} \pi_n = 1 \Rightarrow C\sum_{n=0}^{\infty} x_0^n = 1 \Rightarrow C\frac{1}{1-x_0} = 1 \Rightarrow C = 1 - x_0 : n \geq 0 \tag{12}$$

On the other hand:

$$\pi_0 = cx_0^0 = c = 1 - x_0 \tag{13}$$

So we have:

$$\pi_n = (1 - x_0)x_0^n : n \geq 0, 0 < x_0 < 1 \tag{14}$$

Therefore, amount of L´,W´(mean of people's number and waiting time in each branch of system) will be calculated easily as below:

$$w^{'} = \frac{x_0}{\lambda^{'}(1-x_0)} \tag{15}$$

$$L^{'} = \frac{x_0}{1-x_0} \tag{16}$$

Amount of $L_q^{'}$ and $W_q^{'}$ (Mean of people number and waiting time in each branch's of queue) can be calculated with using LITTLE relations (Table 1).

Model developing from row to whole system and calculating effective measures: Measurement criteria $L_q^{'}, w_q^{'}, L^{'}, w^{'}$ for one branch was measured but calculating these parameters are done for whole queuing system that we show them like W, L_q, W_q, L. as servicing condition in i different row in existing system is identical and similar to each other and even the arrival flow is similar, therefore calculation is done as below:

For parameters like W_q and W that have time nature, According to arrival flow of each branch and with using

Jackson's network approach, we should calculate mean weight base on arrival flow. So (waiting time of each customer in i_{th} branch in queuing and system is W'_{q_i}, W'_i):

$$W = \frac{\lambda'_1}{\lambda} w'_1 + \frac{\lambda'_2}{\lambda} w'_2 + \cdots + \frac{\lambda'_k}{\lambda} w'_i \tag{17}$$

$$W_q = \frac{\lambda'_1}{\lambda} w'_{q1} + \frac{\lambda'_2}{\lambda} w'_{qi} \tag{18}$$

But regarding to the last part we have $\lambda'_i = \lambda' = \frac{\lambda}{i}$ and as the rows are same so it is $W'_1 = W'_2 = \cdots = W'_i = W'$ and $W'_{q1} = W'_{q2} = \cdots = W'_{qi} = W'_q$, *therefore*, we will have $W_q = w'_q$, $W = w'$. In other words waiting time in queue and in system for every branch is equal to waiting time in queue and in the system for whole system and it is obtained from the relation that is calculated before.

Based on quantitative nature of L, L_q their Calculation is a little different. If the available number of people in a branch was L´, because all of branches are the same and independent, so there are L´ customer in other branches. In other word:

$$L'_1 = L'_2 = \cdots = L'_i = L'$$

Hereby, base on the Jackson's network approach, with multiplying L´ in all i branches; we can calculate the number of available customer in the whole system (L). This condition is same for number of available customer in queue for whole system (L_q). These parameters can be obtained from the following relations:

$$L_q = L'_q \times i \tag{19}$$

$$L = L' \times i \tag{20}$$

Calculating method of all parameters for whole system is shown in Table 1.
Subject to:

$$E\left(Max(x_1, x_2, ..., x_k)\right) = \frac{1}{k\mu} + \frac{1}{(k-1)\mu} + \frac{1}{(k-2)\mu} + \cdots + \frac{1}{2\mu} + \frac{1}{\mu}$$

x_0 is the only root of below equation between zero and one:

$$\mu' x^{k+1} - \left(\lambda' + \mu'\right)x + \lambda' = 0$$

SUGGESTIONS TO IMPROVE STUDIED SYSTEM

As we said we have departure and substitution constraints in non-standard queuing systems duo to the layout constraint. There is waste of valuable serving capacity because customers leave the system or occupy free servers with delay. In fact this results in delayed serving. For example assume first car in all branches are being served while all the other automobiles have been served but they are not able to leave the system. Know assume one of these branches with 2 servers i.e., k = 2 and have departure constraint. If each of servers serve n customers in a time frame of we can consume by each server to serve each of n customers it will be like Fig. 5.

Time frame	ΔT				
Server 1	1	2	3	n-1	n
Server 2	1	2	3	n-1	n

Fig. 5: Analyis of serving with presence of departure constraint

It is evident from the figure that although second server can serve customers faster but it have to wait and is free unnecessarily. This is true in state of substitution constraint as well but instead of second server, first one has to wait this time. It will be an improvement in performance of fuel stations if we are able to reduce these delayed servings. These improvements can be in terms of:

- Reduction of waiting time in queue and system
- Shorter queues
- Increase of systems' customers
- Induction in pace of serving
- Customer satisfaction improvement
- Increase of presented service in a given

MANAGEMENTS' SUGGESTIONS

In spite of high investment needed for construction of a fuel station, improper location, layout of nozzles, low width of aisles, insufficient space for cars to utilize free nozzles and etc, will cause high waiting time which is main reason of longer queues and high fuel waste duo to the running engines in queues. This fact reveals the necessity of studying these systems.

The proposed model has many applications in a wide range of queuing systems. This queuing model with non-standard state leads to the large queue length and large waiting time. We want to present a scenario in which the system remains empty immediately for the existing cars in the queue after each service accomplishment. Therefore, we could remove the wasting time between each two consecutive services, which is the main defect of non-standard queuing systems. It seems that we cannot increase the productivity of a gas station by simply adding more pumps in the system. In order to study the behavior

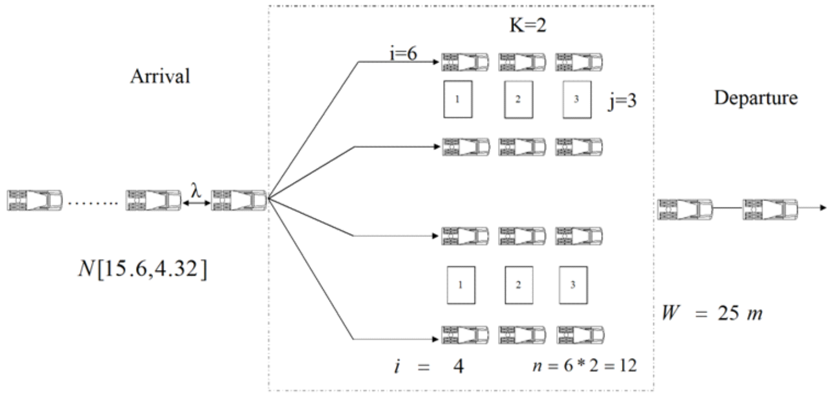

Fig. 6: Schematic of the proposed filling station

Table 2: Comparison between the simulation and the non-standard queuing systems

Measures	Notation	Proposed scenario Value	Existing situation Value	Unite
Mean waiting time in queue	Wq	30	590	Second
Mean queue length	Lq	1	38	Car
Maximum queue length	Lq_{max}	6	68	Car
Utilization	ρ	95	80	Percent
Number of served cars until time *t*	X´(t)	1825	1700	Car
Number of pumps in the station	N	12	12	Number
Number of queue branches	i	4	6	Number
Number of pump in each rows	j	3	2	Number
Number of rows	k	2	3	Number
Station space	S	40*25	40*25	Square meters

of our system, we have used *Show Flow* simulation tool to simulate a scenario in standard state with two rows and three servers in each row (Fig. 6).

It is evident that we need to know some information about system to be able to analyse it like arrival rate and serving status or we have to use sampling to estimate them. However the success of our estimation is dependent on some factors. These factors are classified into major groups. First, are those concerning location of samplings and second are those related to time of samplings. After these factors have been determined, locations and times for sampling, minimum sample size, arrival rate and serving rate were clarified by the help of Design of Experiments (DOE[3]).

Scenario simulation and comparison with proposed model: This scenario is proposed with the same number of servers (6 pumps) and the same station space constraint, which includes two rows with three servers in each row (Fig. 6). Schematic scenario simulation is shown in Fig. 6. Rate of service and rate of arrival that are shown in this figure are obtained using DOE.

The simulation program runs for one working day. The output of management proposal simulation and their comparisons with the existing non-standard queuing model for the gas station are shown in Table 2.

The number of pumps in the station is 12 for both the proposed scenario and the existing situation and the number of queue branches in the proposed scenario is four and for the existing situation is six. The evaluation measures for fuel queuing system are described in Table 2 are *Wq, Lq, Lq max,ρ, X´(t).*

As we can see in Table 2 the systems' performance proposed by the model is better than that the existing non-standard situation, which is modeled in the study. Queue mean waiting time in the proposed scenario is approximately 20 times less than the existing situation and the maximum queuing length in the proposed scenario is also reduced significantly.

Management scenario has better utilization but the gas stations are still built based on the non-standard format.

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CONCLUSION

In this study we have analyzed one of the most important types of queuing systems that is called non-standard duo to some constraints and restrictions. Then we argued why this is not a standard system. by using concepts of queuing theory a hybrid model has been developed to calculate evaluation parameters for this queuing system, a model that can be the foundation for analysis of non-standard queuing systems like fuel stations, assembly lines and production line bottlenecks. Analysis of developed model shows that the performance of this non-standard system is relatively low as a result of mentioned constraints. So we suggested some solutions to improve system efficiency which are trying to eliminate constraints and transform the system to standard type.

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