

A Quick Method of Phase Ambiguity Resolution with X-Ray Pulsar Navigation

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Abstract: In this study, we propose a quick resolution method based on space search method which is based on least square method, space search method and ambiguity covariance method. The results of simulation indicate that this method can effectively improve the speed and efficiency of phase ambiguity resolution and has some certain reference value to the researches which is related to X-ray pulsar navigation.

Keywords: Phase ambiguity, TOA, XPNAVS

INTRODUCTION

X-rays Pulsar-based Navigation System (XPNAVS) detects X-rays light, measures the pulse's Time of Arrival (TOA) and abstracts image information of pulse stars by the X-rays detector installed on the spacecraft. Then, through corresponding signal and data processing, it ascertains the time, position and attitude angle of the spacecraft. Compared with Astronomical Navigation, Inertial Navigation and Satellite Navigation, XPNAVS can realize the autonomous navigation for spacecraft in real sense. It can also provide high-precision navigation parameter information for all kinds of spacecraft like near-earth orbit, deep space detector and inter-star craft. XPNAVS will be the new direction for the development of the next generation of spacecraft navigation and in recent years, the relevant technology is the hot-point among the scholars who specialized in this area.

The basic principle of XPNAVS is similar to that of the modern satellite navigation system. As to TOA, there also lies the problem of phase ambiguity. The resolution method can take example by the quick resolution method of GPS wave phase ambiguity measurement. But the measure principle, observation wave band and distance measurement pattern are different between the two navigation systems. Thus, the phase ambiguity of pulse navigation is essentially different from that of GPS satellite navigation. Therefore, how to get phase ambiguity quickly and precisely become one of the problems which must be solved. Abdel-Hafez *et al.* (2003) propose a high-integrity and efficient gps integer ambiguity resolution method. Mao and Song (2009) analyze the X-ray pulsar navigation phase ambiguity resolution. Shuai (2009) study the principle and method of X-rays Pulsar-Based Navigation system. Xu (1995) have a study of the GPS satellite navigation and precise position. Xie *et al.* (2007) have a research of the X-ray pulsars-based spacecrafts' position determination algorithm based on maximum likelihood estimation. Mao and Song (2009) study the spacecraft position

determination with x-ray pulsar navigation of geometric methods.

In this study, we propose a quick method to resolution phase ambiguity. This method can quickly shrink the searching space and improve the search efficiency effectively. What's more it can get the phase ambiguity of pulse star quickly and precisely, thus it realized the positioning of spacecraft.

THE BASIC METHOD OF ASCERTAINING PHASE AMBIGUITY

X-rays pulse stars' pulse cycle is quite stable and it's easy to abstract the outline of pulse, obtain TOA. However, It is hard to assure which pulse cycle summit the TOA that spacecraft measures belongs to. As to observe course t , the total pulse phase can be expressed as:

$$\Phi(t) = N(t_0) + N(t-t_0) + \phi(t) \quad (1)$$

Here, t_0 is the initial observation course; $N(t_0)$ is the total cycle number of pulse phase in the initial observation course; $N(t-t_0)$ is the total cycle number of pulse phase from t_0 to t ; $\phi(t)$ is the decimal fraction part of total cycle number of pulse phase in t .

In the whole process of XPNAVS, $N(t_0)$ is always a constant, as long as the pulse star is observed continuously. Therefore, on the condition that the pulse signal is not lost, the continuous navigation for spacecraft can be realized, as long as $N(t_0)$ can be ascertained.

Nowadays, the main regular methods to determine XNAV phase ambiguity are as follows:

Least-square method: Least square method is based on the observation which is quite precise. It thinks that in all double D-value phase ambiguities, only 3 are independent. The basic idea is that if the 3 double D-value phase ambiguities can be assured, the other double D-value phase ambiguities can be ensured uniquely. Taking the example of observing 4 stars at the same time, the

simplified observation equation which is expressed in the pattern of phase is:

$$\begin{bmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \hat{n}_3 \\ \hat{n}_4 \end{bmatrix} \Delta x = H \Delta x = \begin{bmatrix} cP_1(\Delta\phi_1 + \Delta N_1) \\ cP_2(\Delta\phi_2 + \Delta N_2) \\ cP_3(\Delta\phi_3 + \Delta N_3) \\ cP_4(\Delta\phi_4 + \Delta N_4) \end{bmatrix} \quad (2)$$

In the equation, Δx is the position of spacecraft which uses time model base point as the origin point, $\Delta\phi_i$ is the D-value between the modified observed pulse phase and predicted value of time model, ΔN_i is the D-value between the phase ambiguity of pulse star relative to spacecraft and the phase ambiguity of pulse star relative to the base point, c is the velocity of light, P_i is the pulse cycle, \hat{n}_i is the unit direction from pulse stars to spacecraft.

Space-searching method: Space search method is a method which chooses a series of phase ambiguity combinations to explore whether each combination meets the requirement or not. This method needs to explore each combination one by one to find out the most correct or most suitable phase ambiguity combination and this caused a relatively large calculation. So, the key of this method is to choose suitable search space to reduce the number of phase ambiguity combinations Abdel-Hafez *et al.* (2003). Generally, phase ambiguity search space is an array which uses an observation reference point as the center and spread along the spreading direction of the pulse stars' signals. In order to make sure the precisely suitable point in the array, that is which phase ambiguity combination is the correct, an extra judgment formula is needed. Take the example of 3 pulse stars, the simplified observation equation which is expressed in the pattern of phase is Mao and Song (2009):

$$\begin{bmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \hat{n}_3 \end{bmatrix} \Delta x = H \Delta x = \begin{bmatrix} cP_1(\Delta\phi_1 + \Delta N_1) \\ cP_2(\Delta\phi_2 + \Delta N_2) \\ cP_3(\Delta\phi_3 + \Delta N_3) \end{bmatrix} \quad (3)$$

According to the phase ambiguity search extent which is set by the orbit of spacecraft, based on the current phase ambiguity, the position of spacecraft can be got by choose a row of phase ambiguity to be measured.

When more than 3 stars are observed, the exactness of a certain row of phase ambiguity can be examined using other pulse star measurement. The examined phase ambiguity can be calculated according to formula (4). Here, round stands for get the integer operation:

$$\Delta N_i = \text{round} \left(\frac{\hat{n}_i}{cP_i} \Delta x - \Delta\phi_i \right) \quad (4)$$

The residual error of the examined pulse star can be calculated according to formula (5). When there are more

than 2 stars, a certain threshold can be set according to the measurement error and the exactness of phase ambiguity can be examined using the norm σ_v of residual error vector:

$$v_i = \frac{\hat{n}_i}{cP_i} \Delta x - \Delta\phi_i - \Delta N_i \quad (5)$$

$$\sigma_v = \text{norm}(v) \quad (6)$$

If more than one combination have passed the threshold examine, more stars and measurements can be added for continue to search; take the phase ambiguity combination which has the smallest residual error as the final result.

Ambiguity covariance method: Ambiguity covariance method regards phase ambiguity parameter as a part of position direction and get the ambiguity covariance array which express phase ambiguity parameter by adjustment all the applicable observation value.

According to the principle that the square sum of residual error, the correct phase ambiguity parameter combination can be got Shuai (2009). The searching procedure is actually the procedure to find the phase ambiguity combination to make the square sum of residual error the smallest.

The merit of this method is that they apply all the observation information by the pattern of *approximate optimum* and is optimized in statistics. Therefore, it usually takes the least course to search and make sure the correct phase ambiguity, especially in the application of short distance positioning.

Among the above 3 methods, least square method takes a small and easy calculation, but is relatively more sensitive to observation errors; ambiguity covariance method is not suitable for long distance positioning; search space method is very suitable when there are many observation errors and it is not easy to encounter the condition that no solution is found, but the calculation is too larger. The key to determine XNAV phase ambiguity is to find out a reasonable threshold and to remove some obviously unreasonable phase ambiguity combination by some ingenious ways to reduce the calculation.

ANALYSES OF THE METHOD OF ASCERTAINING PHASE AMBIGUITY

Actually, ascertaining XNAV phase ambiguity is to find the only combination from a large number of possible phase ambiguity combinations by using relative arithmetic. Once the phase ambiguity combination is ascertained, total cycle number plus the phase of the decimal fraction part directly reflects the distance between spacecraft and the reference coordination system.

Suppose $\hat{n}_i = (x_i, y_i, z_i)$, $r_{sat}(X, Y, Z)$ and the observation equation of observing 4 stars at the same time can be expressed as follow:

$$\begin{cases} x_1X + y_1Y + z_1Z = L_1 \\ x_2X + y_2Y + z_2Z = L_2 \\ x_3X + y_3Y + z_3Z = L_3 \\ x_4X + y_4Y + z_4Z = L_4 \end{cases} \quad (7)$$

$$L_i = cP_i(\Delta\phi_i + \Delta N_i) \quad (8)$$

Here, L_i is the distance between pulse star and spacecraft.

If the correct phase ambiguity is already known, any 3 equations from formula (7) can determine the position of spacecraft, so the determining of phase ambiguity is the key of XNAV positioning.

The general way of space search method is: on the condition of not knowing the phase ambiguity, the superior and inferior limit of phase ambiguity search space can be got by position extent information. Then the Eq. (1), (2) and (3) of formula (7) are used to search in the space one by one and get a series of positioning information of spacecraft. Next, it is examined according to the Eq. (4) and figure out the phase ambiguity of the fourth star as well as the residual error, in a certain confidence interval, the possible integer phase ambiguity combination can be got. If the combination is not unique, more stars should be added to do the examination.

Geometrically, the coefficient $\hat{n}_i(x_i, y_i, z_i)$ in formula (7) is known, L_i is a series of arithmetic progression, in the superior and inferior limit of phase ambiguity search space, Eq. (1), (2) and (3) of formula (7) are used to calculate, the result are supposed to be a series of dispersed points and these points spread regularly in the resolution space (that is 3 coordinate components arrange in the pattern of arithmetical series) Besides, geometrically, Eq. (4) can be seen as limited planes which are parallel to each other in the three-dimensional space. Therefore, calculating the residual error using examine pulse star is actually to figure out the shortest distance between the plane series and a known space position point.

IMPROVED METHOD OF ASCERTAINING PHASE AMBIGUITY

Ascertaining of the assemblage of spacecraft position: According to Cramer's rule, Eq. (1), (2) and (3) of formula (7) can be expressed as:

$$X = \frac{\Delta_1}{|A|}, Y = \frac{\Delta_2}{|A|}, Z = \frac{\Delta_3}{|A|} \quad (9)$$

Here:

$$|A| = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} L_1 & y_1 & z_1 \\ L_2 & y_2 & z_2 \\ L_3 & y_3 & z_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} x_1 & L_1 & z_1 \\ x_2 & L_2 & z_2 \\ x_3 & L_3 & z_3 \end{vmatrix}, \Delta_3 = \begin{vmatrix} x_1 & y_1 & L_1 \\ x_2 & y_2 & L_2 \\ x_3 & y_3 & L_3 \end{vmatrix}$$

Known from formula (8), L_i can be divided into 2 parts. The change of phase ambiguity of a pulse star will influence the result. Take L_1 as an example, when ΔN_1 increases by one unit, the increase of X should be:

$$\Delta X_1 = \frac{\begin{vmatrix} cP_1 & y_1 & z_1 \\ cP_2(\Delta\phi_2 + \Delta N_2) & y_2 & z_2 \\ cP_3(\Delta\phi_3 + \Delta N_3) & y_3 & z_3 \end{vmatrix}}{|A|} \quad (10)$$

Meanwhile, ΔY_1 and ΔZ_1 can be got. According to this, the relationship between phase ambiguity and the coordinate change of observation equation can be established, so that the calculation can be simplified. In actual calculation, one row of spacecraft position needs to be figured out, the positions under other phase ambiguity condition can be got quickly through the above relationship and the observing equation need not be calculated repeatedly, thus the calculation is greatly saved.

Inspection method: Inspection the assemblage of the obtained spacecraft positions using the Eq. (4) in formula (7), a position solution can be got which has a relatively small error.

when inspecting, the distance between spacecraft position point and plane can be used as reference and the value can be got through formula (11):

$$d = \frac{|x_4X + y_4Y + z_4Z - L_4|}{\sqrt{x_4^2 + y_4^2 + z_4^2}} \quad (11)$$

when Z and L_4 are confirmed, a series of spacecraft position assemblages can be screened out which meet the threshold and the requirement of shortest distance by combine different X and Y. These assemblages are one quantity level smaller than the original ones (it is related to the superior and inferior limit of phase ambiguity), thus shrink the search extent greatly. In actual calculation,

formula (11) can be further simplified and can properly arrange the searching order according to the phase ambiguity search space, thus the target of reduce calculation can be achieved.

Algorithm procedure: Through the above analyses, the improved XNAV phase ambiguity is summarized as follows:

- Step 1:** Make sure the superior and inferior limit of phase ambiguity searching space using the foregone information or space extent.
- Step 2:** Figure out a row of spacecraft position information using formula (9), under the condition of a given row of phase ambiguity.
- Step 3:** Figure out the position bias caused by the change of phase ambiguity using formula (10)
- Step 4:** Use the smallest dimensionality of phase ambiguity search space as a cycle, use formula (11) to find out the combination which meet a certain threshold and the requirement of the shortest distance and save them to the assemblage to be chosen
- Step 5:** If there are more X-rays pulse star signals, use formula (6) and calculate them one by one in the assemblage to be chosen and save the phase ambiguity combinations which meet a certain threshold in a new assemblage. If there are no such combinations, jump to Step 7.
- Step 6:** If the volume of the assemblage to be chosen is 1, output the phase combination and terminate; if the volume of the assemblage to be chosen is bigger than 1, jump to Step 5.
- Step 7:** Output the phase ambiguity combination which has the smallest residual error.

SIMULATION EXAMPLES

The simulation of 10 pulse stars is adopted in this essay, which have smaller observation error and higher geometrical precision in DOP calculation. The spacecraft is on the orbit which is 27906.9927 km away from the earth center, the coordinate are $X = 5592953.16$ m, $Y = 27201600.44$ m, $Z = 2625894.20$ m, respectively. Phase ambiguity search space is set in a circle which centered in earth and has a radius of 30000 km. The following formula is used to calculate phase ambiguity:

$$cP(\Delta\phi + \Delta N) = L_{\text{MAX/MIN}} \tag{12}$$

Here, c is the velocity of light, $L_{\text{MAX/MIN}}$ is the superior and inferior limit of search space radius, the results are listed in Table 1.

In the simulation, the first 4 pulse stars in Table 1 is used to figure out the position of space craft and the last 6 stars to examine. Using the 5th pulse star to examine, the extent can be shrunk until the final result is got.

In order to further examine the effect of this arithmetic, different ambiguity threshold can be set and the change of assemblages can be inspected. Table 2 gives different number of examine stars and the volume of the corresponding assemblage. Seen from the result, the assemblage shrinks while the number of examine stars increase. When the ambiguity threshold is 0.1, the only result can be got by using 4 stars. The volume of ambiguity threshold also has influence on the assemblage to be chosen, smaller threshold value means quicker examine work, but it may affect the success rate.

Table 1: Phase ambiguity of pulse stars

Pulse star	Pulse cycle (s)	Observation error (m)	Precise ambiguity
B0531+21	0.0330	62.443	430921
B1821-24	0.0030	350.591	1631545
B1937+21	0.0015	487.515	4461334
J0218+4232	0.0023	877.716	1590072
B1509-58	0.1506	3491.446	41305
J0205+6449	0.0656	4605.216	32612
B0833-45	0.0893	4931.036	30298
J1124-5916	0.1353	5219.205	16288
J1811-1925	0.0646	8269.727	83018
B0540-69	0.0504	8449.806	66054
Pulse star	Inferior limit of search space	Superior limit of search space	Number of row
B0531+21	430920	430927	8
B1821-24	1631484	1631551	68
B1937+21	4461227	4461356	130
J0218+4232	1590052	1590139	88
B1509-58	41304	41306	3
J0205+6449	32611	32615	6
B0833-45	30297	30300	4
J1124-5916	16287	16289	3
J1811-1925	83015	83019	5
B0540-69	66052	66057	6

Table 2: The change of the assemblage to be chosen

The number of checked stars	0	1	2	3	4	5	6
Ambiguity threshold 0.1	7785	885	98	13	1	1	1
Ambiguity threshold 0.15	11323	1815	289	54	8	2	1
Ambiguity threshold 0.2	14849	3123	668	158	35	5	1

CONCLUSION

This study proposes a quick method to resolution phase ambiguity. Through simulation, this arithmetic has merits such as quick and precise compared with regular phase ambiguity search arithmetic and it properly overcome the demerits of traditional search method like slow and low precision. This method is influenced by threshold, the success rate become bigger when threshold become bigger.

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