Group Decision-Making Information Security Risk Assessment Based on AHP and Information Entropy

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Abstract: The phenomenon of over-reliance on subjective assignment is a challenging task in the information security risk assessment process. This study deals with this problem. We have presented a group decision-making information security risk assessment method by combining Analytic Hierarchy Process (AHP) with Information entropy. When AHP is used to assess the security risk of information systems, the elements of the Criteria level are the risk probability, impact and uncontrollability. The priorities of the Alternatives as risk factors with respect to the Criteria level are determined by applying the group decision-making approach. And the experts’ weights are obtained through information entropy. The experts’ judgments are aggregated into a consensus matrix. The consensus matrix reduces the subjectivity of judgments due to the experts’ preferences.

Keywords: Analytic hierarchy process, consensus matrix, entropy, group decision-making, risk assessment, risk factor

INTRODUCTION

AHP (Analytic Hierarchy Process) (Saaty, 1980; Golden et al., 1989; Saaty, 1990) is a frequently used method of Multi-Criteria Decision Making (MCDM) (Belton and Stewart, 2002; Toshztar, 1988) in information system risk assessment (Chivers, 2009). The method can help the decision-maker to deal with a complicated problem relating to multiple conflicting and subjective criteria. By quantifying the decision-makers' experience judgments, AHP can still provide the decision-makers with quantitative decision-making data. Moreover, if one combines qualitative analysis with quantitative analysis to use AHP, he/she can achieve quantitative decision-making.

Individual judgments can be aggregated in different ways. Up to now, there are two typical aggregation methods: the Aggregation of Individual Judgments (AIJ) and the Aggregation of Individual Priorities (AIP) (Forman and Peniwati, 1998). AIJ is to aggregate a series of pair-wise comparison matrices into a pair-wise comparison matrix. AIP is to aggregate the results into the final order of each factor based on the pair-wise comparison matrix of each expert. Thus AIJ is more suitable for the case in which the experts as a team make decisions. However, AIP is more suitable if the experts do as an individual.

When AHP is used to assess the risk of information system, the assessment results rely heavily on the experts’ understanding of the system, which leads to a strong subjectivity since the judgments of one individual expert easily depend on personal preferences. In order to reduce the subjectivity, we proposed a new method by combining with group decision-making (Dong et al., 2010) and AHP. The method contains two phases. First, the experts evaluate the information system. Secondly, all the opinions of experts are aggregated. With our case studies, we chose AIJ to aggregate the experts’ judgments.

For the actual establishment of the hierarchical model, since the Criteria elements do not depend on the specific system, their priorities are more obvious and more objective. Thus we need not use the group decision-making approach on the Criteria layer. We calculate the priorities of the Criteria directly.

The aim of the study is to present the group decision-making method of information security risk assessment. We have showed that how the method is applied to calculate the priorities and aggregate experts’ different judgments. A worked example of the proposed method is given in this study. Although the example is drawn from specific domains, such as information systems, the method is suitable for generic cases, which avoids commitment to any particular domain.

Background: The German physicist Clausius (1865) introduced Entropy as a thermodynamic concept. Planck and Max (1990) indicated the statistical meaning of the entropy by linking up the concept of entropy and the
number of probable micro-states of the system. Shannon (1948) extended the concept of statistical entropy to the information field. Entropy is used to indicate the uncertainty of information source. Shannon’s entropy is defined as follows:

Assume that the system has n different states \( \{ S_1, S_2, \ldots, S_n \} \) and the probability that the system is in state \( S_i \) is \( p_i \). The uncertainty \( H \) of the system is defined as \( H = -C \sum_{i=1}^{n} p_i \ln p_i \), where \( C \) is a constant and \( p_i \) satisfies \( 0 \leq p_i \leq 1 \) and \( \sum_{i=1}^{n} p_i = 1 \).

Proposition (Abramson, 1963) A system has \( n \) different states with the probability distribution \( \{ p_1, p_2, \ldots, p_n \} \), where \( p_i \) denotes the probability of the system in state \( S_i \). Assume that the entropy of the system is \( H \). We have:

\[
H \leq H \left( \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n} \right) = C \ln n
\]

The first step of AHP (Vellore and Olson, 1991) is to stratify the issue, more specifically, to decompose the issue into different elements according to the nature and requirements to reach. The second step is to use all the elements at different hierarchical clustering in accordance with inter-relationship and affiliation of elements to set up a multi-hierarchical analysis structure model and ultimately turn the issue into the lowest nodes weight problem with respect to the top nodes. Before hierarchical model is established, we need analyze the system risks and identify the existing risk factors. Risk analysis (Slovic et al., 2002) includes three elements: the system assets, threats and vulnerabilities. The threat makes use of system vulnerabilities to destruct the assets of the system. If the system is faced with threats, there will be a risk of the system. So the value of the risk can be attributed to the probability of risk events happening, the extent of the impact of asset after the incident and the uncontrollability of risk events.

AHP-BASED INFORMATION SECURITY RISK ASSESSMENT

In this section, we take an instance of computer information systems as an example. According to the system’s security requirements, the risk factors of the system contain attacks on data confidentiality, data integrity destruction, impersonation attacks, unauthorized access and denial of service.

AHP model: The first level of hierarchical model is the Goal. The middle level is the Criteria and the lowest level is the Alternatives (risk factors). Here, the Goal is defined as the importance of risk factors. The elements of the Criteria are the value of the risk probability, impact and uncontrollability. The Alternatives include risk factors. Hierarchical model is shown in Fig. 1.

Pair-wise comparison matrices: Pair-wise comparison matrix represents the relative importance between the nodes of each level with respect to their contribution to the nodes above them. The matrix \( A \) can be written as follows:

\[
A = \begin{bmatrix}
    b_{11} & b_{12} & \ldots & b_{1n} \\
    b_{21} & b_{22} & \ldots & b_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{n1} & b_{n2} & \ldots & b_{nn}
\end{bmatrix}
\]

where \( b_{ij} \) denotes the relative importance of \( b_i \) with respect to \( b_j \) for the first level and their values are from the set \{1, 3, 5, 7, 9\} or their reciprocals.

Criteria vs. the Goal: The priorities are derived from a series of measurements: pair-wise comparisons of all the nodes (Barzilai, 1997). All the nodes at each level are compared, two by two, with respect to their contribution to the nodes of the above level. The results of these comparisons are put into a matrix. When one computes the largest eigenvalue and the corresponding eigenvectors of the matrix, the priorities for all the nodes on the level can be obtained. Here we use the square root method to calculate the node priorities of Criteria with respect to the Goal. Weight vector of each node is:

\[
\omega = \left( \omega_1, \omega_2, \ldots, \omega_n \right)^T, \quad \omega_j = \sqrt{\prod_{j=1}^{n} b_{ij}}
\]

We normalize them as follows:

\[
\omega = \left( \omega_1, \omega_2, \ldots, \omega_n \right)^T, \quad \omega_j = \frac{\omega_j}{\sum_{j=1}^{n} \omega_j}
\]

Then we calculate the largest characteristic value of the matrix:

![Fig. 1: The logic diagram of the assessment elements](image)
Next we compute the consistency index:

\[ \lambda_{\text{max}} = \sum_{i=1}^{n} \frac{(A\omega)_i}{n\omega_i} \quad (3) \]

where, \((A\omega)_i\) represents the \(i\)-th element of the vector \(A\omega\).

Next we compute the consistency index:

\[ CI = \frac{\lambda_{\text{max}} - n}{n - 1}, CR = CI / RI \quad (4) \]

where, \(RI\) is the average random consistency index. If \(CR \leq 0.1\), the consistency of the comparison matrix is acceptable. Otherwise, we should make appropriate adjustments to the comparison matrix until the consistency of the comparison matrix falls within the acceptable range.

**Alternatives vs. criteria:** When we calculate the nodes priorities of the Alternatives with respect to the Criteria, we need a detailed analysis of the system to determine the relative importance between the Alternatives (risk factors). Different experts may have different results. In order to deal with these differences, we adopt a group decision-making approach to determine the node priorities of the Alternatives with respect to the Criteria.

Firstly, each expert first creates three different pair-wise comparison matrices in accordance with three nodes of the Criteria. Secondly, according to the calculated entropy, we determine the weight of each expert. Finally, we run the algorithm of weighted geometric mean to aggregate the pair-wise comparison matrix created by each expert into a new pair-wise comparison matrix, which is called consensus matrix. We use the consensus matrix to calculate the nodes priorities of the Alternatives with respect to the Criteria through the formulae.

**Synthesizing final priorities:** Since we have the priorities of the Criteria with respect to the Goal and the priorities of the Alternatives with respect to the Criteria, we can calculate the priorities of the Alternatives with respect to the Goal from the above priorities. Assume that the hierarchical model contains \(m\) factors in the Criteria layer and \(n\) elements in the Alternatives layer.

By \(w_i = (w_{i1}, w_{i2}, ..., w_{in})^T\), we denote the priorities vector of the Criteria with respect to the Goal. By \(w_j = (w_{j1}, w_{j2}, ..., w_{jn})^T\), we denote the priorities vector of the Alternatives with respect to the Criteria. The priorities vector of the Alternatives with respect to the Goal can be computed as follows:

\[ w = (w_1, w_2, ..., w_n)^T, \quad w_j = \sum_{k=1}^{m} w_j \times w_k, i = 1, 2, ..., n. \]

**Entropy-based information security risk assessment**

In the section, we will combine information entropy with information security risk assessment based on AHP to compute Pair-wise comparison matrices. We will take the AIJ method to aggregate experts’ judgments. We first use the weighted geometric average algorithm to calculate the ratios of the Alternatives priorities and then construct an aggregated comparison matrix. The calculation of the weight of experts’ judgments is involved with two kinds of weights, the static weight and dynamic weights. Static weight is related to experts’ fame, status, their profession and the degree of familiarity with decision-making, while the dynamic weight is determined by the credibility of the results of the experts’ final decision.

Assume that the decision-making experts have the same static weights. Thus, we only need to consider the dynamic weights. Suppose there are \(Q\) experts. The calculation of specific weights and the aggregation of the pair-wise comparison matrices are divided into the following four steps:

- Each expert \(q\) creates a pair-wise comparison matrix \(A_q = (a_{ij})_{n \times n}\), where \(a_{ij}\) represents the relative importance of the element \(i\) with respect to the element \(j\) at the same level.
- We take the \(i\)-th row of the comparison matrix built by the \(q\)-th \((0 \leq q \leq Q)\) expert as the \(j\)-th row to form into a new matrix \(vec(A)\), in the following:

\[ vec(A) = \begin{bmatrix} a_{1i} & a_{2i} & \cdots & a_{ni} \\ a_{1j} & a_{2j} & \cdots & a_{nj} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1q} & a_{2q} & \cdots & a_{nq} \end{bmatrix}, 0 \leq i \leq n. \quad (5) \]

- By normalizing the matrix above by row, we have

\[ vec(A) = \begin{bmatrix} r_{i1} & r_{i2} & \cdots & r_{in} \\ r_{j1} & r_{j2} & \cdots & r_{jn} \\ \vdots & \vdots & \ddots & \vdots \\ r_{q1} & r_{q2} & \cdots & r_{qn} \end{bmatrix} = \begin{bmatrix} r_1^T \\ r_2^T \\ \vdots \\ r_Q^T \end{bmatrix} \quad (6) \]

If all the differences among the elements for \(r\) are small, the judgments of the expert \(i\) are not convincing and thus the expert \(i\) has less contribution to the comprehensive evaluation. If all the difference among the elements for \(r\) are large, the evaluation results are scattered and the judgments of the expert...
i are convincing. Thus the expert judgments have a certain pertinence and higher persuasiveness, which should play a key role in the comprehensive evaluation.

According to the above analysis, it is easy to know that the information entropy can reversely reflect the difference between the index values of each factor. Therefore, we adopt the information entropy to measure the relative importance $H_q$ of expert judgments:

$$H_q = - \sum_{y=1}^{n} r_{xy} \ln r_{xy}$$

(7)

According to the Proposition 1, we have $H_q \leq 1/n$. We normalize $H_q$:

$$e_q = -\frac{1}{\ln n} \sum_{y=1}^{n} r_{xy} \ln r_{xy}$$

(8)

Thus, we obtain the relative importance of the $q$-th expert judgment. We know $0 \leq e_q \leq 1$. A larger $e_q$ has a less contribution to the system risk assessment. In order to make the result positively reflect the importance of the expert judgment, we use $1 - e_q$ to express its importance. And we normalized it:

$$\alpha_q = -\frac{1}{n-\sum_{q=1}^{n} e_q} \left(1 - e_q\right)$$

(9)

Obviously, we have $0 \leq \alpha_q \leq 1$ and $\sum \alpha_q = 1$.

- Let $L_i = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ $\operatorname{vec}(A_i)$. Thus we have a consensus matrix $A = (L_1, L_2, \ldots, L_n)^T$. By $w$ we denote the priorities of the consensus matrix $A$. We calculate $w = (w_1, w_2, \ldots, w_n)^T$ through (1) and (2).

**Worked example:** The hierarchy model of the information system is as shown in Fig. 1. We create a pair-wise comparison matrix of the Criteria with respect to their importance in reaching the Goal. The nodes at the Criteria level are compared, two by two, with respect to their contribution to the Goal. The results of these comparisons are written into a matrix. After the matrix is processed mathematically, we can derive the priorities for all the nodes on the Criteria level. We take a priority matrix $A_B$ of the Criteria with respect to the Goal built by the experts:

$$A_B = \begin{bmatrix} 1 & 1/3 & 5 \\ 3 & 1 & 7 \\ 1/5 & 1/7 & 1 \end{bmatrix}$$

(10)

According to (1) and (2), the eigenvector of the matrix $A_B$ is $w = (0.2790, 0.6491, 0.0719)^T$. Based on (3) and (4), the largest eigenvalue of the matrix is $\lambda_{max} = 3.0649$. The consistency index is $CR = 0.03245 < 0.1$. When $n = 3$, we have $RI = 0.58$. Thus the comparison matrix meets the consistency requirements.

In the following, we will adopt the group decision-making approach to calculate the priorities of the Alternatives with respect to the Criteria. Assume that the decision-making group is composed of three experts.

First, for the probability factor of the Criteria, three experts give their comparison matrices $B_1\_C^1$, $B_1\_C^2$ and $B_1\_C^3$, which represent the priorities of the Alternatives with respect to the probability of the Criteria:

$$B_1\_C^1 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 \\ 2 & 1 & 1/2 & 1/2 & 1/3 \\ 3 & 2 & 1 & 1/2 & 1/2 \\ 4 & 2 & 2 & 1/2 & 1 \\ 5 & 3 & 2 & 2 & 1 \end{bmatrix}$$

(11)

$$B_1\_C^2 = \begin{bmatrix} 1 & 1/3 & 1/5 & 1/7 & 1/7 \\ 3 & 1 & 1/2 & 1/3 & 1/3 \\ 5 & 2 & 1 & 1/2 & 1/2 \\ 7 & 3 & 2 & 1 & 1 \\ 7 & 3 & 2 & 1 & 1 \end{bmatrix}$$

$$B_1\_C^3 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/7 & 1/5 \\ 2 & 1 & 1/2 & 1/4 & 1/3 \\ 3 & 2 & 1 & 1/3 & 1/2 \\ 7 & 4 & 3 & 1 & 2 \\ 5 & 3 & 2 & 1/2 & 1 \\ 

The results demonstrate that the comparison matrices of three experts meet the consistency requirements. We process the matrices $B_1\_C^1$, $B_1\_C^2$ and $B_1\_C^3$ respectively. Thus we obtain a consensus matrix $B_1\_C$:

$$B_1\_C = \begin{bmatrix} 1.0000 & 0.4257 & 0.2739 & 0.1689 & 0.1745 \\ 2.4706 & 10.0000 & 0.5000 & 0.3469 & 0.3333 \\ 3.8678 & 2.0000 & 10.0000 & 0.4497 & 0.5000 \\ 6.0958 & 2.9725 & 2.2739 & 10.0000 & 1.1232 \\ 5.8522 & 3.0000 & 2.0000 & 1.0126 & 10.0000 \end{bmatrix}$$

(12)

According to (1) and (2), the priorities of the Alternatives with respect to the probability of the Criteria are $w = (0.0509, 0.1076, 0.1776, 0.3411, 0.3228)^T$. For the impact factor of the Criteria, three experts give their comparison matrices $B_2\_C^1$, $B_2\_C^2$ and $B_2\_C^3$, which represent the priorities of the Alternatives with respect to the impact of the Criteria:

$$B_2\_C^1 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1/2 & 1 & 2 & 2 & 3 \\ 1/3 & 1/2 & 1 & 2 & 2 \\ 1/4 & 1/2 & 1/2 & 1 & 2 \\ 1/5 & 1/3 & 1/2 & 1/2 & 1 \end{bmatrix}$$

$$B_2\_C^2 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1/2 & 1 & 2 & 2 & 3 \\ 1/3 & 1/2 & 1 & 2 & 2 \\ 1/4 & 1/2 & 1/2 & 1 & 2 \\ 1/5 & 1/3 & 1/2 & 1/2 & 1 \end{bmatrix}$$

$$B_2\_C^3 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1/2 & 1 & 2 & 2 & 3 \\ 1/3 & 1/2 & 1 & 2 & 2 \\ 1/4 & 1/2 & 1/2 & 1 & 2 \\ 1/5 & 1/3 & 1/2 & 1/2 & 1 \end{bmatrix}$$

(13)

(14)

(15)
As above. We have a consensus matrix with respect to the uncontrollability of the Criteria: Similarly, we process the matrices three experts meet the consistency requirements. Then we process the matrices three experts give their comparison matrices are:

Alternatives with respect to the probability of the Criteria.

The results show that the comparison matrices of three experts are:

Thus we have a consensus matrix:

$$B_{3-C^3} = \begin{bmatrix} 1 & 1 & 3 & 4 & 6 \\ 1 / 3 & 1 & 3 & 4 & 6 \\ 1 / 4 & 1 / 4 & 1 / 2 & 1 & 2 \\ 1 / 6 & 1 / 6 & 1 / 3 & 1 / 2 & 1 \end{bmatrix}$$

The results show that the comparison matrices of three experts meet the consistency requirements. We have a consensus matrix with respect to the uncontrollability of the Criteria: three experts give their comparison matrices.

Alternatives with respect to the probability of the Criteria.

The results show that the comparison matrices of three experts are:

$$B_{2-C^2} = \begin{bmatrix} 1 & 2 & 3 & 5 & 7 \\ 1 / 2 & 1 & 2 & 3 & 4 \\ 1 / 3 & 1 / 2 & 1 & 2 & 3 \\ 1 / 5 & 1 / 3 & 1 / 2 & 1 & 2 \\ 1 / 7 & 1 / 4 & 1 / 3 & 1 / 2 & 1 \end{bmatrix}$$

According to (1) and (2), the priorities of the Alternatives with respect to the uncontrollability of the Criteria are:

$$w = (0.4017, 0.2814, 0.1483, 0.1019, 0.0595)^T$$

For the uncontrollability of the Criterion, three experts give their comparison matrices $B_{3-C^3}$, $B_{2-C^2}$ and $B_{1-C}$, which represent the priorities of the Alternatives with respect to the uncontrollability of the Criteria:

$$B_{3-C^3} = \begin{bmatrix} 1 & 3 & 2 & 5 & 5 \\ 1 / 3 & 1 & 1 / 2 & 3 & 3 \\ 1 / 5 & 1 / 3 & 1 / 4 & 1 & 1 \\ 1 / 5 & 1 / 3 & 1 / 4 & 1 & 1 \end{bmatrix}$$

$$B_{2-C^2} = \begin{bmatrix} 1 & 2 & 6 & 5 \\ 1 / 2 & 1 & 1 / 4 & 3 \\ 1 / 6 & 1 / 4 & 1 / 4 & 1 / 2 \\ 1 / 5 & 1 / 3 & 1 / 3 & 2 & 1 \end{bmatrix}$$

$$B_{1-C} = \begin{bmatrix} 1 & 1 & 2 & 4 & 5 \\ 1 & 1 & 2 & 4 & 5 \\ 1 / 2 & 1 / 2 & 1 & 3 & 3 \\ 1 / 4 & 1 / 4 & 1 / 3 & 1 & 1 \\ 1 / 5 & 1 / 5 & 1 / 3 & 1 / 2 & 1 \end{bmatrix}$$

The results show that the comparison matrices of three experts are:

Through (1) and (2), we calculate the priorities of the Alternatives with respect to the uncontrollability of the Criteria:

$$w = (0.3279, 0.2289, 0.2249, 0.0993, 0.0740)^T$$

Finally, according to the rules of synthesizing final priorities, we calculate the priorities of the Alternatives with respect to the Goal. The overall priorities for all the Alternatives are listed in Table 1.

$$w = \sum_{i=1}^{m} w_i \cdot w_{ij}$$

Table 1: The final priorities

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>$B_{1}=0.2790$</th>
<th>$B_{2}=0.6491$</th>
<th>$B_{3}=0.0719$</th>
<th>$w_j = \sum_{i=1}^{m} w_i \cdot w_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.0509</td>
<td>0.4017</td>
<td>0.3729</td>
<td>0.3018</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.1076</td>
<td>0.2814</td>
<td>0.2289</td>
<td>0.2291</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.1776</td>
<td>0.1483</td>
<td>0.2249</td>
<td>0.1620</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0.3411</td>
<td>0.1091</td>
<td>0.0993</td>
<td>0.1731</td>
</tr>
<tr>
<td>$C_5$</td>
<td>0.3228</td>
<td>0.0595</td>
<td>0.0740</td>
<td>0.1340</td>
</tr>
</tbody>
</table>

The above results show that the weight of Confidentiality attacks ($C_1$) is relatively the largest, followed by the Integrity destruction ($C_2$) and Unauthorized access ($C_3$). And the weight of Impersonation attacks ($C_4$) and Denial of service ($C_5$) is relatively smaller. Because there are fewer protection measures in the aspects of the system confidentiality and integrity, the sum of both weights is much higher than that of the other three risk factors. Therefore, there exists higher risk about the confidentiality and integrity in the system.

**CONCLUSION**

When one adopts AHP to quantify the risk factors, the assessment depends on the expert's own experience and understanding of the system. The assessment results are affected by the personal preferences, which lead to some deviations of the system assessment. In this study, we use a group decision-making method to determine the relative weights among the risk factors in a specific system. Since a team of experts are involved in the assessment and the weighted geometric mean algorithm is used to aggregate the pair-wise comparison matrix given by each expert into a consensus matrix, we can reduce the subjectivity due to personal preferences and improve the accuracy of the results of the assessment.

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