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SPI-based Regional Drought Prediction Using Weighted Markov Chain Model

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Abstract: Drought is one of the most serious natural disasters in China. Drought disasters occur frequently and caused huge economic loss in recently. In this paper, a drought prediction model based on weighted Markov Chain is put forward. An application is demonstrated by Anhui province of Huaihe River in China. Based on the precipitation data during 1958-2006 at monthly scale, the different time scales Standardized Precipitation Index (SPI) is computed and the occurrence frequency of extreme drought, severe drought, moderate drought, slight drought and non-drought is obtained. The prediction of SPI is conducted by weighted Markov Chain model and the prediction accuracy is computed for the SPI of different time scales. The results show that weighted Markov Chain model is an effective tool for drought prediction and can provide decision-making for regional drought management.

Keywords: Anhui province, drought prediction, Markov chain, Standardized Precipitation Index (SPI)

INTRODUCTION

Drought is one of the main natural disasters in China. Droughts occur more frequently and affect more region widely in recently. According to the statistics, the affected areas by drought accounted for 60% of the total areas by meteorological disasters from 1951 to 2006 and ranked the first in weather disasters (The China meteorological administration, 2007). At present, there are some studies about drought indicators to analyze the drought degree. For examples, Palmer Drought Severity Index (PDSI), integrated meteorological Drought Index (DI), Z index, precipitation anomaly percentage and Standardized Precipitation Index (SPI) and so on. Yuan and Zhou (2004) compared the Z-index and the Standardized Precipitation Index (SPI) and showed that SPI was better than the Z-index which was widely used in China. The SPI data is easier to access and the SPI can well reflect in the intensity and duration of drought in China. Yan and Lu (2009) combined the advantages of PDSI and SPI and proposed a comprehensive meteorological drought index. The results showed that the comprehensive drought index could well reflect the meteorological drought index of agricultural drought and hydrological drought. Markov model was first proposed by Markov in the early twentieth century, which is widely used in various fields including drought prediction. Zhang and Cui (2010) used Markov Chain to forecast the level of precipitation anomalies based on precipitation. Zhang and Zhou (2010) adopted dual-criteria decision methods and Markov Chain to study the prediction of drought. The existing research on drought prediction mostly directly used the indicator of precipitation. It was still rarely for drought prediction by predicting SPI.

In this study, the weighted Markov models are proposed for drought prediction using Standardized Precipitation Index (SPI). SPI drought indicators of different time scales are used to analyze the spatial and temporal evolution of drought of Anhui province of Huaihe River basin in recent 49 years (1958-2006). Some results are obtained for SPI prediction in Anhui Province of Huaihe River basin, China.

METHODOLOGY

Standardized Precipitation Index (SPI): Standardized Precipitation Index (SPI) was proposed by McKee and be used to assessed the Colorado's drought conditions in the United States in 1993. SPI is the standard deviation between measured precipitation and the rainfall probability distribution function. SPI can better reflect the intensity and duration of drought, making the use of the same drought indicators reflecting different time scales and regional drought conditions possible and therefore it is widely used (McKee *et al*., 1993).

SPI is a precipitation-based index and it can quantify the extent of the lack of rainfall at different times, which are generally 1, 3, 6, 9, 12 and 24 months, respectively. Such as: 3-month time scales SPI represents the standard deviation of precipitation 1-3 months and 12-month time scales SPI represents the standard deviation of precipitation 1-12 months. 3 and 6 month SPI time scales

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may reflect seasonal drought and 9-month, 12-month SPI time scales may reflect a regional long-term drought condition. One month SPI values can be expressed as SPI1, 3-month as SPI3, 6-month (SPI6), 9-month (SPI9), 12-months (SPI12) and 24-month (SPI24), respectively.

For different time scales of SPI indicators, quantitative analysis can be made about the shortage of rainfall in multiple periods and we can respond to the drought of different time periods. If we want to get a standardized precipitation index, we must first find the probability distribution of precipitation and then be able to find the normal standard. Specific solution procedure is as follows (Ma *et al*., 2002):

Step 1: Suppose that a period precipitation is random variable x, the distribution of the probability density function is:

$$f(x) = \frac{1}{\beta^{\gamma}\Gamma(\gamma)} x^{\gamma-1} e^{-x/\beta}, x > 0 \quad (1)$$

where, x is precipitation data, $\bar{x}$ is average rainfall and $\Gamma(\gamma) = \int_0^{\infty} x^{\gamma-1} e^{-x} dx$. $\beta > 0$, $\gamma > 0$, are the scale and shape parameter, respectively. β and γ can be obtained by the maximum likelihood estimation, that is, $\hat{\gamma} = \frac{1+\sqrt{1+4A/3}}{4A}$, $\hat{\beta} = \bar{x}/\hat{\gamma}$ and $A = \lg \bar{x} - \frac{1}{n}\sum_{i=1}^{n} \lg x_i$.

Step 2: Determine the parameters of the probability density function for a given precipitation x_0, the probability of random variable $x < x_0$ is defined as:

$$P(x < x_0) = \int_0^{\infty} f(x)dx \quad (2)$$

When the precipitation is zero, the probability is $P(x = 0) = m/n$, m is the number for zero of the sample precipitation, n is the number total sample.

Step 3: Transform Γ distribution probability into normal standardized, namely:

$$P(x < x_0) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-Z^2/2} dx \quad (3)$$

Solving Eq. (3) by approximate method and then the Z which represents the Standardized Precipitation Index (SPI) can be obtained:

Table 1: Drought level and category of SPI

Drought grade	Category	SPI
1	Non-drought	-0.5<SPI
2	Slight drought	-1.0<SPI≤-0.5
3	Moderate drought	-1.5<SPI≤-1.0
4	Severe drought	-2.0<SPI≤-1.5
5	Extreme drought	SPI≤-2.0

$$Z = S\frac{t - (c_2 t + c_1)t + c_0}{((d_3 t + d_2)t + d_1)t + 1.0} \quad (4)$$

where, $t = \sqrt{1n\frac{1}{P^2}}$, $c_0 = 2.515517$, $c_1 = 0.802853$, $c_2 = 0.010328$, $d_1 = 1.432788$, $d_2 = 0.189269$, $d_3 = 0.001308$. When $P > 0.5$, $P = 1.0 - P$ and S = 1; When $P \le 0.5$, S = − 1. According to the file of Chinese GB/T20481-2006, the drought grade and category based on the SPI are shown in Table 1.

Weighted Markov Chain model: A Markov chain, named after Andrey Markov, is a mathematical system that undergoes transitions from one state to another, between a finite or countable number of possible states (Bolch *et al*., 2001). It is a random process characterized as memoryless: the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical models of real-world processes. In this section, we briefly describe some basic concepts related to Markov Chain. Markov Chain is a method based on random process theory and probability theory (Peng *et al*., 2009; Paulo *et al*., 2005).

A discrete-time stochastic process $\{X_n, n = 0, 1, 2, \ldots\}$, with finite state space $E = \{i_0, i_1, i_2, \ldots\}$, is a Markov chain, if it has the Markov property (memoryless) $p\{X_{n+1} = i_{n+1}/X_n = i_n, X_{n-1} = i_{n-1}, \ldots, X_0 = i_0\} = p\{X_{n+1} = i_{n+1}/ X_n = i_n\}$. The probability of going from state i to state j in n time steps is $p^{(n)}_{ij} = P(X_n = j \mid X_0 = i)$. and the single-step transition is $p_{ij} = P(X_1 = j \mid X_0 = i)$. For a time-homogeneous Markov chain: $p^{(n)}_{ij} = P(X_{k+n} = j \mid X_k = i)$ and $p_{ij} = P(X_{k+1} = j \mid X_k = i)$. The n-step transition probabilities satisfy the Chapman-Kolmogorov equation, that for any k such that $0<k<n$, $p^{(n)}_{ij} = \sum_{r \in E} p^{(k)}_{ir} p^{(n-k)}_{rj}$.

The basic steps of drought prediction using SPI and weighted Markov Chain are shown as follows:

Step 1: Determine the state space $E = \{1,2,\ldots,N\}$ of the Markov Chain.

For drought prediction, the state space represents the SPI classification standard. That is, there are five states, as shown in Table 1.

Step 2: Calculate SPI3, SPI6, SPI9, SPI12 and according to the classification standard to determine the

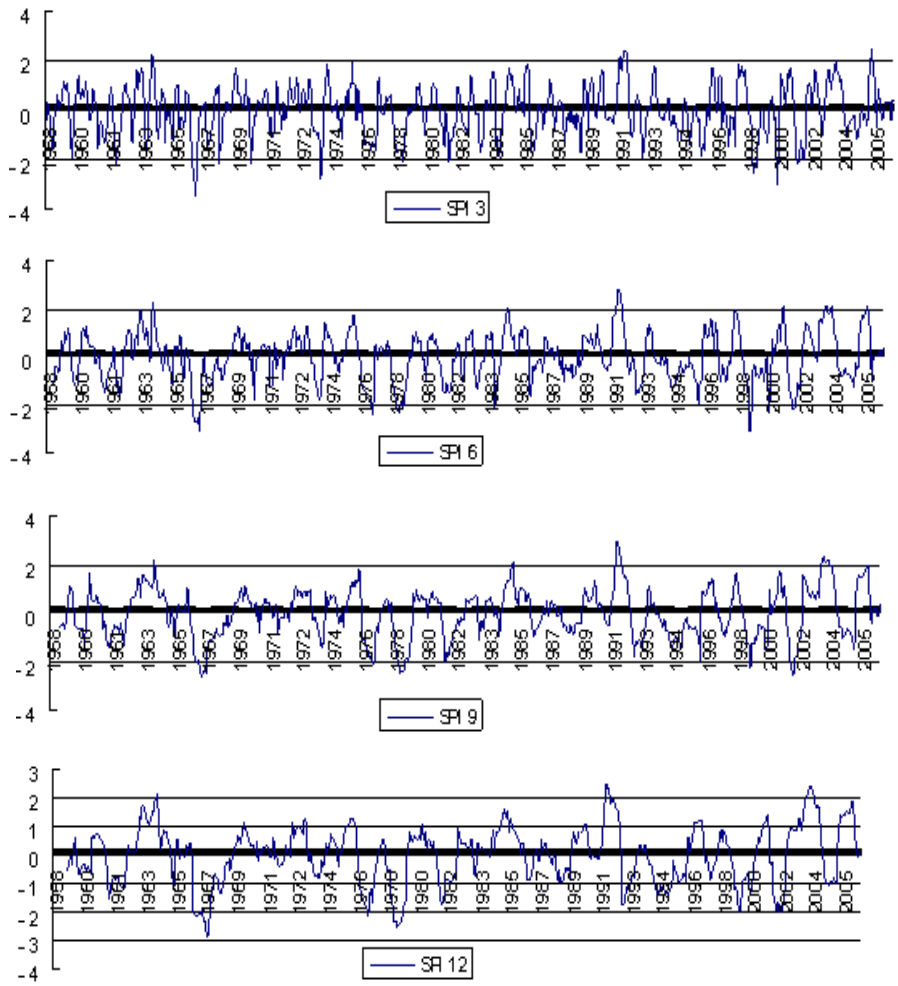

Fig. 1: Different scales SPI change curves of Anhui province from 1958 to 2006

state of SPI every month. Then, construct matrix of Markov Chain transition probability of different lag and do test for Markov characteristics.

Step 3: Calculate self-coefficients r_k:

$$r_k = \sum_{i=1}^{n-k}(x_i - \bar{x})(x_{i+k} - \bar{x}) / \sum_{i=1}^{n}(x_i - \bar{x})^2$$

r_k is related coefficient of k lag, k is step length, $k = 1, 2, \ldots m$. x_i is SPI value of i period time, $\bar{x}$ is the average SPI value, n is the sequence length of SPI. Here, k = 1, 2, 3. The Markov Chain weight of different lag can be computed as follows by standardizing self-coefficients:

$$w_k = |r_k| / \sum_{k=1}^{m}|r_k|$$

Step 4: Forecast the state probability of SPI of the month: $p^{(k)}_i$ according to the foregoing SPI value and corresponding transition probability matrix.

Step 5: Weighting the same state of the predictions probability, the weighted sum is the prediction probability P_i of SPI in the state:

$$P_i = \sum_{k=1}^{m} w_k P_i^{(k)}$$

The future state is corresponding state of the maximum prediction probability P_{imax} .

CASE STUDY

Study region and data: Anhui province is located in southeastern China and the lower reaches of Yangtze

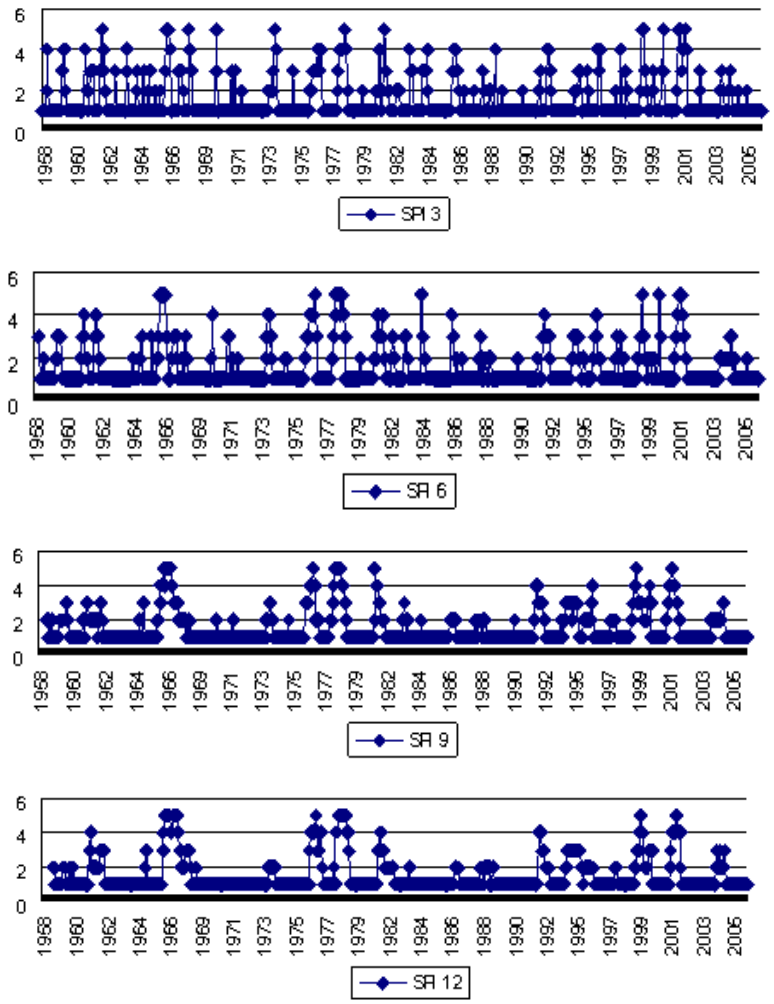

Fig. 2: Drought level of different time scales SPI during 1958-2006

River and the Huaihe River basin on both sides. Anhui province is called the "the land of fish and rice" in China. It belongs to north-south climatic transition zone. About two thirds of Huaihe River Basin is extensive flat plain and there exists a steep climatological gradient from wet southeast to dry northwest regions. The drought disasters occur frequently in Huaihe River Basin. Anhui province of Huaihe River basin is also one of the vulnerable drought regions in China and where the frequency and intensity of droughts have increased significantly in recently. The disasters have made adverse impacts on the regional development of economic and society (Xu *et al*., 2011; Li *et al*., 2009; Moreira *et al*., 2008).

In this study, the regional drought of Anhui region of Huaihe River Basin is studied. The precipitation data of Fuyang, Shouxian, Bengbu, Liuan, Bozhou, Suzhou and Huoshan rainfall stations during 1958-2006 at monthly scale were used. The data comes mainly from the State Information Center and Anhui Meteorological Bureau. Standardized Precipitation Index (SPI) is adopted and the four time scales 3, 6, 9, 12 months are chosen to analyze these data. The SPI3, SPI6, SPI9, SPI12 are computed to describe the different drought degree and then the change law of drought is explored. Anhui region of Huaihe River basin is a frequent drought region, with different drought degree occurring every year. The computing results of different scales SPI are shown in Fig. 1. Figure 1 shows that the drought has certain cyclical.

RESULTS AND ANALYSIS

According to the SPI value, we can determine the drought level, as shown in Fig. 2. The drought intensity, drought frequency and drought conditions during 1958-2006 can be analyzed according to the SPI value.

We analyze the drought law according to 3-month scale SPI value. From 1958 to 2006, only in 1960, 1972,

Table 2: The prediction results by weighted Markov model

Time scales	Index	Non-drought	Slight drought	Moderate drought	Severe drought	Extreme drought	Total
SPI3	Actual	424	59	57	30	16	586
	Prediction	424	15	22	9	7	477
	Accuracy rate	1.00	0.254	0.386	0.300	0.438	0.814
SPI6	Actual	406	78	61	20	18	583
	Prediction	406	21	23	2	2	454
	Accuracy rate	1	0.269	0.377	0.100	0.111	0.779
SPI9	Actual	388	100	54	22	16	580
	Prediction	388	37	15	4	3	477
	Accuracy rate	1	0.370	0.278	0.182	0.188	0.771
SPI12	Actual	401	83	46	26	21	577
	Prediction	401	26	14	4	10	455
	Accuracy rate	1	0.313	0.304	0.154	0.476	0.789

1985, 1993 and 2003, Anhui of Huaihe River basin did not occur any level drought and the other years have occurred different levels drought. According to SPI3 value, there are 162 months of drought with the drought frequency of 27.6% during 1958-2006. Extreme drought frequency is 2.7%, severe drought frequency is 5.1%, moderate drought frequency is 9.7% and slight drought frequency is 10.1%. The analysis revealed that 3-month drought occurred 8 times, 4-month drought 5 times, 5-month drought for 3 times, 6-month drought 4 times, 7-month one time, 8-month one time and 10-month for 2 times.

According to the basic steps of weighted Markov Chain, we can conduct SPI prediction. The Accuracy rate can be computed by the prediction value and actual value, that is, the ratio of number of prediction correct state with number of actual state. Based on the above method, Table 2 shows the accuracy rate of prediction at different scales. It shows that the weighted Markov Chains has quite accuracy in forecasting for different scales of SPI drought. For SPI3, the total accuracy rate is 81.4%. For SPI6, the total accuracy rate is 77.9%. For SPI9, the total accuracy rate is 77.1%. For SPI12, the total accuracy rate is 78.9%.

CONCLUSION

In this paper, a drought prediction model based on weighted Markov Chain is established. Four time scales SPI values of Anhui region of Huaihe River basin during 1958-2006 are computed and analyzed. The different time scales of SPI can reflect the evolution of drought conditions and trends. The short time scales, such as 3 months and 6 months time scales of SPI, can reflect changes in seasonal drought. The long time-scale SPI, such as 9 or 12 months, can provide better long-term continuous monitoring of drought and play an important role in monitoring and evaluating droughts. The results of drought prediction are obtained by weighted Markov Chain model and the prediction accuracy is computed for the SPI of different time scales. The case study shows that the weighted Markov Chain model is effective for drought prediction and can be helpful for regional drought disaster management.

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