

Selective Maintenance Model Considering Time Uncertainty

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Abstract: This study proposes a selective maintenance model for weapon system during mission interval. First, it gives relevant definitions and operational process of material support system. Then, it introduces current research on selective maintenance modeling. Finally, it establishes numerical model for selecting corrective and preventive maintenance tasks, considering time uncertainty brought by unpredictability of maintenance procedure, indetermination of downtime for spares and difference of skill proficiency between staffs. The adaptability and flexibility of selective maintenance model are improved dramatically.

Keywords: Corrective maintenance, maintenance decision, preventive maintenance, selective maintenance model

INTRODUCTION

Equipment maintenance support, is the general designation of all kinds of technical measures and related support activities to maintain and restore technical performance of weapon systems (Yu and Kang, 2008). The operation of support system can be simply abstracted as Fig. 1 shows. Equipment users propose maintain application to Management Office (MO) when weapon equipments breakdown. MO schedules all projects to different maintenance organizations. As maintain task is completed, maintenance organizations submit report to MO and MO informs users to incept equipments. It is obvious that decision making is an important node of operation process. In order to fulfill maintain tasks, MO has to make scientific and rational decisions.

According to the phase partition for maneuver task, as shown in Fig. 2, maintenance decision can be divided into four specific issues, showed in Fig. 3, i.e., Maintenance Task Selection during Interval Phase (MTSIP), Maintenance Task Scheduling Concept Optimization during Interval Phase (MTSCOIP), Maintenance Task Priority Sorting during Implementation Phase (MTPSIP) and Maintenance Task Scheduling Strategy Analysis during Implementation Phase (MTSSAIP).

This study focuses on the problem of MTSIP and establishes an selective maintenance model considering time uncertainty. This model focuses on the influence on maintenance time brought by parts' condition, repaire modes and maintain resources. Consequently, the maintenance decisions are more precise and adaptive.

Moreover, we would probe into the algorithm of this model in the futher.

METHODOLOGY

Current research progress: Whether industrial department, or military organization, need specific system to complete a series of tasks and repair fault components during task interval. But because of the limitation of resources, it's impossible to fulfill all the maintenance tasks. Therefore, MO staff must schedule maintenance activities with certain resource constraints. Allocating maintain resources and making corresponding plan belong to the Selective Maintenance (SM) research field. SM is the process choosing some repair tasks from a group of tasks. It also could be called the Maintenance Task Selection. Research of SM has only spent ten years and mainly focuses on the model and algorithm.

Basic model: Rice *et al.* (1998) proposes SM strategy for the first time. His assumed research object is the system composed of series subsystems. Each subsystem has fixed-failure-rate parts. The system must accomplish a series of repair tasks during interval phase. Due to the limitation of resources, it's impossible to repair all fault parts. Rice established a certain, nonlinear and discrete SM optimization model to maximize the system reliability for next task. The key model variable is the number of parts to be repaired for each subsystem. The model constraint is limited maintenance time. Due to the model complexity, enumeration isn't effective method for model solution. So some scholars put forward the heuristic algorithm.

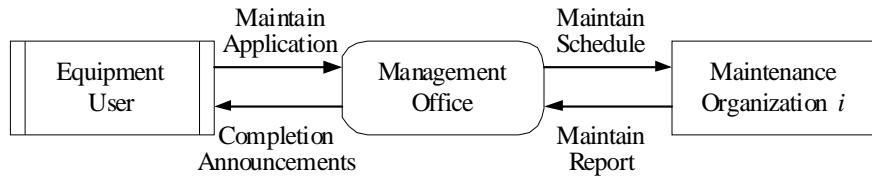


Fig. 1: Operation of maintenance system

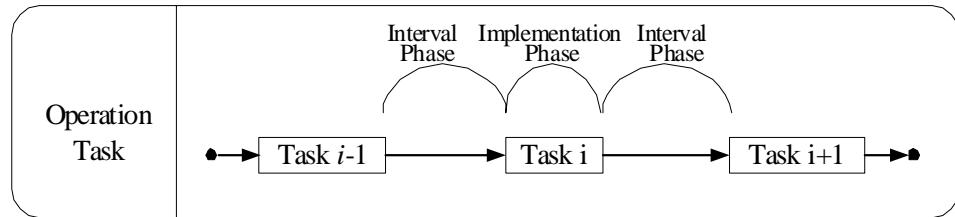


Fig. 2: Mission break and mission duration

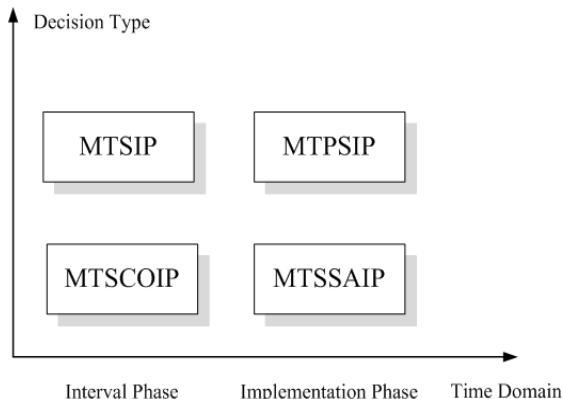


Fig. 3: Four types of maintenance decision

Model improvement: Improvement on system properties: Cassady *et al.* (1998, 2001) develops Rice research further. He considers more complex system composed of series subsystem. Parts coupling of subsystem can be either series or parallel. In addition, repair activities are constrained by maintain time and expenditure. They establish three different Selective

Maintenance Models (SMM):

- Maximum system reliability with time and expenditures constraints
- Minimum maintenance expenditures model with time and minimum reliability constraints
- Minimum maintenance time model with expenditures and reliability constraints.

Chen *et al.* (1999) expands Rice and Cassady research. He proposes minimum expenditures model with

reliability constraints, assuming system has $K+1$ state ($K \geq 1$).

Improvement on task properties: Cassady assumes that a set of system to be maintained are in different state (described by maintenance time vector) and will implement different tasks. Requirements to subsystem are also different according to different tasks. He establishes the mixed integer schedule model to optimize task allocation scheme and maintenance scheduling scheme and proposes three model algorithms (Cassady *et al.*, 2005). Cassady considers the task allocation scheme and maintenance scheduling scheme of multi-equipments simultaneously and gives static and dynamic SMM the of multi-equipments. In the static model, task durations are different and it's the indicator to measure whether task is difficult or not (Cassady *et al.*, 2003). In the dynamic model, not only task durations are different, but start time and end time of tasks are different. Cassady establishes SMM considering continuous multiple tasks and solves the model with stochastic dynamic programming method (Cassady *et al.*, 2004; Schneider *et al.*, 2009).

Improvement on maintenance task properties: Cassady develops Rice's research from two aspects:

- System parts obey Weibull distribution.
- SMM includes three repair activities: minimum repair of fault parts, fault parts replacement and preventive maintenance (Cassady *et al.*, 2001). Liu adds unperfect repair activity in the SMM and imports life-reduction gene as cost function simultaneously. In addition, he imports general generating function to assess dependable probability of single task (Liu *et al.*, 2010).

Improvement on application: Iyoob proposes a method to ascertain the properties of maintenance resources according to a series of tasks. This method resolves redundancy design and resource allocation (Iyoob *et al.*, 2006). His research testifies maintenance strategy can affect equipment design and resource allocation. Schneider calculates the operation efficiency of weapon system with selective maintenance strategy, using simulation and selective maintenance conjunctly (Schneider and Cassady, 2004).

RESULT ANALYSIS

Model algorithm: Rainwater proposes two exhaustive searching algorithms for SMM and compares their flexibility and computing time (Rainwater *et al.*, 2004). Rajanand and Cassady (2004, 2006) proposes several methods to improve the efficiency of the exhaustive searching algorithm. According to the SMM in reference (Simon, 1972), Lust (Hirsch *et al.*, 1968) proposes precise algorithm based on the branch demarcation and tabu search algorithm.

SMM considering time uncertainty: Current researches assume that ingredients of SMM are explicit and do not consider the indeterminacy of maintenance time. The unpredictability of maintenance procedure, indetermination of downtime for spares and difference of skill proficiency between staffs are all the factors leading to the indeterminacy of maintenance time. So, it's difficult to get correct decision using explicit SMM in practice. Therefore, it is necessary to establish the SMM considering the indeterminacy of maintenance time.

Model hypothesis:

- Equipment combat unit is composed of some independent systems. The number of system is q . Each system is composed of independent series subsystem. The number of system is m . Subsystem j contains n_j independent parallel parts. The life-span of parts obeys Weibull distribution. B_j indicates parts' shape parameter. η_j indicates scale parameter. System, subsystem, parts have only two states: work or fault. Work [fault] state can be defined as 1 [0]; $X_i(k)$ [$Y_i(k)$] indicates the state of system i when task k begins [over]; $X_{ij}(k)$ [$Y_{ij}(k)$] indicates the state of system i 's subsystem j when task k begins [over]; $X_{ijl}(k)$ [$Y_{ijl}(k)$] indicates the states of parts l (component of system i 's subsystem j) when task k begins [over]; $i \in \{1, 2, \dots, q\}$, $j \in \{1, 2, \dots, m\}$, $l \in \{1, 2, \dots, n_j\}$; $r_{ijl}(k)$ indicates the probability of parts l to complete task k and it depends on the age of parts l and the working time requirements for subsystem j in task k ;

$A_{ijl}(k)$ indicates the age of parts l when task k starts; $B_{ijl}(k)$ indicates the age of parts l when task k ends; $L_{ij}(k)$ indicates the working time required for subsystem j in task k . Therefore, $r_{ijl}(k)$ can be expressed as (1):

$$r_{ijl}(k) = \frac{R\left(\left(L_{ij}(k) + A_{ijl}(k)\right)\right)}{R\left(A_{ijl}(k)\right)} \quad (1)$$

Task reliability of parts l when task k ends can be calculated as (2):

$$R_{ijl}(k) = P(Y_{ijl}(k) = 1)r_{ijl}(k).X_{ijl}(k) \quad (2)$$

The reliability of task k uses series system reliability function and can be expressed as (3):

$$\begin{aligned} R(k) &= P(Y(k) = 1) \\ &= \prod_{i=1}^q \prod_{j=1}^m R_{ij}(k) = \prod_{i=1}^q \prod_{j=1}^m \prod_{l=1}^{n_j} R_{ijl}(k) \\ &= \prod_{i=1}^q \prod_{j=1}^m \left(1 - \prod_{l=1}^{n_j} (1 - R_{ijl}(k))\right) \end{aligned} \quad (3)$$

$R_{ij}(k)$ indicates the reliability of subsystem j when task k ends. Since each subsystem only contains parallel parts, the reliability relationship between subsystem and parts uses parallel system reliability function.

- Parts' state is "work" or "fault" when task k ends. For decision makers, there are three options: minimum repair of fault parts, fault parts replacement and preventive maintenance. The effect on performance and life for parts according to different maintenance modes is showed in Table 1.
- **Decision variables:**
 - W_{ijl} : It equals 1 if the maintenance modes of parts l is "minimum repair" between task k and task $k+1$, otherwise it equals 0
 - V_{ijl} : It equals 1 if the maintenance modes of parts l is "replacement" between task k and task $k+1$, otherwise it equals 0
- The model also considers maintenance resources except maintenance time. It's assumed that there are V kind resources. D_v^A indicates the available amounts for resources V ; t^m_j indicates the minimum repair time for fault parts attached to subsystem j and it's stochastic variables whose probability distribution

Table 1: Effects of maintenance policy

Maintenance mode	Parts performance when task k ends	Parts performance when task k+1 starts	Parts life when task k+1 starts
Minimum repair	$Y_{ijl}(k) = 0$	$X_{ijl}(k+1) = 1$	$A_{ijl}(k+1) = 1$
Replacement	$Y_{ijl}(k) = 0$	$X_{ijl}(k+1) = 1$	$A_{ijl}(k+1) = 1$
Preventive maintenance	$Y_{ijl}(k) = 1$	$X_{ijl}(k+1) = 1$	$A_{ijl}(k+1) = 0$

function is A_{ijl}^f . d_{jv}^m indicates the amounts of resource V for minimum repair, $v \in \{1, 2, \dots, V\}$. t_j^f indicates the time required to replace normal parts in subsystem j and it's stochastic variables whose probability distribution function is $\Phi_j t_j^f$. d_{jv}^f indicates the amounts of resource V to replace normal parts, $v \in \{1, 2, \dots, V\}$. t_j^r indicates the time required to replace fault parts in subsystem j and it's stochastic variables whose probability distribution function is $\Phi_j t_j^r$. d_{jv}^r indicates the amounts of resource V to replace fault parts in subsystem j , $v \in \{1, 2, \dots, V\}$.

The total time required by all maintenance activities before the next task can be calculated by (4):

$$T(k) = \sum_{i=1}^q \sum_{j=1}^m \sum_{l=1}^{n_j} t_j^m \cdot W_{ijl}(k) + t_j^r \cdot V_{ijl}(k) \cdot (1 - Y_{ijl}(k)) + t_j^f \cdot V_{ijl}(k) \cdot Y_{ijl}(k) \quad (4)$$

The total resources required by all maintenance activities before the next task can be calculated by (5):

$$D_v(k) = \sum_{i=1}^q \sum_{j=1}^m \sum_{l=1}^{n_j} d_{jv}^m \cdot W_{ijl}(k) + d_{jv}^r \cdot V_{ijl}(k) \cdot [1 - Y_{ijl}(k)] + d_{jv}^f \cdot V_{ijl}(k) \cdot Y_{ijl}(k) \quad (5)$$

Mathematic mode:

$$\text{Max } R(k+1)$$

$$= \prod_{i=1}^q \prod_{j=1}^m \prod_{l=1}^{n_j} \left(\exp \left(- \left(\left(\frac{L_{ij}(k+1) + A_{ijl}(k+1)}{\eta_j} \right)^{\beta_j} - \left(\frac{A_{ijl}(k+1)}{\eta_j} \right)^{\beta_j} \right) X_{ijl}(k+1) \right) \right) \quad (6)$$

s.t

$$P_r\{T(k) \leq T_0(k)\} \geq \alpha \quad (7)$$

$$D_v(k) \leq D_v^A(k) \forall v \quad (8)$$

$$W_{ijl}(k) + V_{ijl}(k) \leq 1 \forall i, j, l \quad (9)$$

$$Y_{ijl}(k) + W_{ijl}(k) \leq 1 \forall i, j, l \quad (10)$$

$$A_{ijl}(k+1) = B_{ijl}(k) - B_{ijl}(k)V_{ijl}(k) \forall i, j, l \quad (11)$$

$$X_{ijl}(k+1) = Y_{ijl}(k) + W_{ijl}(k) - V_{ijl}(k) \\ [1 - Y_{ijl}(k)] \forall i, j, l \quad (12)$$

$$W_{ijl}(k), V_{ijl}(k) \in \{0, 1\} \forall i, j, l \quad (13)$$

Equation (7) indicates all maintenance activities must be completed within the prescriptive period of time under the confidence limit α .

Equation (8) indicates maintenance resources consumed can not exceed the available amount.

Equation (9) indicates maintenance mode only can be minimum repair or replacement between task k and $k+1$.

Equation (10) indicates maintenance mode of normal parts can't be minimum repair.

Equation (11) limits the age change of parts between task k and $k+1$.

Equation (12) limits the performance change of parts between task k and $k+1$.

Equation (13) indicates the numerical domain of decision variables is 0-1.

CONCLUSION

According to the deficiency of traditional models, this study establishes an selective maintenance model considering time uncertainty. This model focuses on the influence on maintenance time brought by parts' condition, repaire modes and maintain resources. Consequently, the maintenance decisions are more precise and adaptive. We would probe into the algorithm of this model in the futher.

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