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# Investigation of Creep Phenomenon in Metal Matrix Composites with Whiskers

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**Abstract:** A new mathematical model based on the exponential, logarithmic and polynomial (mixed) functions is presented for determination of some unknowns such as displacement rate in outer surface of unit cell and strain rate of short fiber (whisker) composites with elastic fiber in steady state creep under axial loading. In addition, effective factor or effect coefficient is introduced for determination of creep displacement rate in outer surface. Also, radial, axial displacement rates, equivalent and shear stresses will be determined by new method. Aim of this study is using the mathematical modeling instead of time consuming and costly experimental methods. On the other hand, unknowns are determined by polynomial, exponential and logarithmic functions instead of some theories, simply. These analytical results are then validated by the Finite Element Analysis (FEA). Interestingly, good agreements are found between analytical and numerical predictions for creep strain rate and displacement rate.

**Keywords:** Composite, effective factor, mathematical model, mixed functions, steady state creep, whisker

## INTRODUCTION

Recently, high stress and temperature deformation of composites has been argued in scientific societies, so creep studies become more important in various industries. Many researchers have studied the steady state creep behavior by analytical and experimental methods, but, some of them have analyzed steady state creep problems by long and time consuming methods with various assumptions. Sometimes, these intricate approaches were very difficult and impossible for solving the problem, then, they have used different assumptions for determination of unknowns in second stage creep problems. Of course, some their results were coincided the experimental and finite element methods well. In this study, some unknowns in steady state creep were obtained by some functions for crept matrix in short fiber composites. Results of the new function for determination of creep strain rate are very exact and marvelous. Additionally, effective factor or effect coefficient is presented for determination of creep displacement rate in outer surface in second stage creep in short fiber composites. Recently, extensive investigations have been performed to obtain the creep behavior of short fiber composites in steady state creep. Finite Element Analysis (FEA) is a strong and precise method in complex and intricate geometrical modeling, providing solutions to many complicated problems in various fields of engineering (Mustapha *et al*., 2011; Monfared, 2011). Also, creep of fibrous composite materials and tensile properties of fiber reinforced metals have been studied completely in references (Lilholt, 1985; Kelly and Tyson, 1966). Advanced shear-lag model applicable to discontinuous fiber composites was presented (Fukuda and Chou, 1981). Creep of dispersion reinforced aluminum based metal matrix composite has been studied (Greasly, 1995). Efficiency of densification process in preparation of carbon-carbon composites has been studied (Klučáková, 2006). Stress analysis of a composite material with short elastic fibre in power law creep matrix was presented (Lee *et al*., 1990). Steady state Creep deformation and its behavior of metal-matrix composites has been analysed (McLean, 1985; Mileiko, 1970; Mohamed *et al*., 1992). Prediction of mechanical behavior of PZT and SMA was studied by finite element analysis (Monfared and Khalili, 2011). Second stage creep of silicon carbide whisker/6061 aluminum composite at 573 K was analyzed (Morimoto *et al*., 1988). Creep rupture of a silicon-carbide reinforced aluminum composite has been investigated (Nieh, 1984). Analysis of Mechanical Properties of Composite Materials Made of Palm Fruit Fiber and Sawdust has been investigated (Sosu *et al*., 2011). Mechanical properties of composites prepared from post consumer high-density polyethylene mixed with plant natural fibers as the dispersed phase and having either epoxy or polyurethane binders have been investigated (Wambua *et al*., 2012). Analytical modeling of creep of short fiber reinforced ceramic matrix composite was presented (Wang and Chou, 1992). In the present new study, an analytical and mathematical solution based on effective factor and exponential, polynomial and logarithmic (mixed) functions for prediction of the steady state creep behavior of short fiber composites with elastic fiber without using of some theories instead of costly and time-consuming experimental method is suggested and developed. Here, an axisymmetric unit cell

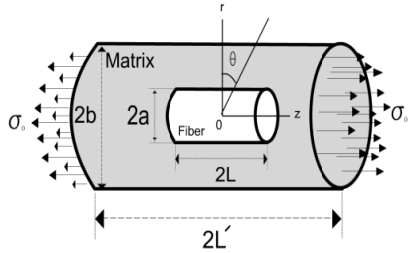


Fig. 1: Presentation of unit cell model

representing a fiber with its surrounding matrix as two coaxial cylinders is assumed. For verification of the solution method, the SiC/6061Al composite is selected as a case study and the results will be compared with the FEM, analytical and experimental available results in Morimoto *et al*. (1988). Therefore, novel mathematical model based on mixed function is introduced for determination of some parameters such as displacement rate in outer surface of unit cell and strain rate of short fiber composites in steady state creep subjected to axial loading, because other methods are intricate and difficult and sometimes are impossible for solution. In addition, effective factor or effect coefficient is introduced for determination of creep displacement rate in outer surface. Also, radial, axial displacement rates, equivalent and shear stresses will be determined. One of the advantages of this method is using mathematical modeling instead of time consuming and costly experimental methods. On the other hand, some unknowns are obtained by polynomial, logarithmic and exponential functions instead of some theories, simply. Perfect fiber-matrix interface is assumed and the steady state creep behavior of the matrix is explicated by an exponential law. These analytical results are then validated by the Finite Element modeling (FEA). Interestingly, numerical (FEA) and presented analytical results match well. In this research, purpose of creep analysis is suitable composite design. That is, creep analysis should be studied for preventing failure and defect in short fiber composites arising from creep phenomenon. Objective of the study is presentation of simple and comprehensive approach in order to determine steady state creep behavior.

## MATERIALS AND METHODS

This study has been performed in department of mechanical engineering of Islamic Azad University, Zanjan, Iran; between December 2011 to May 2012. The unit cell model shown in Fig. 1 has been used to model a short fiber composite.The volume fraction and aspect ratio of the fiber are defined as f and s = L/*a* respectively. Also, in this study, k = (L′ /b) / ( L/*a*) is considered as a parameter related to the geometry

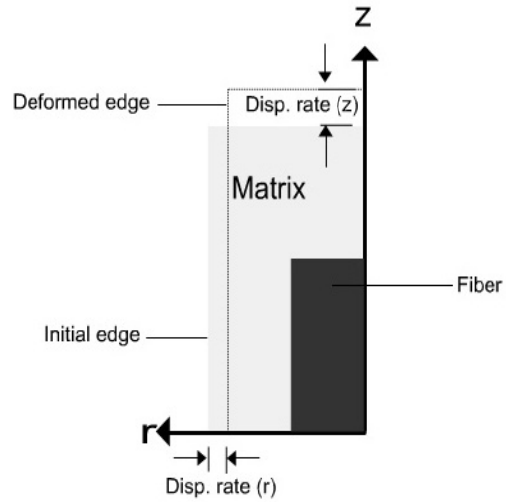


Fig. 2: Undeformed and crept matrix edges in second stage creep

of the unit cell. An applied axial stress $\sigma_0 = \sigma_{app}$ is uniformly induced on the end faces of the unit cell (at $z = \pm L'$). Creep behavior of matrix is described by an exponential law as follows in Eq. (1):

$$\dot{\varepsilon}_{eq} = c_1 \exp\left(\sigma_{eq} / c_2\right) \tag{1}$$

For verification of the solution method, the $SiC_f$ / $Al_m$ composite is selected as a case study and the results will be compared with the FEA, mathematical and analytical and experimental results. For the composite used here, $SiC_f$ / $Al_m$ the volume fraction of fibers is 3/20 and the fibers have an aspect ratio of 7.4 and k = 3/4, which are in accordance with the suggestions made in Morimoto *et al*. (1988). Deformed an undeformed shape of unit cell was presented due to steady state creep and elastic behavior in Fig. 2, respectively.

In the next section, New formulation based on effective factor and mixed functions method are presented for determination of steady state creep behavior of short fiber composites. Also, Finite Element Analysis (FEA) is presented for validation of results. The generalized constitutive equations for small creep deformation of the matrix material in $r$, $\theta$ and $z$ directions are given as below:

$$\dot{\varepsilon}_I = \frac{\dot{\varepsilon}_{eq}}{2\sigma_{eq}}\left[2\sigma_I - \sigma_J - \sigma_K\right], \dot{\gamma}_{IK} = \frac{3\dot{\varepsilon}_{eq}}{\sigma_{eq}}\tau_{IK},$$

Table 1: Boundary conditions in crept unit cell under axial tensile load

| | | | |
|---|---|---|---|
| $\dot{u}(0,z)=0\ ,l\le z\le l'$ | $\dot{u}(b,z)=\dot{u}_b\ ,0\le z\le l'$ | $\dot{w}(r,0)=0\ ,\ a\le r\le b$ | $\dot{w}(a,z)=0$ |
| $\tau_{rz}(b,z)=\tau_{rz}(r,l')=0\ ,0\le z\le l'$ | | $\tau_{rz}(r,0)=\tau_{rz}(r,l')=0,0\le r\le b$ | |
| $\dot{u}(r,l)=\dot{w}(r,l)=0,0\le r\le a$ | | $\dot{u}(a,z)=0$ | $\dot{w}_{l'}=-2l'\dot{u}_b/b$ |
| $\dot{w}(b,l)=-2l\dot{u}_b/b,a\le r\le b$ | | $w_{l'}=-2l'u_b/b$ | $\dot{\varepsilon}_c=\dot{w}_{l'}/l'=-2\dot{u}_b/b$ |

$$I,J,K=r,\theta,z \tag{2}$$

In addition, incompressibility condition should be satisfied, that is:

$$\dot{u}_r+\frac{\dot{u}}{r}+\dot{w}_z=0 \tag{3}$$

where, the equivalent stress $\sigma_{eq}$ and equivalent strain rate $\dot{\varepsilon}_{eq}$ are given by:

$$\sigma_{eq}=\sqrt{\frac{(\sigma_I-\sigma_J)^2+(\sigma_J-\sigma_K)^2+(\sigma_K-\sigma_I)^2}{2}+3\tau_{IK}^{\ 2}} \tag{4}$$

$$\dot{\varepsilon}_{eq}=\sqrt{\frac{2\left[(\dot{\varepsilon}_I-\dot{\varepsilon}_J)^2+(\dot{\varepsilon}_J-\dot{\varepsilon}_K)^2+(\dot{\varepsilon}_K-\dot{\varepsilon}_I)^2\right]}{9}+\frac{4}{3}\dot{\varepsilon}_{IK}^{\ 2}} \tag{5}$$

The applied boundary conditions are given by Table 1.

**Theory and calculation:** Behavior of Second stage creep of silicon carbide whisker/6061 aluminum composite at 573 K has been investigated by experimental method (Morimoto *et al*., 1988). In this section, new mathematical and analytical model based on mixed (polynomial, exponential and logarithmic) functions is presented for displacement rate analysis in external edge of unit cell and prediction of the second stage creep strain rate and stresses of short fiber composites under an axial load. Also, Some unknowns are determined by this mixed functions for obtaining of radial and axial displacement rates, equivalent and shear stress in steady state creep. One of the abilities and profits of this model is using analytical modeling instead of time consuming and costly experimental methods. Additionally, this approach is very simple and short for determination of some mentioned unknowns. Also, a full and complete fiber-matrix interface is supposed and the steady state creep behavior of the matrix is explained by an exponential law. The results determined from the proposed analytical solution satisfy the equilibrium and governing and constitutive creep equations. These analytical results are then validated by the Finite Element Analyzing (FEA). Engagingly, good agreements are found between the

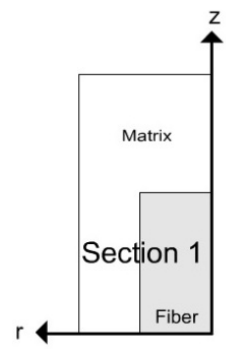


Fig. 3: Two important sections for creep problem

analytical and numerical predictions for all the stress and displacement rate components. This new approach is based on polynomial, logarithmic and exponential (mixed) displacement functions or methods. Also assumed displacement functions $\dot{u}(r,z)$ and $\dot{w}(r,z)$ satisfies the B.Cs. (Fig. 3):

$$\dot{u}(r,z)=ke^{ar}+cLnbr+hr^2+pr+d \tag{6}$$

Also, $\dot{w}(r,z)$ is obtained as follows:

$$\dot{w}(r,z)=-(\frac{k}{r}e^{ar}+ake^{ar}+\frac{c}{r}Lnbr+\frac{1}{r}(c+d)+3hr+2p)z \tag{7}$$

where, the coefficients of $c_i's$ are determined by boundary conditions, Table 1. In addition, shear stress in matrix is presented by using of Eq. (1) and (2) formulation:

$$\tau_{IK}^{m}=\frac{c_2Ln(\frac{\dot{\varepsilon}_{eq}}{c_1})\dot{\gamma}_{IK}}{3\dot{\varepsilon}_{eq}} \tag{8}$$

Also, by using of Eq. (1) and experimental results (Morimoto *et al*., 1988), have:

$$\log\dot{\varepsilon}_c=-10.7+0.06\sigma_{app} \tag{9}$$

Generally, Creep strain rate in exponential form is more accurate than the polynomial and power law forms.

Reason high ability of exponential form is expanding of this below function, that is:

$$\dot{\varepsilon}_{eq} = Aexp\left(\frac{\sigma_{eq}}{B}\right)$$

$$A\left[1 + \frac{\sigma_{eq}}{B} + \frac{\sigma_{eq}^2}{2!\,B^2} + \frac{\sigma_{eq}^3}{3!\,B^3} + \frac{\sigma_{eq}^4}{4!\,B^4} + \cdots\right]$$

$$= \left[A + \frac{A\sigma_{eq}}{B} + \frac{A\sigma_{eq}^2}{2!B^2} + \frac{A\sigma_{eq}^3}{3!B^3} + \frac{A\sigma_{eq}^4}{4!B^4} + \cdots\right]$$

$$=[A + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots]$$

$$\cong \sum_{k=0}^{n} a_k x^k \tag{10}$$

where, A = $a_0$. Thus polynomial and power law functions are just a small part of exponential function expansion. So we can use polynomial and power law functions instead of exponential law generally. Now, displacement rate $\dot{u}_b$ is determined by exponential form of crept matrix approximately. Also displacement rate is obtained without using some theories in steady state creep in short fiber composites. Also effective factor or coefficient is introduced for determination of creep displacement rate. By using of Eq. (1) have:

$$\log\dot{\varepsilon}_c = -10.7 + 0.06\sigma_{app} \tag{11}$$

$$\dot{\varepsilon}_c = 10^{-10.7+0.06\sigma_{app}} \tag{12}$$

where, $\dot{\varepsilon}_c = \frac{\dot{w}_{l'}}{l'} = -\frac{2\dot{u}_b}{b}$ , so by using of Eq. (11) and (12) have:

$$\dot{u}_b = -\left[\frac{b}{2}(10^{-10.7+0.06\sigma_{app}})\right]\times\varphi \tag{13}$$

$$\dot{u}_b = c_1\exp\left(\frac{\sigma_{app}}{c_2}\right)\times\varphi \tag{14}$$

where, $\varphi$ is effective factor, have:

$$\varphi \propto \varphi(\sigma_{app}, f, k, s) \tag{15}$$

Then $\varphi$ is obtain by above information, yields:

$$\varphi \cong \exp(-\sigma_{app}\times\Psi) \tag{16}$$

So,

$$\dot{u}_b = c_1\exp\left(\frac{\sigma_{app}}{c_2}\right)\times\exp(-\sigma_{app}\times\Psi) \tag{17}$$

By using of geometrical relations have:

$$\dot{\varepsilon}_c = -\frac{2}{b}\left[c_1\exp\left(\frac{\sigma_{app}}{c_2}\right)\times\exp(-\sigma_{app}\times\Psi)\right] \tag{18}$$

Behavior of parameter $\Psi$ depends on the variations of axial and radial displacement rates. This parameter is determined by obtaining displacement rates with considerations of boundary conditions. Where volume fraction and aspect ratio of the fiber are defined as f and s = L/a respectively. Also, in this research, k = L′a / Lb is considered as a parameter related to the geometry of the unit cell.

## RESULTS AND DISCUSSION

For comparison purpose, the finite element numerical calculations of creep behavior of this short fiber composite are also performed using the finite element commercial code of ANSYS (The axisymmetry method with non-linear quadratic element is employed for Finite Element Analysis (FEA), This element is a higher order eight-node element and has creep modeling ability), (Note that, $c_1$ = exp(-24.7), $c_2$ = 6.47).The axisymmetry approach with nonlinear quadratic element is used for FEA analysis. Creep strain rate function in steady state crept matrix was determined in short fiber composites with elastic fiber according to the obtained results. The mentioned function was obtained in steady state crept matrix by the exponential forms in short fiber composites, generally. This mathematical approach shown that results of creep strain rate are the same as experimental forms, FEA and analytical results by exponential functions. Exponential and polynomial functions are similar by reason of these Taylor expansions. Next, effective factor or effective coefficient was introduced for determination of creep displacement rate in second stage creep in short fiber composites. This effective factor is obtained by displacement rates behavior in unit cell in crept matrix that results of its effect are reliable and authentic; also, its results are similar to the finite element analysis. Next, shear and equivalent stress results were determined by the new method and FEA that comparison between analytical new work and finite element method results shown in Fig. 4 and 5.

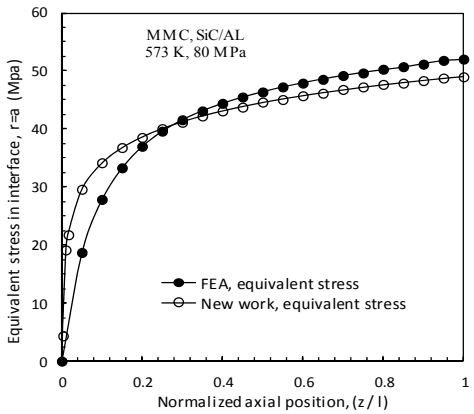


Fig. 4: Equivalent stress in interface at r = a

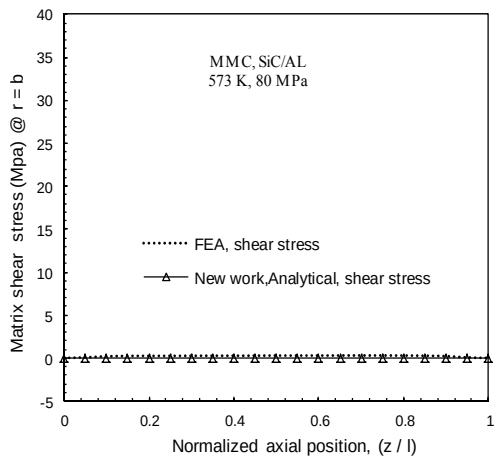


Fig. 5: Matrix shear stress at r = b

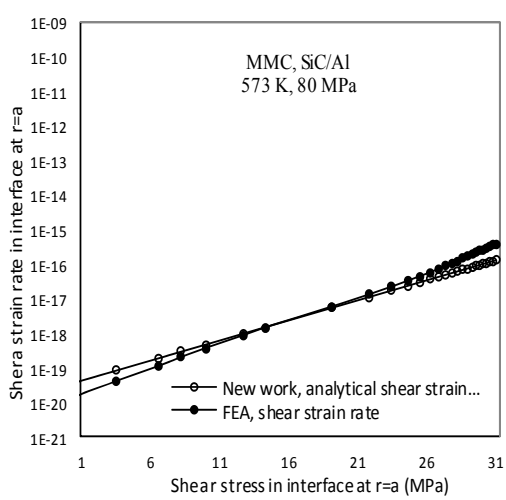


Fig. 6: Shear strain rate vs. shear stress in interface at r = a

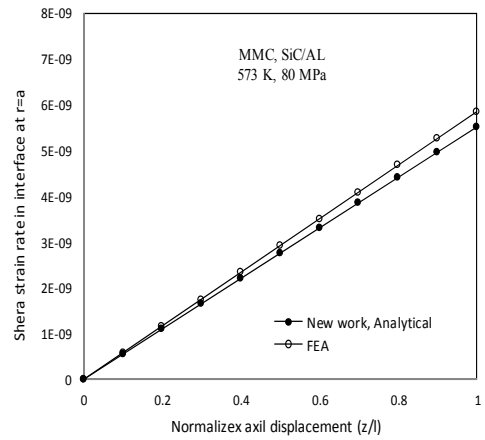


Fig. 7: Shear strain rate vs. normalized axial position in interface at r = a

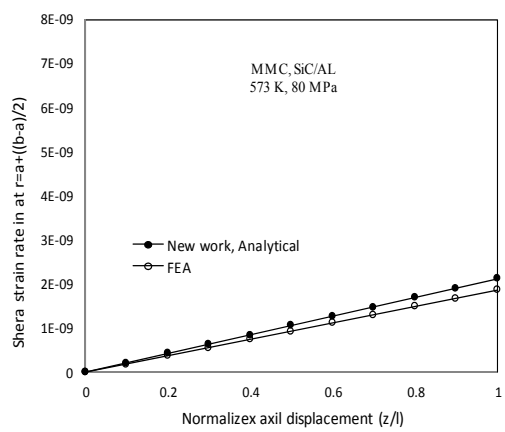


Fig. 8: Shear strain rate vs. normalized axial position in interface at r = a+((b-a)/2)

Also, comparison between analytical and FEA results for determination of shear strain rate in unit cell interface shown in Fig. 6.

In addition, shear strain rate curves with respect to normalized axial position have been shown in Fig. 7 and 8. These curves shown that shear strain rate behaviors are linear in interface and other matrix regions and sections.

## CONCLUSION

It is conclude that results of creep strain rate with respect to creep stress by new effective method in crept matrix are similar to the results of the FEA and analytical results in steady state creep in short fiber composites. In addition, radial and axial displacement rates, radial displacement rate in outer surface and shear and equivalent stress were determined by new mixed

function method. One of the advantages of mentioned method is application of mathematical models instead of time consuming and costly experimental methods or complex method. On the other hand, creep strain rate, some mentioned parameters were determined by mixed (sum of polynomial, logarithmic and exponential) functions instead of some difficult theories, simply. Finally, effective factor or effective coefficient was presented for specification of creep displacement rate in steady state creep in short fiber composites. Additionally its results are similar to the numerical predictions such as FEA. Eventually, we can rely on these effective factor and mixed functions for determination of steady state creep in short fiber composites.

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