

Numerical Study and Analytical Solution of P-Wave Attenuation Insensitive Underground Structure

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Abstract: Stability the underground sensitive structures in order to reduce the damage of explosion on these structure including nuclear and military installations as well as energy site is an issue of great importance which covers all the substructures and vital centers. The focus of this study is on the energy attenuation of one-dimensional p-wave in rock mass. As a proposal to strengthen the structures applying virtual space in rock mass, analytical (based on wave reflection and transmission) and numerical study on P-wave attenuation are used. Analytical and numerical results have shown that any vacuum or cavity in rock mass greatly increases P-wave attenuation and that any rock material with strength mechanical properties in virtual space decreases attenuation and raises P-wave energy transmission. Moreover, these results can be a good source for intelligent designing of these structures.

Keywords: Analytical solution, numerical analysis, virtual space, wave attenuation

INTRODUCTION

The first investigation regarding rock dynamic properties might be those laboratory studies which Barton said that in his book on the early researchers of this field (Barton, 2006).

The potential effect on density on P-wave velocity and attenuation was the issue which Aikada investigated using blast hole (Keda, 1993).

His studies showed that the main reason for an increase in P-wave velocity and a decrease in P-wave attenuation as the depth of the ground increases is due to an increase in rock mass density and a decrease in an isotropic rock mass. In addition, many researchers have investigated the effect of density on P-wave velocity using laboratory studies and insitu tests (Live, 1996; Yamamoto *et al.*, 1995; Shikawa *et al.*, 1995).

We can also refer to Grogik & Koda studies on two types of rock, namely Marl and Predotit, which led them to linear equation of $V_p = 4/75y - 7/3$, of great important result in all the studies conducted is that as rock density increases and rock mass properties get close to isotropic rock, the velocity of P-wave increases.

Considering dynamic from a different perspective, we can refer to the effect of porosity and uniaxial strength of rock on P-wave transmission.

The results clearly illustrate that an increase in porosity and a decrease in uniaxial strength which are in some way related to each other, make P-wave velocity

greatly decrease while P-wave attenuation increase. For further studies refer to Wilkens *et al.* (1984; 1988; 1991) and Won and Raper (1997).

Most of these studies have been concerned with cracks (i.e., micro-cracks and micro-voids) of a small size relative to the seismic wavelength, based on the wave scattering models (Doherty and Anstey, 1977; Spencer *et al.*, 1998; Banik *et al.*, 1985; Bedford and Drumheller, 1994).

In most modeling using numerical methods, it was assumed that fracture or joint be very small in relation to dynamic wave length. The latest studies made in this regard have been mostly conducted by Zhao Zhian and et al as well as Zl Wany's research team (Chen and Zhao, 1998; Zhao, 2010; Wang *et al.*, 2010).

Applying a coefficient called wave transmission coefficient, professor Zhao Zhian investigated the effect of fracture spacing on wave propagation.

Comprehensive scientific research which have been published in a number of papers.

It should be stated that in recent year's Z.L. Wang's research team has attained remarkable results using analytical and numerical methods. Their latest studies are also concerned with the enhancement of underground structures under dynamic loading underground civil structure are essentially considered as a protector or passive defense against destructive effects of explosion, whose main objective is to prevent these structures from failing.

In the last few decades, due to civil structures enhancement wave attenuation studies has turned into one of the main issues in dynamic rock behavior.

Increasing developments in modern weaponry and their destructive effects have made engineers compute to find new method of reducing the damage of explosive weapons.

One of the methods of achieving this goal is to increase wave attenuation while dispersing the energy produced by explosion. P-wave transmission occurs when the first element is compressed this makes p-wave transmit into the next element. As wave transmission continues, the volume material is compressed. This process is called p-wave transmission.

P-wave has some features as follows:

- It has energy
- The environmental elements vibrate or tremble around P-wave

But there is a possibility that p-wave may not be transmitted completely. While p-wave is propagating in continuous environment, the environment particles vibrate with the same frequency, but these vibrations have intervals in the direction of p-wave.

The environment concerned in this study is a discontinuous environment. When waves propagate in rock mass, as a result of some discontinuities in rock mass such as fracture, joint and crack, remarkable wave attenuation in produced the numerical results have shown that low frequency wave (long wavelength) increase wave transmission.

This means that discontinuities have little effect on wave attenuation. These results exactly illustrate geophysical findings as well.

Opening, closing and sliding discontinuities when they are under normal stress and shear stress makes rock mass displace.

It should be mentioned that when the p-wave propagates in fractures, first, wave energy traveled in continuous area or rock material. Second displacement happens in discontinuities which cause wave refractions and reflection.

Analytical solution of energy propagation and attenuation of wave in rock mass: Figure 1 shows that virtual space of a continuous and homogenous environment with Δz thickness is separated from its surrounding rock mass by two fractures.

If one-dimensional P-wave propagates from the left and from the AB surface and when it is transmitted into the virtual space; as the virtual space has different physical properties, wave is affected by reflection, attenuation or transmission.

According to wave equation and using Newton's second law and supposing that the applying force is in the x-direction, wave energy transmission happens as follows:

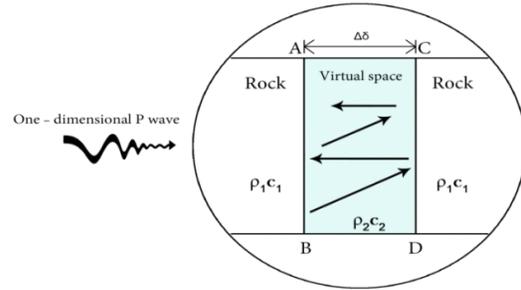


Fig. 1: Shows that virtual space of a continuous and homogenous environment

$$E_{Inc} + E_{Ref} = E_{Tr}^* \quad (1)$$

$$V_{Inc} + V_{Ref} = V_{Tra} \quad (2)$$

Wave energy equation is as follows:

$$E = m\omega^2 A^2 \quad (3)$$

where the factors A, w and m represent wave amplitude, angular frequency and mass respectively. Angular frequency, frequency, wave velocity and wave length are related to each other as illustrated in the following equation:

$$\omega = 2\pi \frac{v}{\lambda} \quad (4)$$

Combining Eq. (4) and (3), we obtain:

$$E = 4m \frac{\pi^2 v^2}{\lambda^2} A^2 * \quad (5)$$

On the other hand we have $m = \rho V$ which equals density multiplied by volume. As analytical solution is considered for one-dimensional phase, we suppose that $V = 1m^3$, so $m = \rho$ then by simplifying the Eq. (5) we can rewrite it as:

$$E = 4\rho m \frac{\pi^2 v^2}{\lambda^2} A^2 * \quad (6)$$

Finally, by combining Eq. (1), (2) and (6) we obtain:

$$E_{Tra} = \frac{2k}{1+k} E_{Inc} = M_E E_{Inc} * \quad (7)$$

$$E_{Ref} = \frac{k-1}{k+1} E_{Inc} = N_E E_{Inc} * \quad (8)$$

where Inc, Ref, Tra represent the variables associated to incident, reflected and transmitted waves, respectively.

It should be mentioned that in these equations we have $Q = \frac{\rho_2}{\rho_1}$, likewise, when the wave propagates from the inclusion to rock, the following relation hold:

$$E_{Tra} = \frac{2}{1+k} E_{Inc} = M'_E E_{Inc} \quad (9)$$

$$E_{Ref} = \frac{1-k}{1+k} E_{Inc} = N'_E E_{Inc} \quad (10)$$

where M'_E and N'_E are the corresponding transmitted and reflected coefficients respectively.

When wave is transmitted through the virtual surface, the new boundary surface CD causes wave reflection and transmission with different shapes.

When the reflected wave hits the AB boundary, it produces several wave reflections. This dynamic process continues until wave energy gradually attenuates.

If we suppose that the total number of wave reflections in virtual surface equals n, therefore, we will have (n+1) number of internal wave interactions. The jth wave energy clearly state as follows:

$$E_i = E_{Tra} (N'_E)^{j-1} \quad (11)$$

After the interaction of n+1 waves in the inclusion, the resultant stress σ within elastic the elastic range can be expressed as:

$$E = E_i (1 + g + g^2 + g^3 + \dots + g^n) = \frac{1 - g^{n+1}}{1 - g} \sigma_r \quad (12)$$

It should be stated that, in reality, the number of wave reflections depends on blasting conditions, angular wave strike, the geometry of the region, rock mechanical properties and dip joint region geometry.

If n is an even number, it would have $\frac{n}{2}$ waves transmitted at the surface B'C', whereas there exit (n+1)/2 transmitted waves if n is an odd number, the resultant stress of the transmitted waves after the surface M'N' can be expressed:

$$\hat{E} = \begin{cases} \frac{1 - g^n}{1 - g^2} h E_{Inc} & (n \text{ is even}) \\ \frac{1 - g^{n+1}}{1 - g^2} h E_{Inc} & (n \text{ is odd}) \end{cases} \quad (13)$$

In addition, this parameter can be shown a different way as follow:

$$h = \frac{4k}{(1+k)^2} \quad (14)$$

The above- mentioned equations can be used to show velocity factor in analytical solution.

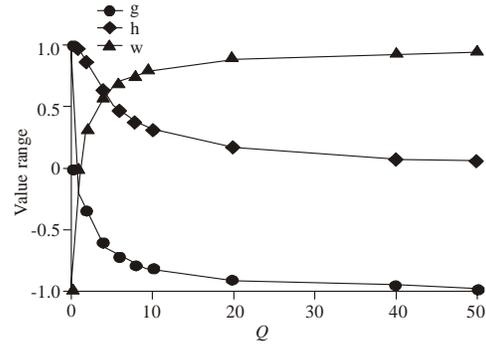


Fig. 2: Change rules of four parameters

$$V_j = V_{Tra} (N'_V)^{j-1} \quad (15)$$

$$V = \frac{1 - w^{n+1}}{1 - w^2} V_{Tra} \quad (16)$$

$$\hat{V} = \begin{cases} \frac{1 - w^n}{1 - w^2} dv_{Inc} & (n \text{ is even}) \\ \frac{1 - w^{n+1}}{1 - w^2} dv_{Inc} & (n \text{ is odd}) \end{cases} \quad (17)$$

where V and V_j are the resultant velocity and strength of jth reflection in the inclusion layer, respectively; \hat{V} is the total strength of transmitted waves after the surface A'B'.

The changes of parameters W, G, h are graphically illustrated if Fig. 2. These changes lead to the following results.

- When $Q = 1$, the entire energy of wave is transmitted $E = E_{Inc}$ or $V = V_{Inc}$. This clearly shows that is no attenuation on the energy of wave; therefore, the virtual surface has no effect and the ground acts completely as a continuous environment, whose density is the same as the environment's.
- When $\frac{\rho_2}{\rho_1} = 0$ or $Q = 0$, the complete energy absorption of the wave happens while there is no wave transmission at all. This means that there is high wave attenuation. Here, the virtual surface has the highest in fluence on the wave. This condition can be due to the vacuum existing in the virtual surface. This is what considered in this study.
- When $\frac{\rho_2}{\rho_1} = \infty$ or $Q = \infty$ it means that we have $E = 0$ again.

NUMERICAL MODELING

Discrete element method: The Discrete Element Method (DEM), originally proposed by Cundall (1971), is a totally discontinuum-based numerical method specially designed to solve discontinuity problems in fractured rock masses. Different from the continuum-based numerical

methods, DEM considers the fractured rock masses as an assemblage of multiple discrete blocks and the fractures as contact interfaces between blocks. The blocks representing rock material can be moved, rotated or deformed. The fractures separating the blocks may be closed, opened or slipped resulting in the displacement discontinuity.

It can be seen that the fracture model in the DEM is essentially the same as the displacement discontinuity model used in the theoretical study.

Fractures is a common discontinuity in rock mass and its presence plays an important part in the responses to plastic, elastic or dynamic loading.

Continuously Yielding Joint model is assumed for the joint in calculations. This law works in a similar fashion both for contacts both for contacts between rigid blocks and contacts between deformable blocks. Typical values are assumed for joint properties in the present analysis: normal stiffness $K_n = 8$ GPa, shear stiffness $K_s = 18$ GPa, cohesion $C = 4$ MPa friction angle $\phi = 28$ and no tensile strength is considered. The following equation can be used to estimate the applied velocity on the left non-reflecting boundary:

$$V_n = \frac{\sigma_n}{\rho C_p}$$

where, $C_p = \sqrt{(K + \frac{4G}{3}) / \rho}$ is the velocity of longitudinal wave with K and G being bulk and shear moduls.

Energy calculation: Energy changes determined in DEM are performed for the intact rock, the joints and for the done on boundaries. Since DEM an incremental solution procedure, the equations of motion are solved at each mass point in the body at every time step. The incremental change in energy components is determined at each time step as the system attempts to come to equilibrium.

The total energy balance can be expressed in terms of the released energy (E_r). This method calculation of

released energy can be made based on kinetic energy, mass damping work, the work performed at viscous boundaries and the strain energy in excavated material:

$$E_r = U_k + W_k + W_v + U_m \tag{18}$$

where,

- U_k = Current value of kinetic energy
- W_k = Total work dissipated by mass damping
- W_v = Work done by viscous (nonreflecting)
- U_m = Total strain energy in excavated material.

This method from of the released energy is particularly useful for dynamic problems in that the released kinetic energy is easily calculated.

Figure 2 shows graphically the sum of energy obtained.

Numerical analysis of energy wave attenuation: In this part, the effects of virtual space properties on energy wave attenuation are explored numerically. For the three sections (density, virtual space, properties of joint) to be explored, the specifications of the computational models are similar to each other. In each case, the incident wave is applied normally at the left boundary and propagates across inclusion layer along the X-direction, as illustrated in Fig. 3. The virtual space is assumed to be located $x = m$ from left boundary and is parallel to the both side of the rock. Viscous boundaries are placed at the upper, lower and right boundaries. The wave amplitude and frequency are 120 MPa and 3000 Hz, respectively. The rock material properties are assumed constantly as follow: the density (d) is 2.7 g/cm³, elastic modulus $E = 90.0$ GPa and Poisson's ratio $\mu = 0/16$, the compressional wave velocity (V_p) is 4460 m/s.

Effect of density rigid body and elastic modulus on energy wave attenuation to quantify the energy wave attenuation of dynamic loading due to a density in a rock mass, a dimension less variable, Fall Factor (FF), is defined as follows:

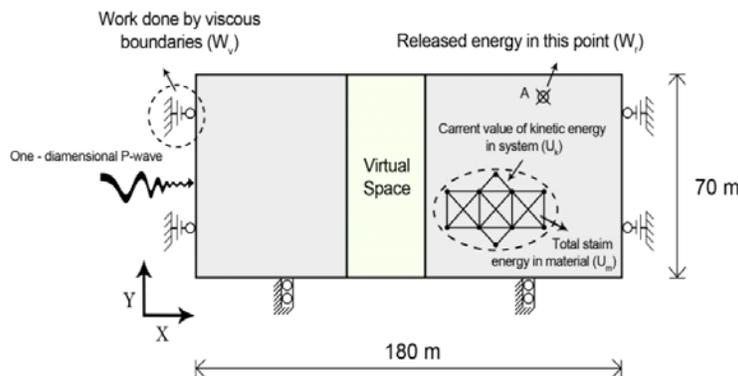


Fig. 3: UDEC models geometry of wave propagation in rock masses

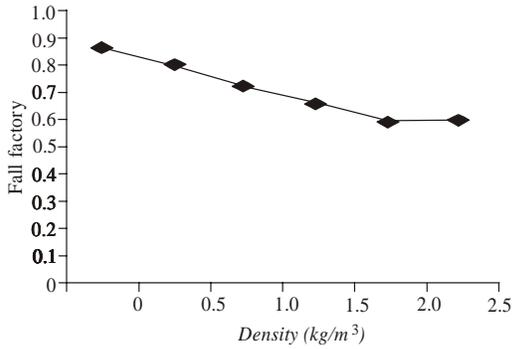


Fig. 4: Plot of fall factor versus thickness

$$F.F = 1 - \frac{E_z}{E_w}$$

where E_z represent the peak value of axial energy at a specified position with virtual space in the rock and E_w is the peak value of the component at the same position without a virtual space in rock mass.

It can be seen from Fig. 4, the density has great influence on the wave attenuation. For instance, the ration of peak energy wave is 0/88 when $\rho = 1.8 \frac{g}{cm^3}$, while it declines to 0/59 for $\rho = 3.2 \frac{g}{cm^3}$

Effect of air or vacuum on energy wave attenuation: It is assumed that the filled layer is free air corresponding to the foregoing case, namely $\frac{\rho_2}{\rho_1} = 0$. Four widths of cavity are chosen in the calculations: 0.4, 0.8, 1.2, 2, 2.4, 3 m, respectively.

Effect of joint wave attenuation: Joint is a common discontinuity in rock mass. The effect of natural dry joint on wave propagation and attenuation in rock mass is examined herein. The DEM modeling is specially adopted, which has been recognized to be a better alternative for problems.

Continuously yielding joint model is assumed for the joint in calculations. This law works in a similar fashion both for contacts between rigid blocks and contacts between deformable blocks.

CONCLUSION

The performance of an underground protective structure is a key issue in the design of civil defense engineering. This paper focuses on the propagation and attenuation of one-dimensional P-wave in rock mass.

The effect of virtual space in rock mass on wave attenuation is explored analytically. Then, the effects of three kind of virtual space on energy wave transmission

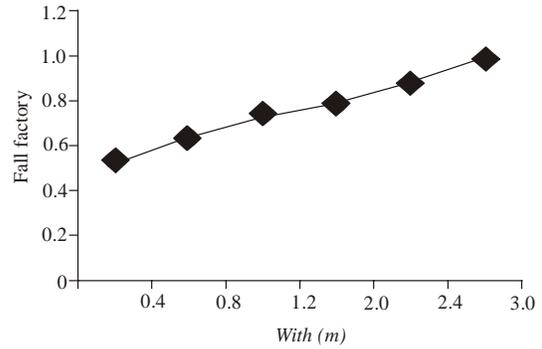


Fig. 5: Plot of fall factor versus with cavity

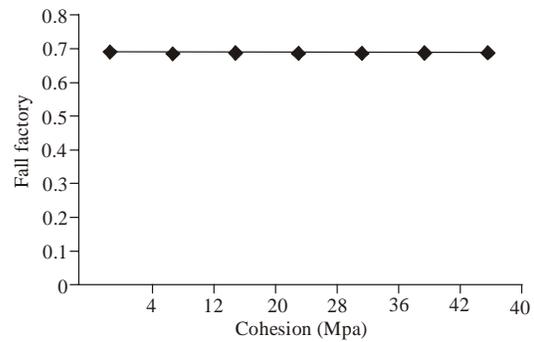


Fig. 6: Plot of fall factor versus with cohesion of joint

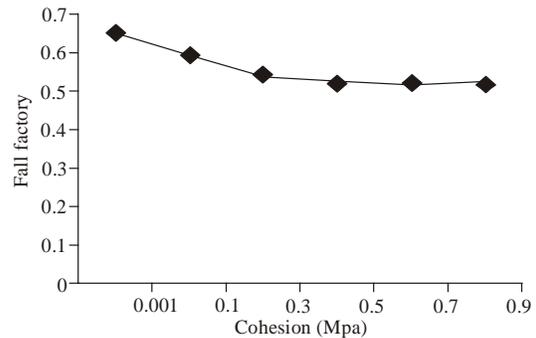


Fig. 7: Plot of fall factor versus with roughness of joint

and attenuation are numerically investigated and discussed via the distinct element modeling. Based on the present work, the following general conclusions may be draw:

- As illustrated in analytical method if $k = \frac{F_2}{F_1} = 1$, that is, when the virtual space has the same density as its surrounding environment, the energy of wave attenuation reaches its minimum value.
- In analytical method, the sensitive analysis showed that when $k = \frac{F_2}{F_1} = 0$, the maximum wave attenuation

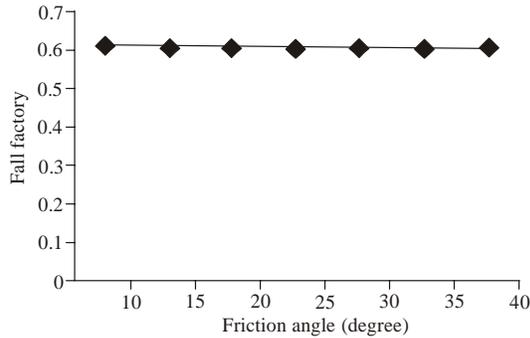


Fig. 8: Plot of fall factor versus with fraction angle of joint

happens as shown in Fig. 5, in numerical modeling when there is any vacuum or cavity, as the diameter of vacuum grows larger wave attenuation greatly increase

- Figure 7 show that joint roughness have very little influence on wave attenuation, while Fig. 6 and 8 illustrate that cohesion and friction angle apply no effect on wave attenuation.

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