Research Journal of Applied Sciences, Engineering and Technology 4(21): 4200-4205, 2012
ISSN: 2040-7467

Submitted: December 18, 2011 Accepted: April 23, 2012 Published: November 01, 2012

Intuitionisitic Fuzzy B-algebras

Jiayin Peng
School of Mathematics and Information Science, Neijiang Normal University, Neijiang, CO 641112 China

Abstract: The notion of intuitionsitic fuzzy B-algebras is introduced and their some properties are investigated. How to deal with the homomorphic image and inverse image of intuitionsitic fuzzy ideals are B-algebras. The relations between a intuitionsitic fuzzy B-algebra and a intuitionsitic fuzzy B-algebra in the product B-algebras are given.

Keywords: B-algebra, homomorphism, intuitionsitic fuzzy B-algebra, product B-algebra

INTRODUCTION

In 1966, (Imai and Iseki, 1966) defined a class of algebras of type (2, 0) named BCK-algebras. At the same time, (Iseki, 1966) generalized another class of algebras of type (2, 0) called BCI-algebras. Hu and Li (1983) introduced the concept of BCH-algebras and gave examples of proper BCH-algebras (Hu and Li, 1985). Neggers and Kim (2001) introduced and investigated a class of algebras, viz., the class of B-algebras, which is related to several classes of algebras of interest such as BCH/BCI/BCK- algebras and which seems to have rather nice properties without being excessively complicated otherwise. Xi (1991) applied the notion of fuzzy sets (Zadeh, 1966) to BCK/BCI-algebras. Jun (1993), Meng, (1994) and Jun and Roh (1994) investigated fuzzy ideals of BCK/BCI-algebras. Young *et al.* (2002) introduced notions of fuzzy B-algebras and investigated its properties.

In 1986, (Atanassov, 1986) introduced the concept of intuitionistic fuzzy sets, which is a significant extension of fuzzy set theory by Zadeh (1966).Li and Wang (2000) studied intuitionistic fuzzy group and its homomorphisic image and subsequently discussed the product and extension principle of the intuitionistic fuzzy sets (Li and Wang, 2002). In this study, we introduce notion of intuitionistic fuzzy B-algebras and discuss their properties. Some properties of the homomorphic (isomorphic) image and inverse image of intuitionistic fuzzy B-algebras will be studied, the notions of intuitionistic fuzzy relation, strongest intuitionistic fuzzy relation and Cartesian product of intuitionisitic fuzzy sets will be given, some relations between intuitionistic fuzzy B-algebras and its product B-algebras are exposed.

PRELIMINARIES

Definition 1: Atanassov (1986) Let X be a nonempty set. A set $A = \{< x, \mu_A(x) \ \nu_A(x)>| \ x \in X \}$ is said to a intuitionsitic fuzzy set of X if mapping μ_A: → [0, 1] and ν_A : X→ [0, 1] satisfy $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in X$.

A collection of intuitionsitic fuzzy sets of X is denoted by $IFS(X)$. Let $A, B \in IFS(X)$. $A = \{< x, \mu_A(x) \ \nu_A(x)>| \ x \in X \}$ and $B = \{< x, \mu_B(x) \ \nu_B(x)>| \ x \in X \}$.Consider the relation between A and B as follow:

- $A \subseteq B$ if and only if $\mu_A(x) < \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$.
- $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.
- $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x) \nu_A(x) >| x \in X\}$
- $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >| x \in X\}$

Otherwise,

- $\bigcap_{j\in J} A_i = \left\{ \left\langle x, \bigwedge_{j\in J} \mu_{A_j}(x), \bigvee_{j\in J} \nu_{A_j}(x) \right\rangle \middle| x \in X \right\}$
- $\bigcap_{j\in J} A_i = \left\{ \left\langle x, \bigvee_{j\in J} \mu_{A_j}(x), \bigwedge_{j\in J} \nu_{A_j}(x) \right\rangle \middle| x \in X \right\}$
- $\Box A = \{< x, \mu_A(x) \ 1- \mu_A(x) >| \ x \in X \}$
- $\Diamond A = \{< x, 1- \nu_A(x), \nu_A(x)>| \ x \in X \}$

Definition 2: (Li and Wang, 2000) Let f be a mapping from the set X to the set Y and $B = \{< x, \mu_B(x), > | x \in X \}$. a intuitionsitic fuzzy set of Y. The inverse image of B, denoted by $f^{-1}(B)$, is the intuitionsitic fuzzy set of X defined by:

$$f^{-1}(B) \triangleq \{\langle x, \mu_B(f(x)), \nu_B(f(x))\rangle | x \in X\}$$

Conversely, let $A = \{< x, \mu_A(x), \nu_A(x), >| \ x \in X \}$. be a intuitionsitic fuzzy set of X. Then the image of A, denoted by $f(A)$, is the intuitionsitic fuzzy set of Y given by:

$f(A) \triangleq \{\langle y, f(\mu_A)(y), \hat{f}(\nu_A)(y)\rangle | y \in Y\}$

where,

$$f(\mu_A)(y) = \begin{cases} \bigvee_{x \in f^{-1}(y)} \mu_A(x), & f^{-1}(y) \neq \varnothing \\ 0, & f^{-1}(y) \neq \varnothing \end{cases}$$

$$f(\nu_A)(y) = \begin{cases} \bigvee_{x \in f^{-1}(y)} \nu_A(x), & f^{-1}(y) \neq \varnothing \\ 1, & f^{-1}(y) \neq \varnothing \end{cases}$$

Definition 3: Neggers and Kim (2001) An algebra of type $(X;*,0)$ is called a B-algebra if it satisfies the following axioms:

(B1) $x * x = 0$

(B2) $x * 0 = x$

(BCH3) $(x* y)* z = x* (z* (0 * y))$

for all $x, y, z \in X$.

In Young *et al.* (2002), the concept of fuzzy B-algebras is introduced. A fuzzy set μ in B-algebra X is said to be an anti-fuzzy B-algebra of X if for all $x, y \in X$, $\mu(x*y) \leq \mu(x) \vee \mu(y)$.

INTUITIONSITIC FUZZY B-ALGEBRAS

Definition 1: Let $(X; *, 0)$ be a B-algebra. A intuitionsitic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ of X is said to be a intuitionsitic fuzzy B-algebra if it satisfies $\mu_A(x*y) \wedge \mu_A(x)\ \mu_A(y) \wedge$ and $\nu_A(x*y) \leq \nu_A(x) \vee \nu_A(y)$ for all $x, y \in X$:

Proposition 1: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ be a intuitionsitic fuzzy B-algebra. Then , $\mu_A(0) \geq, \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x) \leq$, for all $x \in X$.

Proof: Since $x*x = 0$ for all $x \in X$, we havet hat $\mu_A(0) = \mu_A(x*x) \geq \mu_A(x) \wedge \mu_A(x) = \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x) \vee \nu_A(x) = \nu_A(x)\ \nu_A(0) \leq$ .This completes the proof.

For any elements x and y of X, let us write $\prod_n x*y$ for $x*(\ldots*(x*(x*y)))$ where, x occurs n times.

Proposition 2: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ be a intuitionsitic fuzzy B-algebra and let $n \in N$. Then for all:

$\mu_A(\prod_n x*x) \geq \mu_A(x)$ and

$\nu_A(\prod_n x*x) \leq \nu_A(x)$ Whenever n is odd

$\mu_A(\prod_n x*x) = \mu_A(x)$ and

$\nu_A(\prod_n x*x) = \nu_A(x)$ Whenever n is even.

Proof: Let $x \in X$ and assume that n is odd. Then $n = 2k-1$ for some positive interger k. Observe that $\mu_A(x*x) = \mu_A(0) \geq \mu_A(x)$ and $\nu_A(x*x) = \nu_A(0) \leq \nu_A(x)$. Suppose that $\mu_A(\prod_{2k-1} x*x) \geq \mu_A(x)$ and $\nu_A(\prod_n x*x) \leq \nu_A(x)$ for a positive integer k. Then:

$$\mu_A(\prod_{2(k+1)} x*x) = \mu_A(\prod_{2k+1} x*x)\mu_A(\prod_{2k-1} x*(x*(x*x)))$$
$$= \mu_A(\prod_{2k-1} x*x) \geq \mu_A(x)$$

$$\nu_A(\prod_{2(k+1)} x*x) = \nu_A(\prod_{2k+1} x*x) = \nu_A(\prod_{2k-1} x*(x*(x*x)))$$
$$= \nu_A(\prod_{2k-1} x*x) \leq \nu_A(x)$$

which proves (1). Similarly we obtain the second part.

Proposition 3: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ be a intuitionsitic fuzzy B-algebra, then for all $x, y \in X$:

(IFB1) $\mu_A(0*x) \geq \mu_A(x)$ and) $\nu_A(0*x) \leq \nu_A(x)$,
(IFB2) $\mu_A(x*(0*y)) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x*(0*y)) \leq \nu_A(x) \vee \nu_A(y)$

Proof: For any $x, y \in X$, we have $\mu_A(0*x) \geq \mu_A(0) \wedge \mu_A(x), = \mu_A(x)$ and $\mu_A(x*(0*y)) \geq \mu_A(x) \wedge \mu_A(x)\ (0*y) \geq \mu_A(x) \wedge \mu_A(y)$.

Similarly we have that $\nu_A(0*x) \leq \nu_A(x)$ and $\nu_A(x*(0*y)) \leq \nu_A(x) \vee \nu_A(y)$.

Corollary 4: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$be a intuitionsitic fuzzy B-algebra, then for all $x \in X$, $\mu_A(0*x) = \mu_A(x)$ and $\nu_A(0*x) = \nu_A(x)$.

Proof: Since $x = 0*(0*x)$, if A is a intuitionsitic fuzzy B-algebra, then $\nu_A(x) = \nu_A(0*(0*x)) \leq \nu_A(0) \vee \nu_A(0*x) = \nu(0*x)$, i.e., $\nu_A(0*x) = \nu_A(x)$. Similarly we can prove that $\mu_A(0*x) = \mu_A(x)$. for all $x \in X$.

Theorem 5: If a intuitionsitic fuzzy set A of X satisfies (IFB1) and (IFB2), then A is a intuitionsitic fuzzy B-algebra.

Proof: Suppose that A satisfies (IFB1) and (IFB2) and let $x, y \in X$. Then,

$\nu_A(x * y) = \nu_A(x * (0 * (0 * y)))$

$\leq \nu_A(x) \vee \nu_A(0*y)$

$= \nu_A(x) \vee \nu_A(y)$

Similarly we have that $\mu_A(x * y) \geq \mu_A(x) \wedge \mu_A(y)$ Hence A is a intuitionsitic fuzzy B-algebra.

Theorem 6: If A and B are intuitionsitic fuzzy B-algebra of X, then so is $A \cap B$.

Proof: For all $x, y \in X$, we get:

$$\begin{aligned}\mu_{A\cap B}(x*y) &= \mu_A(x*y) \wedge \mu_B(x*y)\\ &\geq [\mu_A(x) \wedge \mu_A(y)] \wedge [\mu_B(x) \wedge \mu_B(y)]\\ &= [\mu_A(x) \wedge \mu_B(x)] \wedge [\mu_A(y) \wedge \mu_B(y)]\\ &= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)\end{aligned}$$

similarly, we have that:

$$\nu_{A\cap B}(x*y) \leq \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y)$$

and hence $A \cap B$ is a intuitionsitic fuzzy B-algebra.

Corollary 7: Let A_i be intuitionsitic fuzzy B-algebra of X for all $i \in J$. Then $\bigcap_{i\in j} A_i$ is also a intuitionsitic fuzzy B-algebra of X. Similarly, we have:

Theorem 8: Let A and B are intuitionsitic fuzzy B-algebras of X . Then $\Box A$ and $\Diamond A$ are intuitionsitic fuzzy B-algebras of X.

Definition 2: A intuitionsitic fuzzy set A of X has sup-inf property if, for any $T \subseteq X$, there exist x_0, y_0, $\in T$ such that $\mu_A(x_0) = \mathrm{SUP}_{z\in T}\, \mu_A(z)$ and $\nu_A(y_0) = \inf_{z\in T} \nu_A(z)$.

Theorem 9: Let f be a homomorphism from a B-algebra X into a B-algebra Y and A a intuitionsitic fuzzy B-algebra of X with sup-inf property. Then the image $f(A)$ of A is a intuitionsitic fuzzy B-algebra of Y.

Proof: Let $A = \{< x, \mu_A(x), \nu_A(x), >| \, x \in X\}$ and let $y_1, y_2 \in Y$. we consider the following two cases:

Case (1): If $f^{-1}(y_1) = \emptyset$ or $f^{-1}(y_2) = \emptyset$, then $f^{-1}(y_1 * y_2) = \emptyset$. And so $f(\mu_A)(y_1 * y_2) = 0$ and $\hat{f}(\nu_A)(y_1 * y_2) = 1$. Thus $f(\mu_A)(y_1 * y_2) = 0 = f(\mu_A)(y_2) \wedge$ and:

$$\hat{f}(\nu_A)(y_1 * y_2) = 1 = \hat{f}(\nu_A)(y_1) \vee \hat{f}(\nu_A)(y_2)$$

Case (2): If $f^{-1}(y_1) = \emptyset$ and $f^{-1}(y_2) = \emptyset$, then let $x_{10}, x_{20} \in X$ such that $\mu_A(x_{10}) = \inf_{z\in f^{-1}(y_1)} \mu_A(z)$, $\mu_A(x_{20}) = \inf_{z\in f^{-1}(y_2)} \mu(z)$ and:

$\mu_A(x_{10} * x_{20}) \quad \inf_{z\in f^{-1}(y_1*y_2)} \mu(z)$. Then $f(\mu_A)(y_1 * y_2) = \sup_{z\in f^{-1}(y_1*y_2)} \mu_A(z) = \mu_A(x_{10} * x_{20}) \geq \mu_A(x_{10}) \wedge \mu_A(x_{20})$

$$= (\inf_{z\in f^{-1}(y_1)} \mu_A(z)) \wedge (\inf_{z\in f^{-1}(y_2)} \mu_A(z)) = f(\mu_A)(y_1) \wedge f(\mu_A)(y_2).$$

Similarly, we have that:

$$\hat{f}(\nu_A)(y_1 * y_2) \leq \hat{f}(\nu_A)(y_1) \vee \hat{f}(\nu_A)(y_2)$$

Therefore, $f(A)$ is a intuitionsitic fuzzy B-algebra of Y.

Theorem 10: Let f be a homomorphism from a B-algebra X onto a B-algebra Y and A a intuitionsitic fuzzy B-algebra of Y. Then the preimage $f^{-1}(A)$ of A is a intuitionsitic fuzzy B-algebra of X.

Proof: Let $x,y \in X$. Since A is a intuitionsitic fuzzy B-algebra of Y , we have that:

$$\begin{aligned}\mu_{f^{-1}(A)}(x*y) &= \mu_A(f(x*y))\\ &= \mu_A(f(x) * f(y))\\ &\geq \mu_A(f(x)) \wedge \mu_A(f(y))\\ &= \mu_{f^{-1}(A)}(x) \wedge \mu_{f^{-1}(A)}(y)\end{aligned}$$

Similarly,

$$\nu_{f^{-1}(A)}(x*y) \leq \nu_{f^{-1}(A)}(x) \vee \nu_{f^{-1}(A)}(y)$$

Hence $f^{-1}(A)$ is a intuitionsitic fuzzy B-algebra of X .

Corollary 11: Let f be a homomorphism from a B-algebra X onto a B-algebra Y. Then the following conclusions hold:

- If for all $j \in J$, A_j, are intuitionsitic fuzzy B-algebras of X , then $f(\bigcap_{j\in J} A_j)$ is intuitionsitic fuzzy B-algebra of Y.
- If for all $t \in T$, B_t are intuitionsitic fuzzy B-algebras of Y, then $f^{-1}(\bigcap_{t\in T} B_t)$ is intuitionsitic fuzzy B-algebras of X.

- If A is a intuitionsitic fuzzy B-algebra of X, then f(A) and $f(\Diamond A)$ are intuitionsitic fuzzy B-algebra of Y.
- If B is a intuitionsitic fuzzy B-algebra of Y, then f^{-1}(B) and $f^{-1}(\Diamond B)$ are intuitionsitic fuzzy B-algebras of X.

Theorem 12: Let f be an isomorphism from a B-algebra X onto a B-algebra Y. If A is a intuitionsitic fuzzy B-algebra of X, then $f^{-1}(f(A)) = A$.

Proof: For any $x \in X$, let $f(x) = y$, since f is an isomorphism, $f^{-1}(y) = \{x\}$. Thus $= f(\mu_A)(f(x)) = f(\mu_A)(x)) = f(\mu_A)(y) = \bigvee_{x \in f^{-1}(y)} \mu_A(x) = \mu_A(x)$ and:

$$f^{-1}(\hat{f}(\nu_A))(x)\ \hat{f}(\nu_A)(f(x)) = \hat{f}(\nu_A)(y)$$

$$= \bigwedge_{x \in f^{-1}} \nu_A(x) = \nu_A(x)$$

therefore $f^{-1}(f(A)) = (A)$.

Corollary 13: Let f be an isomorphism from a B-algebra X onto a B-algebra$Y$. If B is a intuitionsitic fuzzy B-algebra of Y, then $f^{-1}(f(B)) = (B)$.

Corollary 14: Let $f : X \to X$ be an automorphism. If A is a intuitionsitic fuzzy B-algebra of X, then:

$$f(A) = A \Leftrightarrow f^{-1}(A) = A$$

A intuitionsitic fuzzy set: R = {⟨(x, y), μ_R (x, y), ν_R (x, y)⟩|x∈X, y∈Y}∈ IFS(X×Y) is called a binary intuitionsitic fuzzy relation (Lei *et al.*, 2005) from X into Y. A binary intuitionsitic fuzzy relation from X into Y is said to be a binary intuitionsitic fuzzy relation on X if $X = Y$.

Definition 3: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ IFS[X] A binary intuitionsitic fuzzy relation:

$$R = \left\{\left\langle (x,y), \mu_R(x,y), \nu_R(x,y)\right\rangle \middle| x, y \in X\right\}$$

on X is called a intuitionsitic fuzzy relation on A if $\mu_R(x, y) \le \mu_A(x) \wedge \mu_A(y)$ and $\nu_R(x,y) \ge \nu_A(x) \vee \nu_A(y)$ for all $x, y \in X$.

Lemma 15: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x), >| x \in X\}$ be intuitionsitic fuzzy sets of X. A Cartesian product of A and B defined by:

$$A \times B\left\{\left\langle (x,y), \mu_{A\times B}(x,y), \nu_{A\times B}(x,y)\right\rangle \middle| x, y \in X\right\}$$

where, $\mu_{A\times B} = \mu_A(x) \wedge \mu_B(y)$, $\nu_{A\times B} = \nu_A(x) \vee \nu_B(y)$. Then A×B is a binary intuitionsitic fuzzy relation on X.

Theorem 16: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x), >| x \in X\}$ be intuitionsitic fuzzy B-algebra of X. Then A×B is a intuitionsitic fuzzy B-algebra of $X \times X$.

Proof: Since A, B are intuitionsitic fuzzy B-algebras of X, we have:

$$\mu_{A\times B}((x,y)*(x',y'))$$

$$= \mu_{A\times B}(x*x', y*y')$$

$$= \mu_A(x*x') \wedge \mu_B(y*y')$$

$$\ge [\mu_A(x) \wedge \mu_A(x')] \wedge [\mu_B(y) \wedge \mu_B(y')]$$

$$= [\mu_A(x) \wedge \mu_B(y)] \wedge [\mu_A(x') \wedge \mu_B(y')]$$

$$= \mu_{A\times B}(x,y) \wedge \mu_{A\times B}(x',y')$$

for all:

$$(x,y),(x',y') \in X \times X$$

Similarly,

$$\nu_{A\times B}((x,y)*(x',y')) \le \nu_{A\times B}(x,y) \vee \nu_{A\times B}(x',y')$$

for all $(x, y), (x', y')$ X×X. Hence A×B is a intuitionsitic fuzzy B-algebra of X×X.

Theorem 17: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$and $B = \{< x, \mu_B(x), \nu_B(x), >| x \in X\}$ be intuitionsitic fuzzy sets of a B-algebra X such that A×B is a intuitionsitic fuzzy B-algebra of X×X . Then,

- Either $\mu_A(x) \le \mu_A(0)$ or $\mu_B(x) \le \mu_B(0)$ for all $x \in X$.
- Either $\nu_A(x) \le \nu_A(0)$ or $\nu_B(x) \ge \nu_B(0)$ for all $x \in X$.
- If $\mu_A(x) \le \mu_A(0)$ for all $x \in X$, then $\mu_A(x) \le \mu_B(0)$ or $\mu_B(x)$ $\mu_B(0)$.
- If $\nu_A(x) \le \nu_A(0)$ for all $x \in X$, then $\nu_A(x) \ge \nu_B(0)$ or $\nu_B(x) \ge \nu_B(0)$.
- If $\mu_B(x) \le \mu_B(0)$ for all $x \in X$, then $\mu_A(x) \le \mu_A(0)$ or $\mu_B(x) \le \mu_A(0)$.
- If $\nu_B(x) \le \nu_B(0)$ for all $x \in X$, then $\nu_A(x) \ge \nu_A(0)$ or $\nu_B(x) \ge \nu_A(0)$.
- Either μ_A or μ_B is a fuzzy B-algebra (Young *et al.*, 2002) of X.
- Either ν_A or ν_B is an anti-fuzzy B-algebra of X.'

Proof: (i) Suppose that $\mu_A(x) \le \mu_A(0)$ and $\mu_B(y) > \mu_B(0)$ for some $x, y \in X$. Then:

$$\mu_{A\times B}(x,y) = \mu_A(x) \wedge \mu_B(y)) > \mu_A(0) \wedge \mu_B(0) = \mu_{A\times B}(0,0).$$

This is a contradiction. Hence (i) holds.
(ii) is by similar method to part (i).
(iii) Assume that there exist $x, y \in X$. such that $\mu_A(x) > \mu_B(0)$ and $\mu_A(y) > \mu_B(0)$. Then $\mu_{A\times B}(x,y) = \mu_A(x) \wedge \mu_B(y) > \mu_B(0) = \mu_B(0)) = \mu_{A\times B}(0,0)$, which is a contradiction. Hence (iii) holds.
(vi), (v) and (vi) are by similar method to part (iii).
(vii) Since by (i) either $\mu_A(x) \le \mu_A(0)$ or $\mu_B(x) > \mu_B(0)$ for all $x \in X$, without loss of generality we may assume that $\mu_B(x) \le \mu_B(0)$ for all $x \in X$. From (v), it follows that $\mu_A(x) \le \mu_A(0)$ or $\mu_B(x) > \mu_A(0)$. If $\mu_B(x) > \mu_A(0)$, then $\mu_{A\times B}(0, x) = \mu_A(0) \wedge \mu_B(x) = \mu_B(x)$. Let (x_1, x_2), (y_1, y_2), $\in X\times X$. Since $A\times B$ is a intuitionsitic fuzzy B-algebra of , X×X we have:

$$\text{(IFB3)} \quad \mu_{A\times B}((x_1,x_2)*(y_1,y_2)) \ge \mu_{A\times B}((x_1,x_2) \wedge \mu_{A\times B}(y_1,y_2))$$

$$= [\mu_A(x_1) \wedge \mu_B(x_2)] \wedge [\mu_A(y_1) \wedge \mu_B(y_2)]$$

If we take $x_1 = y_2 = 0$, then

$$\mu_B(x_2 * y_2) = \mu_{A\times B}(0, x_2 * y_2)$$

$$\ge [\mu_A(0) \wedge \mu_B(x_2)] \wedge [\mu_A(0) \wedge \mu_B(y_2)]$$

$$= \mu_B(x_2) \wedge \mu_B(y_2)$$

This proves that μ_B is a fuzzy B-algebra of X. Now we consider the case $\mu_A(y) \le \mu_A(0)$ for all $x \in X$. Suppose that $\mu_B(y) > \mu_A(0)$ for some $y \in X$. Then $\mu_A(0) < \mu_B(y) \le \mu_B(0)$. Since $\mu_A(x) \le \mu_A(0)$ for all $x \in X$, we have $\mu_B(0) > \mu_A(x)$. for all $x \in X$. Hence $\mu_{A\times B}(x,0) = \mu_A(x) \wedge \mu_B(0) = \mu_A(x)$. Taking $x_2, y_2 = 0$ in (IFB3), then:

$$\mu_{A\times B}(x_1 * y_1) = \mu_{A\times B}(x_1 * y_1, 0)$$

$$\mu_{A\times B}((x_1,0)*(y_1,0))$$

$$\ge [\mu_A(x_1) \wedge \mu_B(0)] \wedge [\mu_A(y_1) \wedge \mu_B(0)]$$

$$= \mu_A(x_1) \wedge \mu_A(y_1))$$

Therefore μ_A is a fuzzy B-algebra of X.
(viii) is by similar method to part (vii) and the proof is completed.

From the proof of Theorem 17 (vii) and Theorem 17 (viii), the following results hold up.

Theorem 18: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x), >| x \in X\}$ be intuitionsitic fuzzy sets of a B-algebra X such that $A\times B$ is a intuitionsitic fuzzy B-algebra of $X \times X$. Then:

- If $\mu_A(x) \le \mu_B(0) \wedge \mu_B(0)$ and $\nu_A(x) \le \nu_B(0) \vee \nu_B(0)$ for all $x \in X$, then A is a intuitionsitic fuzzy B-algebra of X.
- If $\mu_B(x) \le \mu_A(0) \wedge \mu_B(0)$ and $\vee$for all $x \in X$, then B is a intuitionsitic fuzzy B-algebra of X.

Definition 4: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\} \in$ IFS(X). A intuitionsitic fuzzy relation:

$$R = \left\{\langle (x,y), \mu_R(x,y), \nu_R(x,y)\rangle \middle| x, y \in X\right\}$$

on X is called a strongest intuitionsitic fuzzy relation on A if:

$\mu_B(x, y) = \mu_A(x) \wedge \mu_A(y)$ and $\nu_R(x, y) = \nu_A(x) \vee \nu_A(y)$

for all $x, y \in X$.

Proposition 19: For a given intuitionsitic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ of a B-algebra X, let R be a strongest intuitionsitic fuzzy relation on A. If R is a intuitionsitic fuzzy B-algebra of $X \times X$, then $\mu_A(x) \ge \mu_A(0)$ and $\nu_A(x) \ge \nu_A(0)$ for all $x \in X$.

Proof: Since R is a intuitionsitic fuzzy B-algebra, we have $\mu_R(x, x) \le \mu_R(0,0)$ and $\nu_R(x, x) \ge \nu_R(0,0)$ for all $x \in X$, that is, $\wedge$ and $\nu_A(x) \wedge \nu_A(x) \ge \nu_A(0 \vee \nu_A(0)$. So $\mu_A(x) \le \mu_A(0)$ and $\nu_A(x) \ge \nu_A(0)$ for all $x \in X$.

Theorem 20: Let $A = \{< x, \mu_A(x), \nu_A(x), >| x \in X\}$ Let be a intuitionsitic fuzzy set of B-algebra X and:

$$R = \left\{\langle (x,y), \mu_R(x,y), \nu_R(x,y)\rangle \middle| x, y \in X\right\}$$

strongest intuitionsitic fuzzy relation on A. Then A is a intuitionsitic fuzzy B-algebra of X if and only if R is a intuitionsitic fuzzy B-algebra of $X \times X$.

Proof: Assume that A is a intuitionsitic fuzzy B-algebra of X. Then:

$$\mu_R((x_1,x_2)*(y_1,y_2)) = \mu_R(x_1 * y_1, x_2 * y_2)$$

$$= \mu_A(x_1 * y_1) \wedge \mu_A(x_2 * y_2)$$

$$\geq [\mu_A(x_1) \wedge \mu_A(y_1)] \wedge [\mu_A(x_2) \wedge \mu_A(y_2)]$$

$$= [\mu_A(x_1) \wedge \mu_A(x_2)] \wedge [\mu_A(y_1) \wedge \mu_A(y_2)]$$

$$= \mu_R(x_1, x_2) \wedge \mu_R(y_1, y_2)$$

for all $(x_1, x_2), (y_1, y_2) \in X \times X$. Similarly, we have that $\nu_R(x_1, x_2),*(y_1, y_2)) \leq \nu_R(x_1, x_2) \vee \nu_R(y_1, y_2)$ for all $(x_1, x_2), (y_1, y_2) \in X \times X$. Hence R is a intuitionsitic fuzzy B-algebra of $X \times X$.

Conversely, suppose that R is a intuitionsitic fuzzy B-algebra of $X \times X$ and let $x, y \in X$, Then,

$$\mu_A(x*y) = \mu_R(x*y, x*y)$$

$$= \mu_R((x, x)*, (y, y))$$

$$\geq \mu_R(x, x)* \wedge = \mu_R(\wedge y, y))$$

$$= \mu_R(x) \wedge \mu_R(y)$$

Similarly, we have $\nu_A(x*y) \leq \nu_A(x) \vee \nu_A(y)$. And completes the proof.

REFERENCES

Atanassov, K., 1986. Intuitionistic fuzzy sets. Fuzzy Sets Syst., 20(1): 87-96.

Hu, Q.O. and X. Li, 1983. On BCH-algebras. Math. Sem. Notes, 11: 13-320.

Hu, Q.O. and X. Li, 1985. On proper BCH-algebras. Math. Japan., 30: 659-661.

Imai, Y. and K. Iseki, 1966. On axiom systems of propositional calculi XIV. Proc. Japan Acad., 41: 19-22.

Iseki, K., 1966. An algebra related with a propositional calculus XIV. Proc. Japan Acad., 42: 26-29.

Jun, Y.B., 1993. Characterization of fuzzy ideals by their level ideals in BCK (BCI)-algebras. Math. Japan., 38: 67-71.

Jun, Y.B. and E.H. Roh, 1994. Fuzzy commutative ideals of BCK-algebras. Fuzzy Sets Syst., 64: 401-405.

Li, X.P. and G.J. Wang, 2000. Intuitionistic fuzzy group and its homomorphisic image. Fuzzy Syst. Math., 14(1): 45-50.

Li, X.P. and G.J. Wang, 2002. The extension operations of the intuitionistic fuzzy sets. Fuzzy Syst. Math., 16(1): 40-46.

Lei, Y.J., B.S. Wang and Q.G. Miao, 2005. On the intuitionsitic fuzzy relations with compositional operations. Syst. Eng. Theor. Pract., 2: 113-133.

Neggers, J. and H.S. Kim, 2001. On B-algebras. Int. J. Math. Sci., 27: 749-757.

Meng, J., 1994. Fuzzy ideals of BCI-algebras. SEA Bull. Math., 18: 65-68.

Xi, O.G., 1991. Fuzzy BCK-algebras. Math. Japan., 36: 935-942.

Young, B.J., H.R. Eun and J. Chin, 2002. On fuzzy B-algebras. Czechoslovak Math. J., 52(127): 375-384.

Zadeh, L.A., 1966. Fuzzy sets. Inform. Contr., 8: 338-352.