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Application of Probabilistic Analysis in Finite Element Modeling of Prestressed Inverted T-Beam with Web Openings

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Abstract: Recent trends of structural mechanics applications in finite element analysis demonstrate an increasing demand for efficient analysis tools. This paper presents a probabilistic analysis approach applied in finite element analysis for modeling prestressed inverted T-beams with web openings structure used in building service system (mechanical, electrical, communications, and plumbing). The experimental program reported in this paper tested four prestressed inverted T-beams with circular web openings to failure to evaluate the openings' effect on various beam behaviors. Using ANSYS, finite element models were developed to simulate beam deflection behavior. Comparison of analytical results with the available experimental results for load-deflection relationships showed good agreement between both results. Probabilistic analysis methodology could predict the response (i.e., deflection, stress, strain etc) due to various combination of input variables (i.e., Poisson`s ratio, modulus of elasticity, etc). In reality, uncertainties exist in a system and environment that may make the application of deterministic design unreliable which causes the values of the variables that are acting on the system cannot be predicted with certainty. As such, probabilistic approach was applied to the model after deterministic analysis. In this study, the probabilistic analysis approach was applied to account for the variability in fabrication. Probabilistic methodology applied in finite element modeling provides another alternative ways of structural analysis of preststressed inverted T-beams with web openings to achieve a robust and reliable design in a more efficient way. In this study, Monte Carlo simulation was used to analyze the effect of parameter uncertainty for the prestressed inverted T-beams with web openings. From the analysis results, it was observed that the changes in prestressing force, elastic modulus of prestressing steel, ultimate tensile strength of prestressing steel and beam width tend to be the most influencing parameters, which need to be tightly controlled. As a result, from deterministic analysis and probabilistic analysis, it was found that probabilistic analysis tends to be closer to reality than deterministic methods and gives a way of designing for quality.

Key words: Finite element modeling, monte carlo simulation, prestressed inverted t-beams, probabilistic analysis

INTRODUCTION

A precast prestressed inverted T-beam with circular web openings allow building services (mechanical, electrical, communication and plumbing) to cross the beam, reducing a building`s floor to floor height and overall height of the structure. Those height reductions have a potential to improve the competitiveness of total precast concrete structures versus other type of building system. However, introducing an opening into the web of a prestressed concrete beam reduces stiffness and leads to more complicated behavior. Therefore, the effect of openings on strength and service ability must be considered in the design process. Numerous investigations such as (Mansur, 1988) have been carried out on reinforced concrete beams with opening. The first published work on prestressed beams with web openings was conducted by (Regan and Warwaruk, 1967). Since then, several other researchers (Sauve, 1970; Kennedy and Abdalla, 1992) have investigated prestressed beams with web openings. Based on their researches, the design procedures for prestressed concrete beams with web openings were developed. While they have not considered simplifications and probabilistic response of prestressed beams with web openings, therefore an experimental and analytical study on effect of opening and strength of the prestressed inverted T-beam with web openings is required to provide data and probabilistic analysis which will be used in the design guideline of the beams. Hence, an understanding of the structural behavior of inverted prestressed T-beam with circular web opening under influence of locations and size of the openings, the effect of multiple web opening in both constant moment and constant shear region as well as probabilistic analysis are useful to provide data which could be helpful to develop satisfactory design guideline of the beams into use.

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Uncertainty exists in most engineering structures, such as bridges and buildings which are essential parts of our modern lives. Uncertainty of structural parameters may be resulted from manufacturing, construction tolerances or caused by the aggressive environmental agents such as progressive deterioration of concrete and corrosion of steel. In engineering applications, uncertainty also exists in determining external loads, such as wind forces, earthquake motions and automobiles on highways. Nondeterministic analysis of engineering structures with uncertainties in system parameters and inputs has been attracted considerable attention in the last two decades (Chen *et al*., 1992; Rao and Berke, 1997; Ma *et al*., 2006; Falsone and Ferro, 2007; Stefanou, 2009). Usually probabilistic analysis is used to deal with the uncertainty. In probabilistic methods, uncertain parameters are modeled as random variables and uncertainties of loads are described by random processes. As very useful tools, probabilistic methods have been widely used to predict the static, dynamic and random responses of structural systems with uncertainty (Stefanou and Papadrakakis, 2004; Val and Stewart, 2005; Gao and Kessissoglou, 2007; Singh *et al*., 2009). These methods can provide not only the mean value but also the standard deviation and even the probability density for structural responses. Monte Carlo simulation method (Figiel and Kaminski, 2009; Yu *et al*., 2010) has been developed to analyze random data structures. The probabilistic methods, however, are only applicable when information about an uncertain parameter in the form of a preference probability function is available. If the uncertain information or statistical data of system parameters are not sufficient to give their satisfactory stochastic characterization, only the non-probabilistic approaches can be applied (Falsone and Ferro, 2007). On the contrary, if it is possible to characterize the uncertain parameters stochastically, then the probabilistic method are the most suitable ones. In this study, the probabilistic analysis approach is applied to account for the variability in fabrication. Probabilistic methodology applied in finite element analysis (Fazilat *et al*., 2002; Giuseppe, 2011; Taejun and Tae, 2008) provides another alternative ways of structural analysis of prestressed inverted T-beams with web openings to achieve a robust and reliable design in a more efficient way.

METHODOLOGY

Probabilistic design is an analysis technique for assessing the effect of uncertain input parameters and assumptions on deterministic model. In this study, ANSYS probabilistic design system help to determine the extent to which uncertainties in the model affect the results of a finite element analysis. An uncertainty is a parameter whose value is impossible to determine at a given point in time (if it is time-dependent) or at a given location (if it is location-dependent) (SAS, 2009).

Monte Carlo simulation and Latin hypercube simulation: Monte Carlo Simulation is the most common and traditional method for a probabilistic analysis. A fundamental characteristic of the Monte Carlo Simulation method is the fact that sampling points are located at random locations in the space of the random input variables. There are various techniques available in literature that can be used to evaluate the random locations of the sampling points (Hammersley and Handscomb, 1964). The direct Monte Carlo simulation and Latin Hypercube Sampling are two approaches. The direct Monte Carlo Simulation method is not used because its random sampling has no memory. Instead of that the Latin Hypercube Sampling technique is implemented and the range of all random input variables is divided into *n* intervals with equal probability.

Simulation is the process of replicating the real world phenomenon based on a set of assumptions and conceived realistic models. It may be performed theoretically or experimentally. For engineering purpose simulation may be applied to predict or study the performance or response of a structure. With a prescribed set of values for the design variable, the simulation process yields a specific measure of performance or response. A conventional approach to this process is Monte Carlo simulation technique. However, in practice, Monte Carlo simulation may be limited by constrains, computer capability, and significant expense of computer runs in such complex structural system as prestressed inverted T-beam with web openings. An alternative approach is to use a constrained sampling scheme. One such scheme developed by (Iman and Conover, 1980; Iman *et al.*, 1981a; Iman *et al*., 1981b) was Latin Hypercube sampling (LHS) method. By sampling from the assumed probability density function of *X`s* and evaluating *Y* for each sample, the distribution of *Y*, its mean, standard deviation, percentiles and so on, can be estimated.

The LHS methods consist of two steps to obtain a *N* design matrix. The first step is dividing each input variable into *N* intervals. The second step is the coupling of input variables with tables of random permutations of rank numbers. Every input variable, X_k, (where, k =1, 2, K) is described by its known Cumulative Distribution Function (CDF), $Fx_k(x)$ with appropriate statistical parameters. The range of the known CDF, $Fx_k(x)$ of each input variable, X_k is partitioned into *N* intervals with equal probability of 1/*N*.

The representative value in each interval is used just once during the simulation procedure. Therefore there are *N* observations on each of the *K* input variables. They are ordered in the table of random permutations of rank numbers which have *N* rows and *K* columns. For such a

sample, one can evaluate the corresponding value Y_n of the output variable. From N simulations, one can obtain a set of statistical data. $\{Y\} = [Y_1, Y_2, Y_n]^T$. This set is statistically assessed and thus the estimations of some statistically parameters, such as the mean value and standard deviation, are obtained. A more detailed discussion of this sampling method can be found in the papers by some researchers (Nov *et al*., 1998; Iman and Conover, 1982).

Monte Carlo simulation: Monte Carlo Simulation (MCS) results established prestressed inverted T-beam response statistics. Three graphical tools are utilized to determine a best-fit distribution model for each response: probability distribution, empirical Cumulative Distribution Function (CDF), and histogram. Confidence Interval (CI) with 95% confidence level and p-value were also included in the probability distribution. CI represents the intervals covering the estimation. p-value represents the probability of obtaining an observed result and its significance level. The smaller the p-value, the more significant the results are.

Considering various distribution types, a best-fit distribution for each random input variable was determined. The input variable, however, were fit using two distribution types: normal (Gaussian) and lognormal distribution. A normal distribution is represented by mean (μ) and standard deviation (σ) to describe the distribution shape. PDF and CDF for the normal distribution are:

Probability Density Function (PDF) equation:

$$fx(x) = \frac{1}{\sigma\sqrt{2}}\exp\left(-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2\right)(-\infty < x < +\infty) \qquad (1)$$

CDF:

$$Fx(x) = \int_{-\infty}^{x}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2\right)dx \qquad (2)$$

where,
μ = mean value; σ = standard deviation.
PDF and CDF for the lognormal distribution is:

PDF:

$$f_x(x) = \frac{1}{x\sigma_y\sqrt{2}}\exp\left(-\frac{1}{2}\left[\frac{\ln x}{\sigma y}\right]^2\right)(0 < x < +\infty) \qquad (3)$$

CDF:

$$F_x(x) = \int_{-\infty}^{x}\frac{1}{x\sigma y\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left[\frac{\ln x-\mu y}{\sigma y}\right]^2\right)dx \qquad (4)$$

where,

μx = Mean value; $\sigma^2 = \ln\left[1+\left(\frac{\sigma_x}{\mu_x}\right)^2\right]$

and $\mu_y = In(\mu_x) - \frac{1}{2}\sigma_y^2$

It is convenient to describe location, and scale parameters for lognormal distribution. Location and scale have a relationship with mean and standard deviation:

$$Location = 2In(\mu) - \frac{In(\mu^2+\sigma^2)}{2} \qquad (5)$$

$$Scale = \sqrt{In(\mu^2+\sigma^2) - 2In(\mu)} \qquad (6)$$

where,
μ = The desired mean of the lognormal data
σ = The desired standard

Deviation of the lognormal data from Eq. (6), μ and σ can be derived from *location* and *scale* parameters as:

$$\mu = \exp\left(location + \frac{scale^2}{2}\right) \qquad (7)$$

$$\sigma = \exp[2(location + scale^2)] - \mu^2 \qquad (8)$$

Sensitivity analysis: An important step in the probabilistic structural analysis is the sensitivity analysis of structural responses. This helps identify the important parameters. On other hand, sensitivity analysis is also useful in reducing the size of problems with a large numbers of random variables. This is because that, in general, only a few variables have a significant effect on the probabilistic structural response. In this paper, the results of the Latin Hypercube simulations can be used to determine which of the model parameters are most significant in affecting the uncertainty of the design. The so-called Spearman rank-order correlation r_s are frequently applied within the framework of a simulation method (Oh and Yang, 2000). The Spearman rank-order correlation can be defined as:

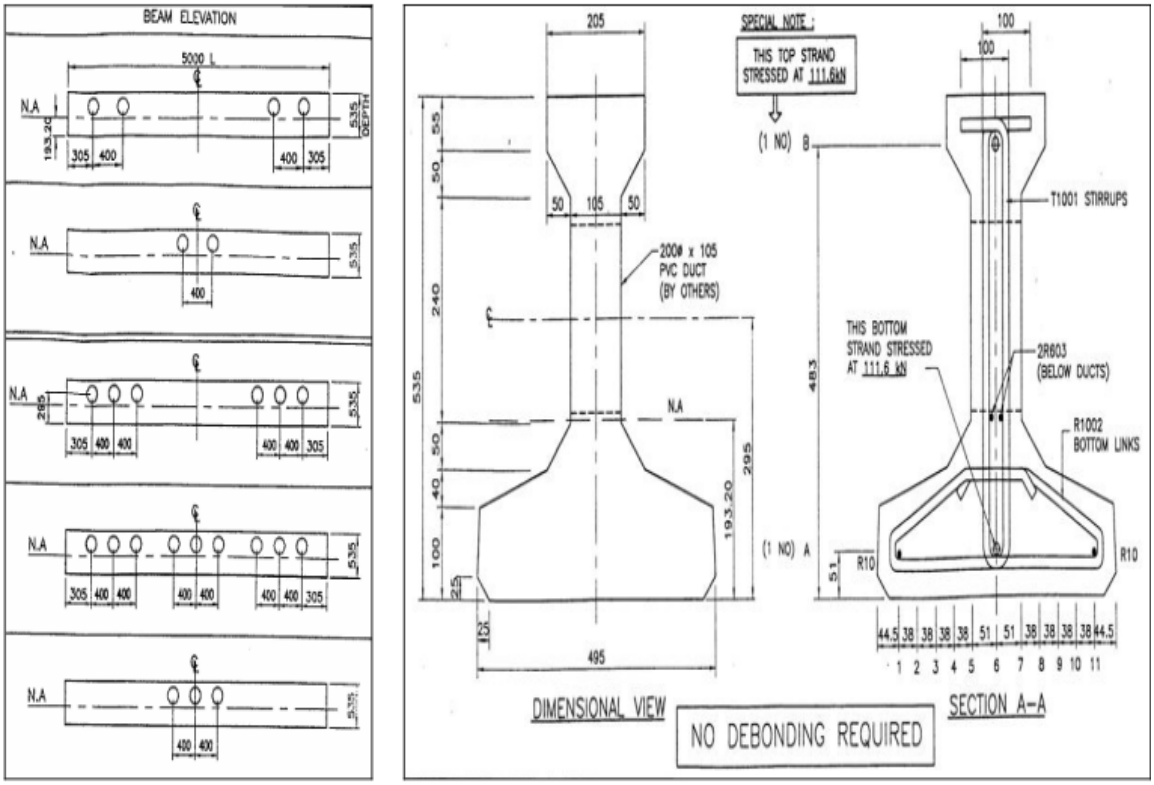

Fig. 1: Configuration of web openings inside the prestressed inverted T-beam and inverted T-beam overall detailed dimensions (Cheng *et al*., 2009)

$$r_i = 1 - \frac{6\Sigma_j (m_{ji} - n_j)^2}{N(N^2 - 1)}, r_i \in [-1,1] \quad (9)$$

where r_i is the order representing the value of random variable X_i in an ordered sample among N simulated values applied in the j th simulation (the order m_i equals the permutation at LHS); and n_j is the order of an ordered sample of the resulting variable for the j th run of the simulation process (m_{ij} - n_j is the different between the ranks of two samples). If the coefficient r_i had a value near to 1 or -1, it would suggest a very strong dependence of the output on the input. Opposite to this, the coefficient with its value near to zero will signalize a low influence. The sensitivity coefficients k_i, defined on behalf of variation coefficients by the relation:

$$k_i = 100\frac{\upsilon_{yi}^2}{\upsilon_y^2}[\%] \quad (10)$$

where v_{yi} is the variation coefficient of the output quantity, assuming that all the input quantities except the ith one (i = 1, 2..., M) where, M is number of input variables) are considered to be deterministic (during the simulation, they are equal to the mean value); v_{yi} is the variation coefficient of the output quantity, assuming that all the input quantities are considered to be random ones.

Based on Eq. (9) and (10) the random input variables that have significant effects on the output variables can be identified. It should be pointed out that the correlation coefficients, defined in Eq. (9) and (10), have been used to evaluate the probabilistic sensitivities in the ANSYS Probabilistic Design System.

Experiment program: The experimental program was designed to investigate the failure behavior of prestressed inverted T-beams with circular web openings under static loading conditions. Testing was intended to evaluate the flexural strength. Therefore, all beams were designed such that shear strength exceeded flexural strength. Accordingly, flexural failure was expected. Figure 1 and Fig. 2 show the configuration of web openings in the prestressed beams as well as cross-section prestressing steel and shear reinforcement detailing. The materials used, design and fabrication of prestressed inverted T-beams are described below (Cheng *et al*., 2009). The experimental program was carried out at the Hume

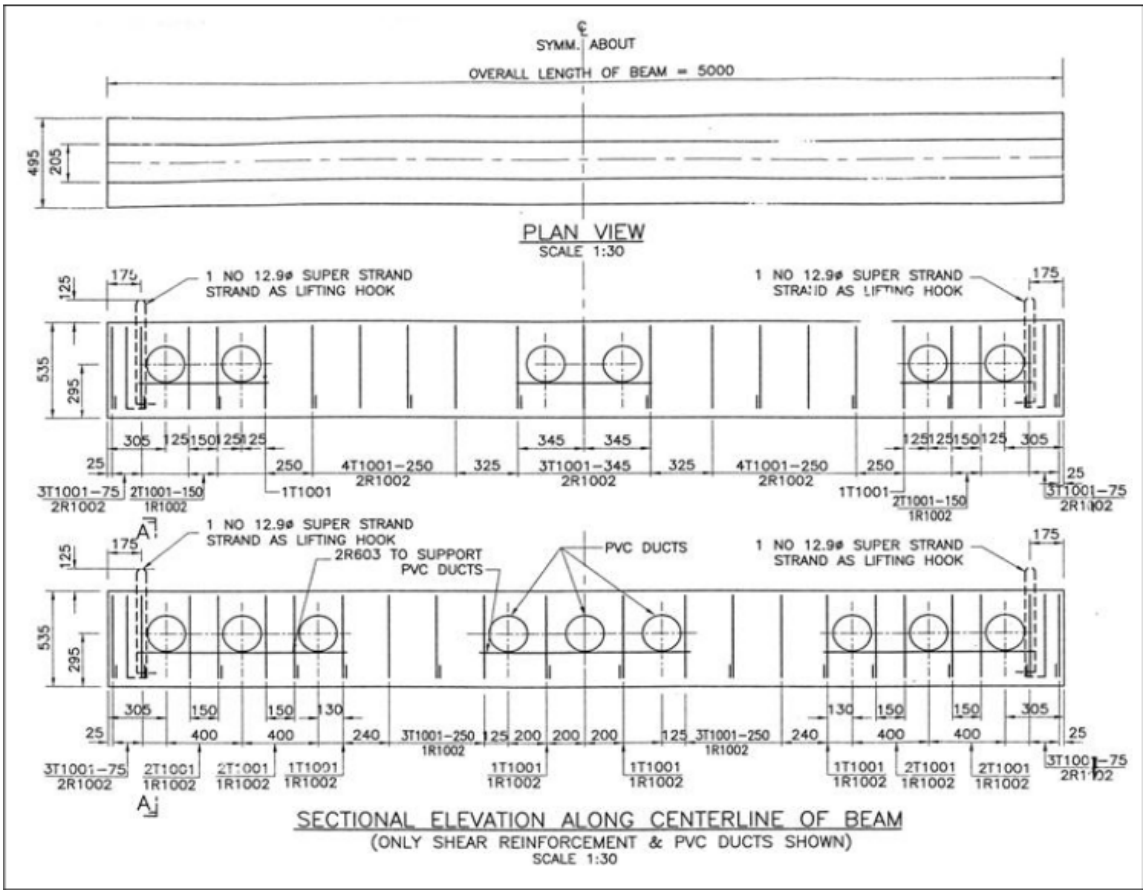

Fig. 2: Detailing and location of prestressing steels and shear reinforcements (Cheng *et al*., 2009)

Fig. 3: Prestrssing steel and shear reinforcement locations (Cheng *et al*., 2009)

Concrete Product Research Centre (HCPRC) and Tenaga Nasional Research Berhad (TNBR) for testing facilities and analysis assistance of prestressed beam with web openings which started from 2009 to 2010. The extended work of this research study, finite element modeling and probabilistic analysis, was studied at University Malaysia Pahang 2011.

Materials: The average 28 days concrete cube strength in compression was 55 MPa for all four beams, as evaluated by tests on three cubes specimens for each beam. Figure 3 show the prestressing steel and shear reinforcement location in the prestressed concrete beams. Straight, 3 show the prestressing steel and shear reinforcement location in the prestressed concrete

Fig. 4: Prestressed inverted T-beams with circular web openings fabricated at factory (Cheng *et al*., 2009)

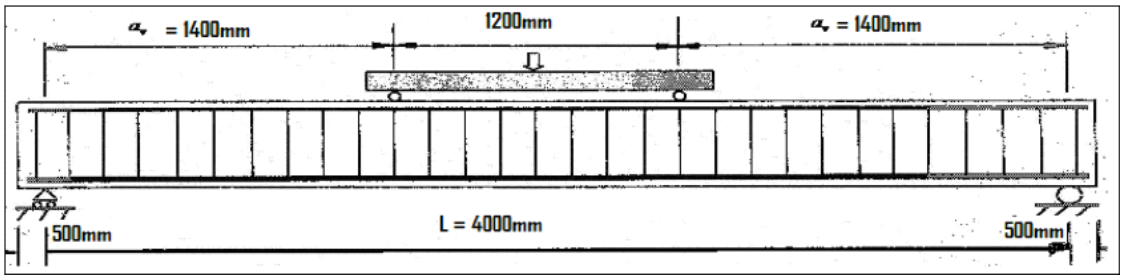

Fig. 5: Four points bending test of prestressed inverted T-beam (Cheng *et al*., 2009)

beams. Straight, bonded, seven-wired super high tensile strand with 12.9 mm diameter were used as prestressing tendons, with ultimate strength of 1,860 MPa. The stirrups for shear reinforcement were made from 10 mm rebars with minimum specified yield strength of 250 N/mm^2.The elastic modulus of the prestressing steel is taken as 195 10^3 N/mm^2. Figure 4 shows the fabricated prestressed beams used in the test program.

Testing set-up: All tests were conducted with a close-loop hydraulic servo-controlled MTS testing system. The 360 KN jack was capable of both displacement and load control for monotonic or cyclic loading. A four-point loading scheme, with an effective span of 4,000 mm and a distance of 1,200 mm between the loads points was used to limit the presence of shear stress in the mid-span zone. Figure 5 shows the layout of four points bending test of prestressed inverted T-beams (Cheng *et al*., 2009).

Finite element models: In this study, the finite element analysis of the model was set up to examine three different behaviors: initial cracking of the beam, yielding of the steel reinforcement, and the strength limit state of the beam. The Newton-Raphson method of analysis was used to compute the nonlinear response. The application of the loads up to failure was done incrementally as required by the Newton-Raphson procedure. After each load increment was applied, the restart option was used to go to the next step after convergence. The two convergence criteria used for the analysis were force and displacement.

Element types: Concrete part was modeled using a three-dimensional solid element, SOLID65, which has the material model to predict the failure of brittle elements. SOLID65 is defined with eight nodes each with three degrees of freedom: translations in nodal x, y and z directions. This element is capable of cracking in tension and crushing in compression. Plastic deformation and creep can also be captured. The cracking is determined by the criterion of maximum tensile stress, called `*tension cutoff*`. Concrete crushes when the compressive principal stress (*Von Mises* stress) on the failure surface surpasses the *Willam-Warnke* failure criterion dependent on five materials parameters (Willam and Warnke, 1974). The SOLID 45 element was used for the supports for the beam. This element has eight nodes with three degrees of freedom at each node with translations in the nodal x, y and z directions. To simulate the behaviors of prestressing steel, a truss element, LINK8, were used to withstand the initial strain attributed to prestressing forces, by assuming perfect bond between these elements and concrete. LINK8 requires users to input `real constants` to define reinforcement geometry, material behavior, and prestressing strain. Note that this truss element cannot resist neither bending moments nor shear forces. The

descriptions for each element type were laid out in the ANSYS element library (SAS, 2009).

Material properties:

Concrete: The SOLID65 element requires linear isotropic and multi-linear isotropic materials properties to properly model concrete. Concrete is a quasi-brittle material and has very different behaviors in compression and tension. The tensile strength of concrete is typically 8-15% of the compressive strength. The ultimate concrete compressive and tensile strengths for each beam model were calculated by Eq. (11) and (12), respectively (ACI Committee 318-99, 1999).

$$f_c' = \left(\frac{E_c}{4730}\right)^2 \tag{11}$$

$$f_r = 0.623\sqrt{f_c'} \tag{12}$$

where,

- E_c = Elastic modulus of concrete
- f_c' = Ultimate compressive strength
- f_r = Ultimate tensile strength (modulus of rupture)

The following Eq. (13) and (14) (Desayi and Krishnan, 1964) are used along with Eq. (15) to construct the uniaxial compressive stress-strain curve for concrete in this study:

$$f = \frac{E_c \varepsilon}{1+\left(\frac{\varepsilon}{\varepsilon_0}\right)^2} \tag{13}$$

$$\varepsilon_0 = \frac{2 f_c'}{E_c} \tag{14}$$

$$E_c = \frac{f}{\varepsilon} \tag{15}$$

where,

f = Stress at any strain ϵ
ϵ = Strain at stress f
E_c = Concrete elastic modulus
ϵ_0 = Strain at the ultimate compressive strength f_c'

strength. After this point, the concrete cracks and the strength decreases to zero. Figure 6 shows the simplified uniaxial stress-strain relationship that is used in this study.

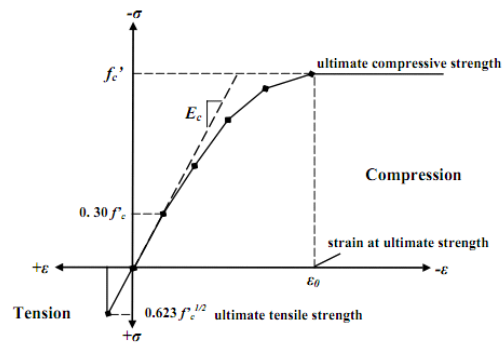

Fig. 6: Simplified uniaxial stress-strain curve for concrete

In tension, the stress-strain curve for concrete is assumed to be linearly elastic up to the ultimate tensile Poisson`s ration for concrete was assumed to be 0.3 and was used for all beams. The value of a shear transfer coefficient, representing conditions of the crack face, used in many studies of reinforced concrete structures varied between 0.05 and 0.25 (Bangash, 1989; Hemmaty, 1998; Huyse *et al*., 1994). The shear transfer coefficient used in this study is equal to 0.2.

Steel reinforcement and prestrssing steel: Steel reinforcement in the experiment beams was constructed with typical steel reinforcing bars f_y = 1000 MPa. Elastic modulus and yield stress for the steel reinforcement used in this FEM study follow the design material properties used for the experimental investigation. The steel for the finite element models is assumed to be an elastic-perfectly plastic material and identical in tension and compression. A Poisson`s ratio of 0.3 is used for the steel reinforcement.

The SOLID 45 element is being used for supports on the beam. Therefore, this element is modeled as linear isotropic element with a modulus of elasticity for the steel E_s = 1000 MPa and Poisson`s ratio v = 0.3. The LINK8 element is being used for all the steel reinforcement in the beam and it is assumed to be bilinear isotropic. Bilinear isotropic material was also based on the *Von Mises* failure criteria.

For prestressing steel, the bi-linear elastic-plastic material models can be used as well as the multi-linear isotropic model from the manufacture`s data. In the present study a multi-linear isotropic stress-strain relationship is adopted for prestressing strands by using Eq. (16) or (17) (Mohamad and Akram, 2010):

$$\varepsilon_{ps} \le 0.008: \qquad f_p = 28{,}000\,\varepsilon_{ps} \tag{16}$$

where:

ϵ_{ps} = Strain of prestressing steel

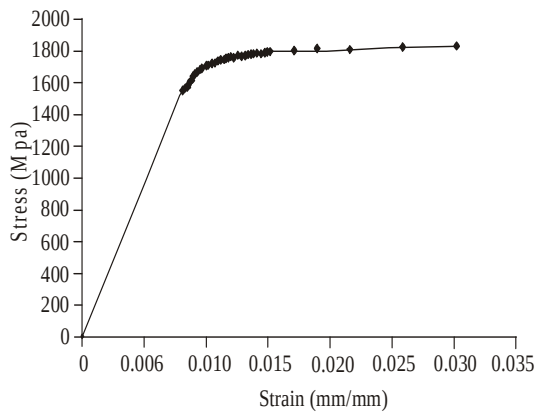

Fig. 7: Stress-strain curves of steel strands φ12.7mm) () (Mohamad and Akram, 2010)

$$\varepsilon_{ps} > 0.008, \quad f_p = 268 - \frac{0.075}{\varepsilon_{ps} - 0.0065} < 0.98 f_p \quad (17)$$

where,

- ϵ_{ps} = Strain of prestressing steel
- f_p = Ultimate strength of prestressing steel

The ultimate stress f_p =1, 860 MPa and the elasticity of modulus of prestressing steel E_p = 195,000 MPa. Poisson`s ration for prestressing steel was also assumed to be 0.3. The adopted stress-strain curve is show in Fig. 7. The initial prestressing strain is taken equal to 0.006 mm.

Analytical model: The purpose of the analysis was to obtain the load-deflection curve of prestressed beams. The

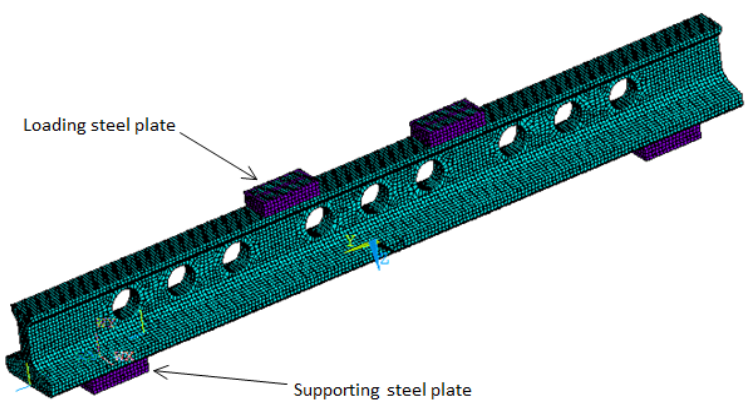

Fig. 8: FEA model of prestressed inverted T-beam with web opening

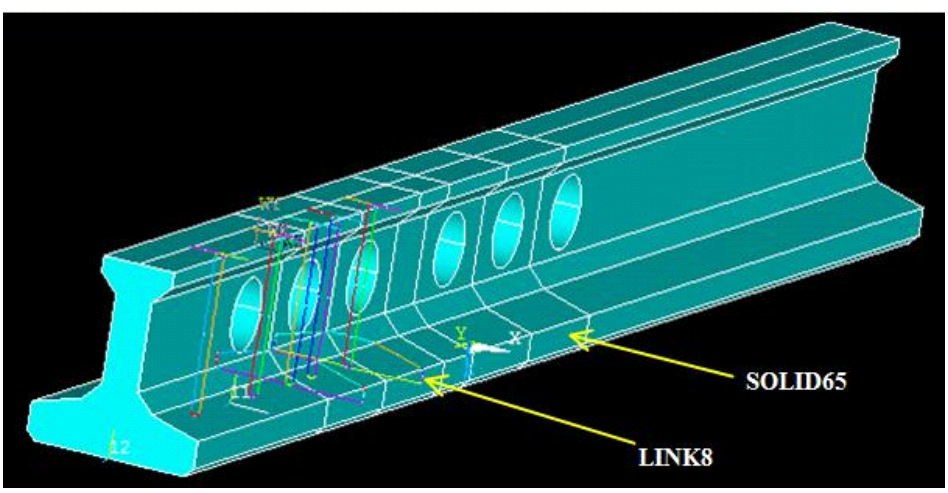

Fig. 9: FEA model of iso-viewing of tendon and stirrup

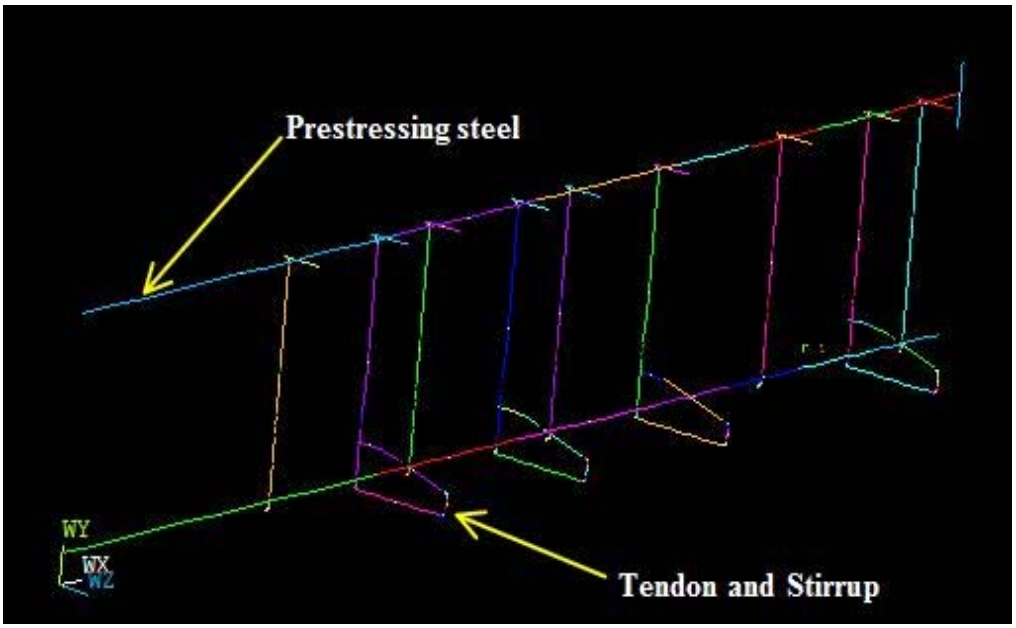

Fig. 10: FEA model of 1/4 beam of tendon and stirrup

beam geometry was modeled and meshed, then the tendons were defined and finally the loads area applied to it. The analyzed beam was 5 m long and had various different types openings configurations along it. It was prestressed by 2 straight tendons localized on the top and bottom sides of it. Figure 8 to 10 show the finite element models for the prestressed inverted T-beam.

Due to the symmetry in cross-section of the prestressed concrete beam and loading, symmetry was utilized in the finite element analysis; only one quarter of the beam was model. This approach reduced computational time and computer disk space requirements significantly. The steel reinforcement is simplified in the model by ignoring the inclined portion of the steel bar present in the test beams. Ideally, the bond strength between the concrete and steel reinforcement should be considered. However, in this study, perfect bond between materials is assumed.

Numerical results: The experimental and numerical load-deflection curves obtained for the beams are illustrated in Fig. 11 to 14. The load-deflection plots for all four beams from the finite element analysis agree quite well with the experimental data. First cracking loads for all four models from the finite element analysis are higher than those from experimental results by 3-8%. After first cracking, the stiffness of the finite element models is agreed well with the experimental beams by 1-2% differences. There are several factors that may cause the higher stiffness in the finite element models at the initial loading periods. Micro-cracks produced by drying shrinkage and handling are present in the concrete to some degree. These would reduce the stiffness of the actual beams, while the finite element models do not include micro-cracks. Perfect bond between the concrete and steel reinforcing is assumed in the finite element analysis, but the assumption would not be true for the actual beams. When bond slip occurs, the composite action between the concrete and steel reinforcing is lost. The overall stiffness of the actual beams could be lower than what the finite element models predict, due to the factors that are not incorporated into the models.

Results from the non-linear finite element analysis were compared to those from the test on the four prestressed inverted T-beam with web openings as well as solid beam. The stress and strain distributions around the opening were determined. The influence of the opening on deflection, cracking and shear distribution was studied. Figure 11 to 14 show a typical comparison between the theoretical and experimental test results for the deflection in the vicinity of web openings in prestressed beam, when the external vertical load is applied at web openings located at constant moment area as well as other locations. The agreement between experiment and finite element analysis is acceptable in all cases.

From the experimental results of beam with 9 web openings analysis data, the maximum value of deflection is 37.9 mm, maximum *Von Mises* strain is 71.4 $\mu\epsilon$ and maximum *Von Mises* stress is 131.733 KN have a quite small difference with solid beams analysis data. Meanwhile, as shown in Table 1, the deterministic analysis data also showed a quite difference between

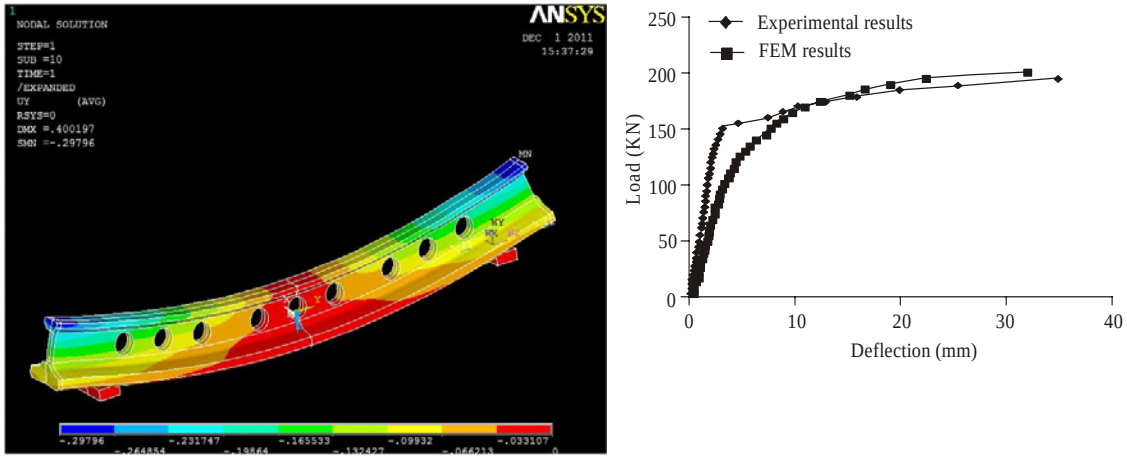

Fig. 11: FE modeling and load-deflection plots for beam with nine web openings

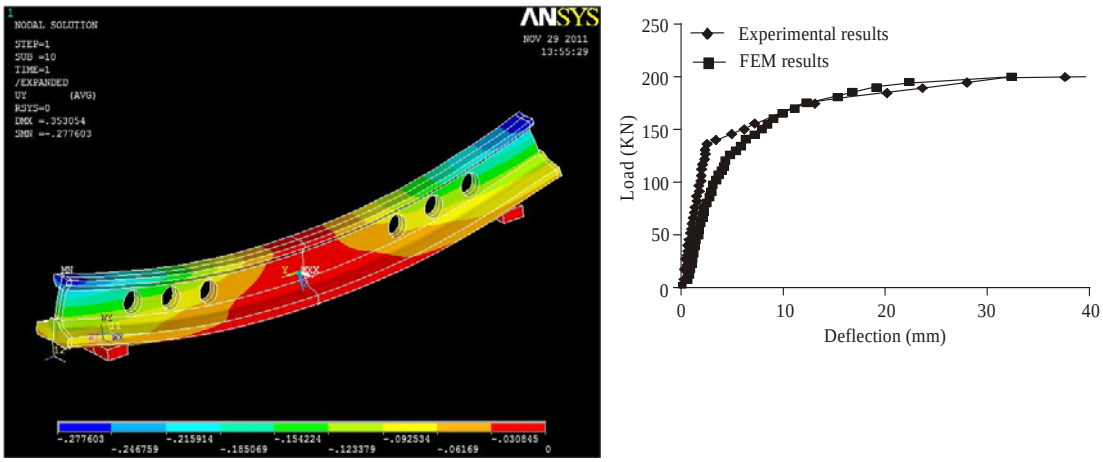

Fig. 12: FE modeling and load-deflection plots for beam with six web openings

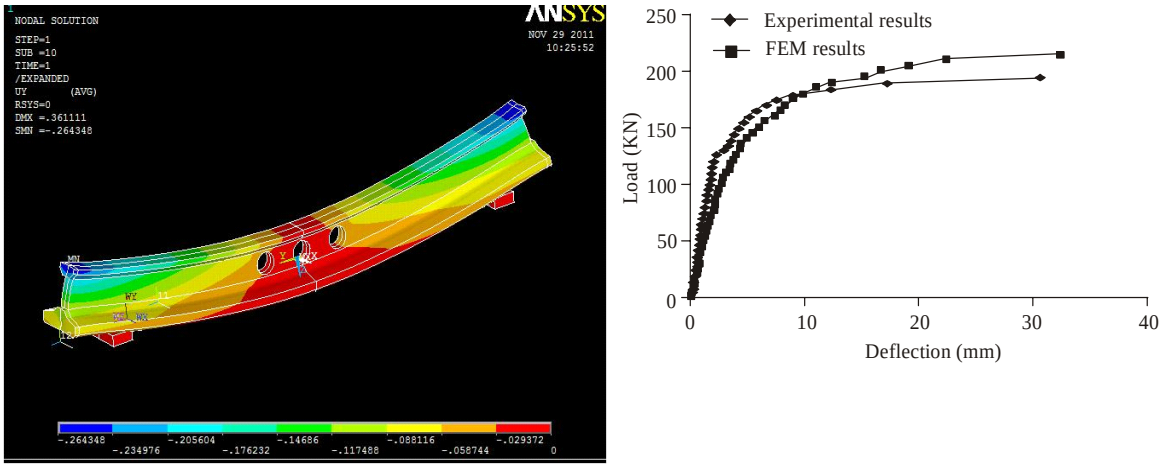

Fig. 13: FE modeling and load-deflection plots for beam with three web openings

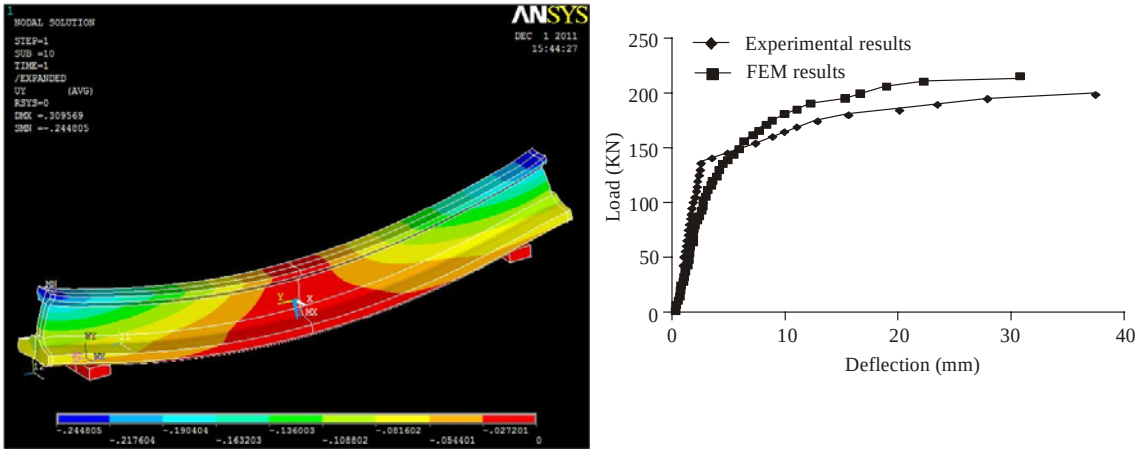

Fig. 14: FE modeling and load-deflection plots for solid beam

Table 1: Comparison of FEM results and experimental results of prestressed beams with different number of web openings

Parameter	Deterministic analysis maximum value	Experimental data maximum value
Beam with 9 web openings		
Deflection (mm)	40	37.9
Von Mises strain (με)	68.1	71.4
Von Mises stress (KN)	123.776	131.733
Beam with 6 web openings		
Deflection (mm)	35.3	35.1
Von Mises strain (με)	68.2	68.2
Von Mises stress (KN)	123.966	125.830
Beam with 3 web openings		
Deflection (mm)	33.8	30.3
Von Mises strain (με)	68.1	66.3
Von Mises stress (KN)	123.859	122.324
Solid beam		
Deflection (mm)	31	35.7
Von Mises strain (με)	68.2	68.4
Von Mises stress (KN)	123.979	125.829

numbers of web openings and solid beam. In consequence of data result from Table 1, the numbers of web openings and solid beam were almost having identical strength property.

Probabilistic analysis design: In probabilistic design system, all these input parameters could be anything ranging from geometry and material properties to different boundary conditions. These parameters are defined as random input variables and are characterized by their distribution types and variables (e.g., mean, standard deviation, etc). The key outputs of the simulation are defined as random output parameters. During a probabilistic analysis, multiple analysis loops are executed to compute the random output parameters as a function of the set of random input variables.The values for the input variables are generated randomly using Monte Carlo simulation.

To illustrate the application of the probabilistic method, four group of experiment prestressed beam with numbers of web opening as describle in experiment program previously was analyzed. The geometric and sectional parameters of the pretressed beam and layout parameters of the prestressing steel are assumed to deterministic since they are relatively predictable in comparison to other parameters such as material strength. The applied load (P), elastic modulus of concrete (E_c), elastic modulus of prestressing steel (E_p), beam depth (l), beam width (b), prestressing force (P_s), Poisson`s ratio ($v$), ultimate compressive strength of concrete ($f_c^{`}$) and ultimate tensile strength of prestressing steel (f_p) were chosen as the random input variables of interest for this study. The statistical descriptions of these random variables are shown in Table 2.

Probabilistic analysis results: As the objective of this research was to study probabilistic response analysis of pretressed concrete beam with web openings, all random parameters in the analysis were based on arbitrary but

Table 2: Statistical analysis of various input random variables for probabilistic design

Random variables	Distribution types	Mean value	SD	Coefficients of variation
P (N)	Gaussian	100000	10000	10%
E_c (Mpa)	Gaussian	45000	6750	15%
E_p (Mpa)	Gaussian	195000	9750	5%
l (mm)	Gaussian	523	10.46	2%
b (mm)	Gaussian	485	9.7	2%
P_s (N)	Lognormal	111600	5580	5%
v	Lognormal	0.3	0.015	5%
$f_c^{`}$ (MPa)	Gaussian	55	9.25	15%
f_p (MPa)	Gaussian	1860	93	5%

Table 3: Statistical analysis of three output parameters

Name	Mean	SD	Min.	Max.
Beam with 9 web openings				
MAXIMUM_DEFLECTION (mm)	40	1.6	35.7	44.8
MAX_VON_MISES_STRAIN (με)	68.8	7.7	54.6	100
MAX_VON_MISES_STRESS(KN)	123.8	6.332	107	142.1
Beam with 6 web openings				
MAXIMUM_DEFLECTION (mm)	35.3	1.7	31.5	40.3
MAX_VON_MISES_STRAIN (με)	68.8	8	53	97
MAX_VON_MISES_STRESS(KN)	123.8	6.271	109.3	143.1
Beam with 3 web openings				
MAXIMUM_DEFLECTION (mm)	33.8	1.6	30.1	38.7
MAX_VON_MISES_STRAIN (με)	68.8	8	49.2	92.5
MAX_VON_MISES_STRESS(KN)	123.9	6.238	109.3	143.4
Solid beam				
MAXIMUM_DEFLECTION (mm)	31	1.7	26.7	38.3
MAX_VON_MISES_STRAIN (με)	69	8.4	50.2	100
MAX_VON_MISES_STRESS(KN)	124.1	6.670	105.8	152

Table 4: Comparative studies of results obtained from deterministic and probabilistic analysis

Output variables	Deterministic analysis maximum value	Probabilistic analysis maximum value
9 web openings		
Deflection (mm)	40	44.8
Von Mises strain (με)	68.1	100
Von Mises stress (KN)	123.776	142.1
6 web openings		
Deflection (mm)	35.3	40.3
Von Mises strain (με)	68.2	97
Von Mises stress (KN)	123.966	143.1
3 web openings		
Deflection (mm)	33.8	38.7
Von Mises strain (με)	68.1	92.5
Von Mises stress (KN)	124.038	143.4
Solid beam		
Deflection (mm)	31	38.3
Von Mises strain (με)	68.2	100
Von Mises stress (KN)	123.979	152

typical values. On the other hand, since the determination of the correlation of the random parameters is a difficult task, using the independence assumption cans greatly simply the probabilistic analysis. Therefore, all random parameters in the paper were treated as stochastically independent from each other. Unless stated otherwise, the statistics of random variables given in Table 2 were used in the present studies.

For the probabilistic analysis, the ANSYS probabilistic design system analysis was looped through 1000 sample points considering the variations defined in the input variables, and the corresponding statistical analysis of the output parameters are given in Table 3.

In this research, the three output parameters value lists in Table 3 which obtained from using probability analysis have a differences value between deterministic analyses. This differences value was examined and tabulated in Table 4. For example, as the specimen of 9 web openings, the maximum deflection value is 40 mm and maximum *Von Mises* stress is 123.776 KN of deterministic results with probabilistic results the maximum deflection value is 44.8 mm and maximum *Von Mises* stress is 142.1 KN, probabilistic results are higher than deterministic ones. So from the results from Table 4, the approach of probabilistic analysis is relatively higher than deterministic analysis.

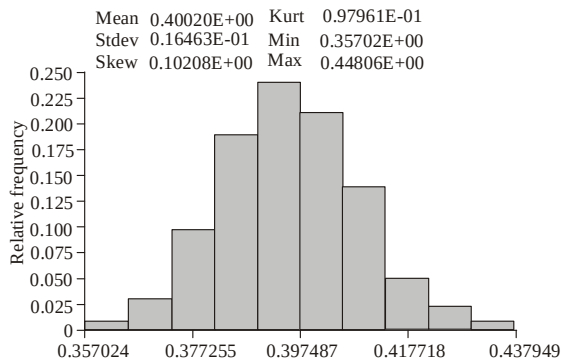

Fig. 15: Histogram of maximum deflection (beam with 9 web openings)

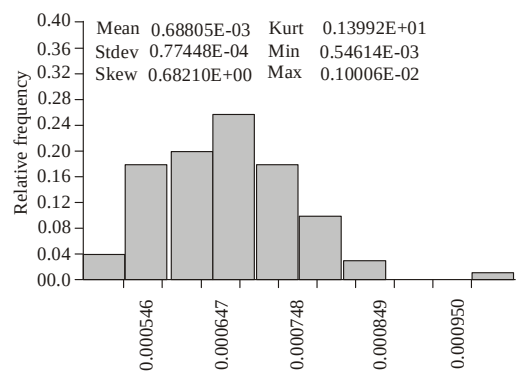

Fig. 16: Histogram of maximum *Von Mises* strain (beam with 9 web openings)

Probabilistic post-processing results of beam with 9 web opening specimen: Figure 15 to 17 graphically depicts a histogram of each output parameters. The values

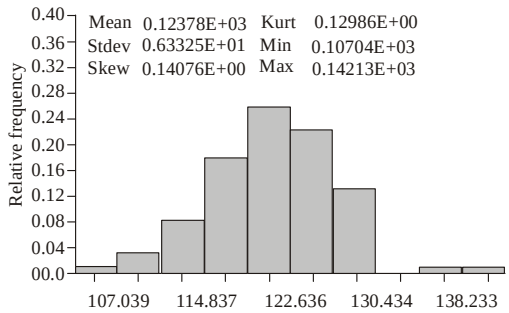

Fig. 17: Histogram of output variable maximum *Von Mises* stress (beam with 9 web openings)

given on each distribution plot were mean value, standard deviation, skewness, kurtosis, minimum value and maximum value, respectively.

Descriptions below provide some technical statistics information on the coefficient of skewness and coefficient of kurtosis which has been broadly used in this research analysis.

Coefficient of skewness, a measure of the asymmetry of the probability distribution of a real-valued random variable. The skewness value can be positive or negative, or even undefined. Qualitatively, a negative skew indicates that the tail on the left side of the probability density function is longer than the right side and the bulk of the values lie to the right of the mean. A positive skew indicates that the tail on the right side is longer than the left side and bulk of the values lie to the left of the mean. A zero value indicates that the value a relatively evenly distributed on both side of the mean, typically but not necessarily implying a symmetric distribution. Normal distributions produce a skewness statistic of about zero. As shown in Fig. 15 to 17, the value of skewness 0.10208, 0.6821 and 0.14076 would be an acceptable skewness value for normally distribution set of test scores because it is very close to zero and is probably just a chance fluctuation from zero.

Coefficient of kurtosis, a measure of the peakedness of the probability distribution of a real-valued random variable, although some sources are insistent that heavy tails, and not peakedness, is what is really being measured by kurtosis. Higher kurtosis means more of the variance is the result of infrequent extreme deviations, as opposed to frequent modestly sized deviations. Normal distributions produce a kurtosis statistic of about zero. In this analysis, a kurtosis statistic of 0.097961, 0.13992 and 0.12986 from Fig. 15 to 17 would be an acceptable kurtosis value for a normal distribution because it is close to zero.

As shown in Fig. 15 to 17, the minimum and maximum of *Von Mises* strain are 54.6 and 100 $\mu\epsilon$,

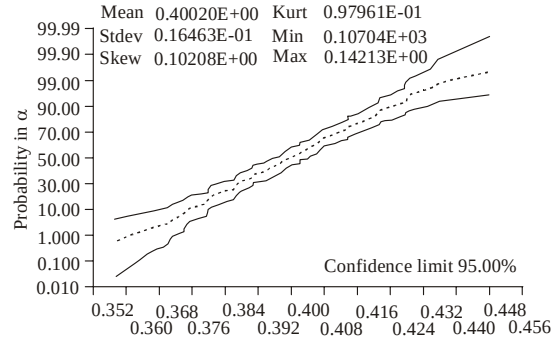

Fig. 18: 95% confidence interval for maximum deflection (beam with 9 web openings)

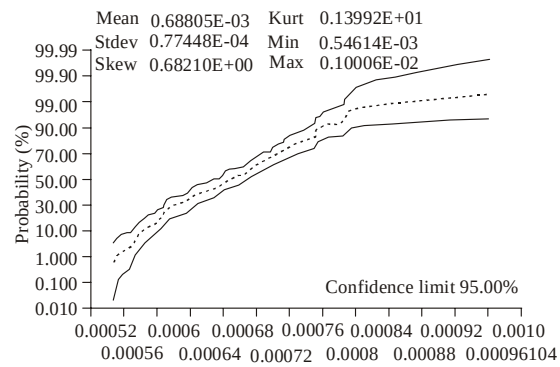

Fig. 19: 95% confidence interval for maximum *Von Mises* strain (beam with 9 web openings)

respectively, and it increases by 83.2%. While the minimum and maximum of deflection are 35.7 and 44.8 mm, respectively, and it increases by 25.5%. Meanwhile the minimum and maximum of *Von Mises* stress are 107.4 to 142.13 KN, respectively, and it increases by 32.3%. It means that output parameter of *Von Mises* strain has more significant influence by input parameter`s variations.

Technical products are typically designed to fulfill certain design criteria based on the output parameters. For example, a design criterion is that the deflection will be above or below a certain limit. The cumulative distribution curve for maximum deflection is 44.8 mm, maximum *Von Mises* stress is 142.1 KN and maximum *Von Mises* strain is 100 $\mu\epsilon$ are shown in Fig. 18 to 20. The line in middle is the probability *P* that the maximum deflection, maximum *Von Mises* stress and maximum *Von Mises* strain remains lower than a certain limit value Maximum deflection is 42.6 mm, maximum *Von Mises* stress is 135 KN and maximum *Von Mises* strain is 95 $\mu\epsilon$ with 95% confidence interval. The complement 1.0 - *P* is the probability that Maximum deflection, maximum *Von Mises* stress and maximum *Von Mises* strain exceed this limit. For the stability of the concrete beam, the reliability of beams is given by the probability that the deflection,

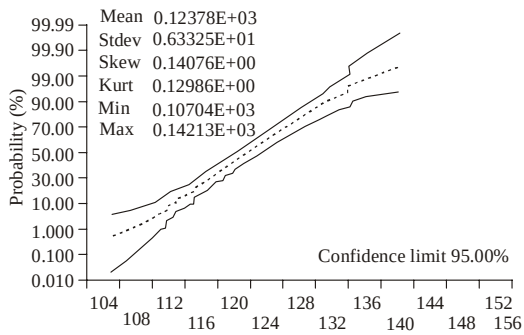

Fig. 20: 95% confidence interval for maximum *Von Mises* stress (beam with 9 web openings)

stress and strain should falls within the range of confidence interval. The upper and lower curves in Fig. 18 to 20 are the confidence interval using a 95% confidence level. The confidence interval quantifies the accuracy of the probability results. After the reliability of the beams has been quantified, it may happen that the resulting value is not sufficient. Then, probabilistic methods can be used to answer the following question: Which input variables should be addressed to achieve a robust design and improve the quality? The answer to that question can be derived from probabilistic sensitivity diagrams plot.

Sensitivity analysis results: The result of the proposed method was Spearman rank-order correlation Eq. (9) and (10) to determine which random parameters are most significant in affecting the uncertainty of the design. The sensitivity analysis results obtained for nonlinear analysis are shown in Fig. 21 to 23. The sensitivities are given as absolute values (bar chart) and relative to each other (pie chart). For easy input these parameters in ANSYS program, here use shortened form of proper name, such as: P = F1, l = B14, b = B1, E_c = E3, E_p = E2, P_s = PREF, v = PRXY4, f_c = SYGJ and f_p = SB. From Figures as shown below, the prestressing force P_s and beam depth l significant influence on the output parameter for maximum deflection. On the other hand, applied load P; the prestressing force P_s; elastic modulus of prestressing steel E_p; ultimate tensile strength of prestressing steel f_p; beam depth l and beam width b have a significant influence on for maximum deflection. On the other hand, applied load P; the prestressing force P_s; elastic modulus of prestressing steel E_p; ultimate tensile strength of prestressing steel f_p; beam depth l and beam width b have a significant influence on the output parameter for maximum *Von Mises* strain. Besides, the ultimate tensile strength of prestressing steel f_p; the prestressing force P_s; beam depth l and beam width b have a significant influence on the output parameter for maximum *Von Mises* stress.

The sensitivity analysis results indicate that applied load, prestressing force, elastic modulus of prestressing steel, ultimate tensile strength of prestressing steel, beam depth and beam width are more sensitive to other input variables because the impact of other input variables on the result is not significant enough to be worth considering. This is a reduction of the complexity of the problem from nine input variables down to six.

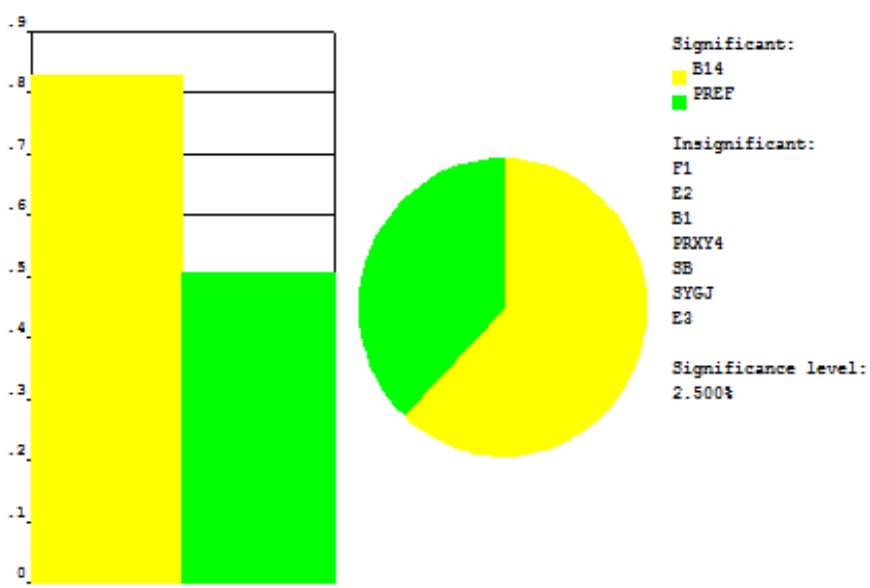

Fig. 21: Sensitivity plot for maximum deflection (beam with 9 web openings)

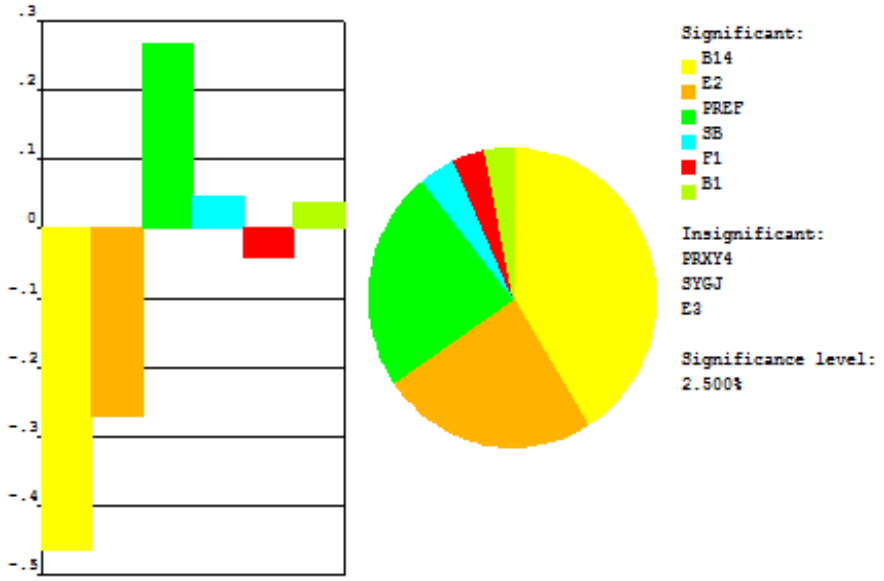

Fig. 22: Sensitivity plot for maximum *Von Mises* strain (beam with 9 web openings)

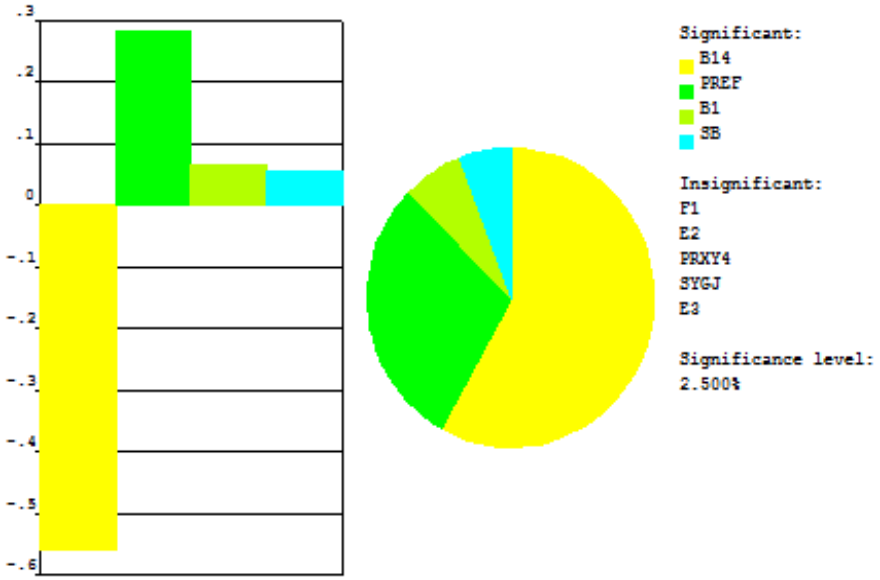

Fig. 23: Sensitivity plot for maximum *Von Mises* stress (beam with 9 web openings)

CONCLUSION

A method of probabilistic and sensitivity analysis to assess the openings` effect on the prestressed inverted T-beam with web openings is proposed. Latin Hypercube simulation technique was used to study the uncertainty of model parameters. The samples are obtained according to underlying probabilistic distributions and the outputs from the numerical simulation are translated into probabilistic distributions. To conduct sensitivity analysis on the realizations of input vectors, a rank transformation was applied to the input and output variables. Multiple linear regression on the ranks is then performed to obtain relationships between the input and output variables. The coefficients of the regression equations are related to the coefficient of determination and can be used to identify the most important model parameters.

The sample of probabilistic studies that have focused on the beam with 9 web openings is very limited. In addition, to enable a probabilistic analysis to be conducted, the model employed was of necessity simplified. However, the present work has shown that the selection of different samples, variables and output parameters leads to varying sensitivity results. Therefore,

future work will focus on a model incorporating beam with 6 and 3 web openings.

LIST OF ABBREVIATIONS

X_k	= input variables of Latin Hypercube sampling method
Fx_k (x)	= F_x(x) cumulative distribution function (CDF)
p-value	= Represents the probability of obtaining an observed result and its significance level
μ,	= Mean value
σ,	= Standard deviation
f_x(x)	= Probability density function
r_i	= The order representing the value of random variable
j th	= Number of simulations
n_j	= The order of an ordered sample of the resulting variable for the j th run on the simulation process
k_i	= Sensitivity coefficients
v_{yi}	= The variation coefficient of the output quantity
P	= Applied load
P_s	= Prestressing force
v	= Poisson`s ratio
E_c	= Elastic modulus of concrete
$f_c^{`}$	= Ultimate compressive strength
f_r	= Ultimate tensile strength (modulus of rupture),
f_y	= Strength of reinforcing steel for stirrup
f	= Stress at any strain ϵ
ϵ	= Strain at stress f
ϵ_0	= Strain at the ultimate compressive strength $f_c^{`}$
ϵ_{ps}	= Strain of prestressing steel
E_s	= Elastic modulus of steel
f_p	= Ultimate strength of prestressing steel
E_p	= Ulastic modulus of prestessing steel

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