

Adaptive Constant Modulus Blind Equalization with Variable Step-Size Control by Error Power

^{1,2}Ying Xiao and ¹Yuhua Dong

¹College of Information and Communication Engineering, Dalian Nationality University, Liaoning, Dalian, China

²Information and Communication Engineering Postdoctoral Station, Dalian University of Technology, Liaoning, Dalian, China

Abstract: A variable step-size control by error power was proposed to improve the performance of adaptive constant modulus blind equalization algorithm based on the analysis of the cost function. In the Constant Modulus Algorithm (CMA), large step-size can achieve faster convergence rate, but the steady-state residual error is big, on the contrary, small step-size can achieve higher convergence precision, but the convergence rate is slow. A triangle inequality can be set up according to the cost function of CMA and the power of error and signal can be estimated by exponential decay window, then the attenuation function can be obtained to control the step-size change. Meanwhile, the threshold is set to reset the step-size to ensure the tracking performance when the channel has burst interference. Computer simulation results prove the effectiveness of the proposed algorithm.

Keywords: Blind equalization, burst interference, constant modulus algorithm, error power, variable step-size

INTRODUCTION

In the digital communication system, time varying and multipath characterize of the channel would lead to Inter-Symbol Interference (ISI) at the receiver (Mendes *et al.*, 2012), which influence the quality of communication. Adaptive equalization technique is one of the effective techniques to eliminate ISI. Compared with traditional adaptive equalization, blind equalization can achieve compensation and tracking of the channel need no training sequence to eliminate the ISI at the receiver (Li *et al.*, 2011), which can improve the quality of the communication, at the same time, save the communication bandwidth. For interception of military information and multi-user broadcast communications, there is no training sequence can be exploited, under this conditions, blind equalization has important practical value. Blind equalization restore the send signal only rely on the statistical properties of the received signal, thus it can maintain the stability of the communication system to avoid the equalizer unlocking. In recent years, blind equalization technique has attracted a lot of experts and scholars to research on the theory and algorithm and made great achievements, in which one of the most mature is still Constant Modulus Algorithm (CMA) for blind equalization (Mahmoud *et al.*, 2012). According to the theory of adaptive filtering, CMA blind equalization algorithm with fixed step-size also exist the contradiction

in convergence rate and convergence precision and the effective way to solve this problem is using variable step-size in the equalizer updating (Yun-Shan *et al.*, 2011). In this paper, a kind of variable step-size algorithm control by error power was proposed based on the analysis of the CMA cost function. In order to guarantee that the improved algorithm is still effective in channel burst interference condition, a threshold is set according to the error power to control the reset of the equalizer weights and the step-size. At last, computer simulation is adopted to verify the performance of the proposed algorithm.

METHODOLOGY

The basic principles of CMA blind equalization: CMA is a special case of the Godard algorithm, also is one of the most commonly used of Bussgang blind equalization algorithms. The schematic diagram of CMA can be shown as Fig. 1.

According to the principle of communication, the signal transmission process can be expressed as:

$$s(n) = x(n)*h(n) \quad (1)$$

$$y(n) = s(n)+n(n) \quad (2)$$

where, $h(n)$ is the impulse response of the unknown channel, $x(n)$ is the send signal and $s(n)$ is the output of

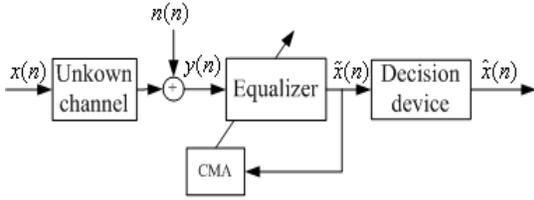


Fig. 1: schematic diagram of CMA blind equalization

the channel, then the observed signal $y(n)$ is got by $s(n)$ plus white Gaussian noise $n(n)$. Blind equalization can restore the send signal $x(n)$ only rely on $y(n)$ under the condition of the send signal $x(n)$ and channel $h(n)$ is unknown. The only priori information need for blind equalization is that the send signal meets non-Gauss distribution which is the digital modulation signal easy to meet. Bussgang blind equalization algorithm is based on the equalizer output of some nonlinear transformations, the nonlinear transformation meets the Bussgang process. As CMA algorithm is one of the special case of Bussgang algorithm and the cost function is Xiao and Yu-Hua (2012):

$$J_D = \frac{1}{2} \left[|\tilde{x}(n)|^2 - R_{CM} \right]^2 \quad (3)$$

where, R_{CM} is constant modulus can be compute according to Eq.4:

$$R_{CM} = \frac{E(|\tilde{x}(k)|^4)}{E(|\tilde{x}(k)|^2)} \quad (4)$$

Let $w(n)$ is the impulse response of equalizer, then the output $\tilde{x}(n)$ can be expressed as:

$$\tilde{x}(n) = w(n) * y(n) = w^T(n) y(n) \quad (5)$$

where, the symbol “ T ” denote Transpose operation. According to Eq.3 and Eq.5 the cost function can be rewritten as:

$$J_D = \frac{1}{2} \left[|w^T(n) y(n)| - R_{CM} \right]^2 \quad (6)$$

CMA algorithm updates the weights $w(n)$ of equalizer according to stochastic gradient descent algorithm to implement the minimization of the cost function, then the updating direction of $w(n)$ can be established based on the partial derivative of the cost function relative to $w(n)$ (Gupta and Mehra, 2012):

$$\frac{\partial J_D}{\partial w} = \left[|w^T(n) y(n)|^2 - R_{CM} \right] w^T(n) y(n) y^*(n) \quad (7)$$

If let the error $e(n)$ is:

$$e(n) = \left[|w^T(n) y(n)|^2 - R_{CM} \right] w^T(n) y(n) \quad (8)$$

Then the updating of $w(n)$ can be expressed as (Ma et al., 2011):

$$w(n+1) = w(n) - \mu e(n) y^*(n) \quad (9)$$

Here μ is the step-size which control the size of amplitude for updating the weights $w(n)$ and μ often set as a fixed value, then there is a contradiction between convergence rate and convergence precision of the algorithm.

Variable step-size CMA blind equalization: From the derivation process of CMA blind equalization can know that only the partial derivative of the cost function relative to $w(n)$ is used and the instantaneous gradient instead of the statistical gradient during the updating of $w(n)$, as a result, the larger step-size can obtain faster convergence rate, but large steady-state error would remain after convergence. In contrast, the smaller step-size can obtain smaller steady-state error after convergence (Sheng and Zhu, 2011), but the convergence rate would be slow. Variable step-size CMA blind equalization algorithm consider that the larger step-size is chosen at the beginning to obtained faster convergence rate and the step-size would gradually reduce during the iterative process to obtain lower steady-state error, which can achieve compromise between convergence rate and convergence precision. Here a new variable step-size method control by the error power of output signal was proposed, defined the control function for the step-size as:

$$\theta(n) = \frac{2|R_{ee}(n)|}{|R_{zz}(n)| + |R_{RR}(n)|} \quad (10)$$

where, R_{ee} , R_{zz} and R_{RR} is estimated as follow:

$$R_{ee}(n) = \alpha R_{ee}(n-1) + (1-\alpha) e(n) e^*(n) \quad (11)$$

$$R_{zz}(n) = \alpha R_{zz}(n-1) + (1-\alpha) |\tilde{x}(n)|^4 \quad (12)$$

$$R_{RR}(n) = \alpha R_{RR}(n-1) + (1-\alpha) R_{CM}^2 \quad (13)$$

According to CMA blind equalization algorithm, Minimize the cost function is equivalent to the minimize the output error, that is $\min J_D$ is equivalent to the

modulus of the output signal $\tilde{x}(n)$ tend to close to R_{CM} . Then set output error $e(n)$ as:

$$e^2(n) = (|\tilde{x}(n)|^2 - R_{CM})^2 \quad (14)$$

According to the triangle inequality relation can know that:

$$e^2(n) \leq |\tilde{x}(n)|^4 + R_{CM}^2 \quad (15)$$

With the iterative process of CMA blind equalization, the modulus of $\tilde{x}(n)$ would tend to R_{CM} and if and only if $|\tilde{x}(n)|$, the right of the inequality (15) get the maximum value, meanwhile, $|e(n)|$ gradually reduce to 0 under ideal conditions. Therefore, Eq.15 has a stable variation of attenuation law and $0 \leq \theta(n) \leq 1$. If initialize $R_{ee}(0) = 1$, $R_{zz}(0) = 1$, $R_{RR}(0) = 1$, then $\theta(0) = 1$, so called $\theta(n)$ is normalized mean square error and the step-size of CMA blind equalization algorithm can control by $\theta(n)$ as follow:

$$\mu(n+1) = \mu_{max} \theta(n) \quad (16)$$

In the actual communications, due to the relative motion of the receiver or communication environment changes suddenly, will make the communication channel to produce burst interference. In order to ensure that the variable step-size CMA blind equalization algorithm is robustness, the step-size and the weights of the equalizer need to reset timely. For normalized mean square error $\theta(n)$ adopt recursive method to estimate during the iterative process, then it is not sensitive to the changes of the output error $e(n)$, so the normalized mean square error information entropy increment is defined to detect the channel burst interference. The normalized mean square error information entropy increment can be expressed as:

$$E(n) = \left(1 - \frac{\theta(n)}{\theta(n=1)}\right) \ln(|\theta(n) - \theta(n-1)|) \quad (17)$$

If the channel meets the burst interference, then $\theta(n)$ must be greater than $\theta(n-1)$, then $E(n)$ is positive, otherwise, $E(n)$ is negative. Logarithmic $\ln(\cdot)$ magnify the change of the normalized mean square error increment. Here set a positive threshold δ , if $0 \leq E(n) \leq \delta$ after a certain iterative times, then reset $R_{ee}(n) = R_{ee}(0)$, $R_{zz}(n) = R_{zz}(0)$, $R_{RR}(n) = R_{RR}(0)$ and the weights of equalizer reset to initialization at the same time to ensure the effective of the algorithm under the channel burst interference condition.

COMPUTER SIMULATIONS

In the simulations, equivalent probability binary sequence is adopted to act as sending signal and QPSK

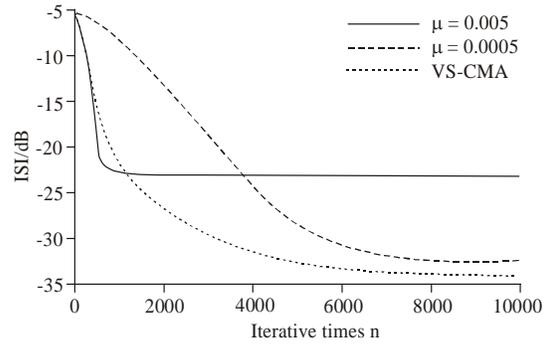


Fig. 2: Residual inter-symbol interference (SNR = 20dB)

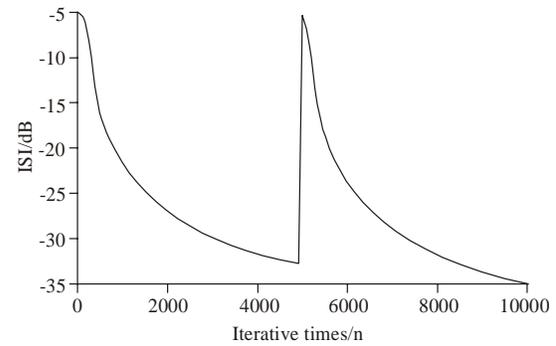


Fig. 3: The algorithm performance under the channel burst interference condition

modulation is utilized. Adding noise is band-limited gauss white noise with zero mean. The channel impulse response is $h = [0.3132, -0.1040, 0.8908, 0.3143]$ which is a mixed phase channel (Feng *et al.*, 2003). The comparison is in terms of residual Inter-Symbol Interference (ISI) which defined as (Hwang and Choi, 2012):

$$ISI = \frac{\sum_i |C_i|^2 - |C_i|_{max}^2}{|C_i|_{max}^2} \quad (18)$$

where, C is the combined impulse response of channel and equalizer. Equalizer order is set to 31. Signal to noise ratio $SNR = 20\text{dB}$. To prove the variable step-size CMA (VS-CMA) blind equalization algorithm proposed in this paper, compared with fixed large step-size $\mu = 0.005$ and fixed small step-size $\mu = 0.0005$ in the simulations. The results of Monte Carlo simulation by 500 times can be shown as Fig. 2. From Fig. 2 can see that the variable step-size blind equalization algorithm has the similar convergence rate with the fixed large step-size, but the residual inter-symbol interference lower about 12dB after convergence. Use the fixed small step-size can obtained the similar residual inter-symbol interference after convergence, but the convergence rate slower more than

3000 iterative times. The results show that VS-CMA blind equalization improve the performance of CMA blind equalization effectively.

For testing the detection effect for channel burst interference by the normalized mean square error information entropy increment, set the channel refracted path phase reversal when the iterative times reach to 5000 to simulation the channel burst interference, that is $h = [0.3132, -0.1040, 0.8908, -0.3143]$ when iterative times equal to 5000. Signal to noise ratio $SNR = 22\text{dB}$. The results of Monte Carlo simulation by 500 times can be shown as Fig.3. Figure 3 shows that the variable step-size CMA blind equalization with the normalized information entropy increment detecting the channel burst interference is effective.

CONCLUSION

In this study, a variable step-size CMA blind equalization algorithm was proposed, step-size vary with normalized mean square error according to output error power, then it has a stable variation of attenuation law. Simulation results show that this method improves the CMA blind equalization algorithm effectively and keep robustness under the channel burst interference condition.

ACKNOWLEDGMENT

This paper is funded by the Fundamental Research Funds For the Central Universities (No. DC10040103), Scientific and Technological Research Project for Education Department of Liaoning Province(2010046)

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