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Analytical Solution of the Point Reactor Kinetics Equations for One-Group of Delayed Neutrons for a Discontinuous Linear Reactivity Insertion

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Abstract: The understanding of the time-dependent behaviour of the neutron population in a nuclear reactor in response to either a planned or unplanned change in the reactor conditions is of great importance to the safe and reliable operation of the reactor. It is therefore important to understand the response of the neutron density and how it relates to the speed of lifting control rods. In this study, an analytical solution of point reactor kinetic equations for one-group of delayed neutrons is developed to calculate the change in neutron density when reactivity is linearly introduced discontinuously. The formulation presented in this study is validated with numerical solution using the Euler method. It is observed that for higher speed, r = 0.0005 the Euler method predicted higher values than the method presented in this study. However with r = 0.0001, the Euler method predicted lower values than the method presented in this study except for t = 1.0 s and 5.0 s. The results obtained have shown to be compatible with the numerical method.

Keywords: Analytical solution, extraneous neutron source, linear reactivity, neutron population, point reactor kinetic, time-dependent

INTRODUCTION

The base of a reactor model is a set of ordinary differential equations known as the point reactor kinetics equations. These equations which expresses the time-dependence of the neutron population and the decay of the delayed neutron precursors within a reactor are first order and linear and essentially describe the change in neutron population within the reactor due to a change in reactivity.

One of the important properties in a nuclear reactor is the reactivity, due to the fact that it is directly related to the control of the reactor. For safety analysis and transient behaviour of the reactor, the neutron population and the delayed neutron precursor concentration are important parameters to be studied.

The start-up process of a nuclear reactor requires that reactivity is varied in the system by lifting the control rods discontinuously. In practice, the control rods are withdrawn at time intervals such that reactivity is introduced in the reactor core linearly, to allow criticality to be reached in a slow and safe manner.

Under simplified conditions, an analytical solution to the point reactor kinetic equations can be obtained, for example, such as there being no extraneous neutron source, assuming a prompt jump approximation (Zhang *et al*., 2008; Chen *et al*., 2006, 2007; Li *et al*., 2007) and a constant neutron source (Nahla, 2009). Aboanber and Hamada (2003) solved the point reactor kinetic equations analytically in the presence of Newtonian temperature feedback for different types of reactivity input using a straightforward recurrence relation of a power series. Zhang *et al*. (2008) described an analytic method study of point reactor kinetic equation when cold start-up. Daniel *et al*. (2009) presented analytical solution of the point kinetic equations for linear reactivity variation during the start-up of a nuclear reactor. However these analytical solutions can only be used for qualitative rather than quantitative computation.

For quantitative computations, a numerical calculation is used. Furthermore, a very small step size has to be adopted for stability due to the problem of stiffness in the differential equations when applying a general numerical method such as the Euler, Runge-Kutta, or Adams method. Many numerical methods have been proposed in literature, such as the Runge-Kutta procedure (Sánchez, 1989), quasistatic method (Koclas *et al*., 1996), piecewise polynomial approach (Hennart, 1977), stiffness confinement method (Chao and Attard, 1985), power series solution (Aboanber and Hamada, 2002, 2003; Sathiyasheela, 2009), Padé approximation (Aboanber, 2004; Aboanber and Nahla, 2002), CORE (Quintero-Leyva, 2008) and Kinard and Allen (2004).

In this study, an analytical solution of point reactor kinetic equations for one-group of delayed neutrons is

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developed to calculate the change in neutron density when reactivity is linearly introduced discontinuously. Also the neutron density responses with the speed of lifting control rod and the duration have also been studied and the results compared with the solution using the Euler method and that of Zhang *et al*. (2008).

Zhang *et al*. (2008) developed an analytical expression for the calculation of neutron density response based on the kinetics equations with one-group of delayed neutron precursors:

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \lambda C(t) + q \tag{1}$$

$$\frac{dC(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda C(t) \tag{2}$$

with the reactivity insertion expressed as:

$$\rho(t) = \begin{cases} \rho_0 + rt & 0 \le t < t_0 \\ \rho_0 + rt_0 & t \ge t_0 \end{cases} \tag{3}$$

where, n(t) is the neutron density, ρ(t) is the reactivity, Λ is the prompt neutron generation lifetime, λ is the radioactive decay constant of the delayed neutron precursor, β is the total fraction of delayed neutrons, C(t) is the delayed neutron precursor density, q is the external neutron source intensity, r is the velocity of inserted reactivity, t is time, t_0 is the duration of each lifting of control rod and ρ_0 is the sub-critical reactivity.

By assuming the prompt jump approximation and a constant neutron source, Zhang *et al*. (2008) obtained the following expression for the neutron density:

$$n(t) = \begin{cases} \dfrac{\beta n_0 + q\Lambda}{\beta + |\rho_0| - rt}, & 0 \le t < t_0 \\ \dfrac{q\Lambda}{|\rho_0| - rt_0}\left[1 - e^{-\frac{\lambda(|\rho_0| - rt_0)}{\beta + |\rho_0| - rt_0}(t - t_0)}\right] \\ + \dfrac{\beta n_0 + q\Lambda}{\beta + |\rho_0| - rt_0} e^{-\frac{\lambda(|\rho_0| - rt_0)}{\beta + |\rho_0| - rt_0}(t - t_0)}, & t \ge t_0 \end{cases} \tag{4}$$

MATHEMATICAL FORMULATION

Differentiate Eq. (1) with respect to t, we get:

$$\frac{d^2 n(t)}{d^2 t} = \frac{n(t)}{\Lambda}\frac{d\rho(t)}{dt} + \frac{\rho(t) - \beta}{\Lambda}\frac{dn(t)}{dt} + \lambda \frac{dC(t)}{dt} \tag{5}$$

Substituting Eq. (2) into (5), we get:

$$\frac{d^2 n(t)}{d^2 t} = \frac{n(t)}{\Lambda}\frac{d\rho(t)}{dt} + \frac{\rho(t) - \beta}{\Lambda}\frac{dn(t)}{dt} + \lambda\left[\frac{\beta}{\Lambda} n(t) - \lambda C(t)\right] \tag{6}$$

From Eq. (1) and (6) and eliminating λC(t), we get:

$$\frac{d^2 n(t)}{d^2 t} = \frac{n(t)}{\Lambda}\frac{d\rho(t)}{dt} + \frac{\rho(t)}{\Lambda}\frac{dn(t)}{dt} + - \frac{\beta}{\Lambda}\frac{dn(t)}{dt} + \lambda\frac{\beta}{\Lambda} n(t) + \lambda q + \frac{\lambda\rho(t)}{\Lambda} n(t) - \frac{\lambda\beta}{\Lambda} n(t) - \lambda\frac{dn(t)}{dt} \tag{7}$$

Further simplification of Eq. (7) leads to:

$$\frac{d^2 n(t)}{d^2 t} = \frac{n(t)}{\Lambda}\frac{d\rho(t)}{dt} + \frac{\rho(t) - \beta - \lambda\Lambda}{\Lambda}\frac{dn(t)}{dt} + \frac{\lambda\rho(t)}{\Lambda} n(t) + \lambda q \tag{8}$$

When $t \ge t_0$, $\rho(t) = \rho_0 + rt_0$, $\frac{d\rho(t)}{dt} = 0$, so that Eq. (7) reduces to:

$$\frac{d^2 n(t)}{d^2 t} + \frac{\beta + \lambda\Lambda - rt_0 - \rho_0}{\Lambda}\frac{dn(t)}{dt} - \frac{\lambda(\rho_0 + rt_0)}{\Lambda} n(t) = \lambda q \tag{9}$$

Let $\rho_m = \beta + \lambda\Lambda - rt_0 - \rho_0$ and $\rho_k = rt_0 + \rho_0$, so that:

$$\frac{d^2 n(t)}{d^2 t} + \frac{\rho_m}{\Lambda}\frac{dn(t)}{dt} - \frac{\lambda\rho_k}{\Lambda} n(t) = \lambda q \tag{10}$$

The solution to Eq. (10) is:

$$n(t) = A_1 \exp\left(\frac{-\rho_m + \sqrt{\rho_m^2 + 4\lambda\rho_k\Lambda}}{2\Lambda} t\right) + A_2 \exp\left(\frac{-\rho_m - \sqrt{\rho_m^2 + 4\lambda\rho_k\Lambda}}{2\Lambda} t\right) - \frac{q\Lambda}{\rho_k} \tag{11}$$

Similar to the derivation of Eq. (10), we obtain:

$$\frac{d^2 C(t)}{d^2 t} + \frac{\rho_m}{\Lambda}\frac{dC(t)}{dt} - \frac{\lambda\rho_k}{\Lambda} C(t) = \frac{\beta q}{\Lambda} \tag{12}$$

The solution to Eq. (12) is:

$$C(t) = B_1 \exp\left(\frac{-\rho_m + \sqrt{\rho_m^2 + 4\lambda\rho_k\Lambda}}{2\Lambda} t\right) + B_2 \exp\left(\frac{-\rho_m - \sqrt{\rho_m^2 + 4\lambda\rho_k\Lambda}}{2\Lambda} t\right) - \frac{\beta q}{\lambda\rho_k} \tag{13}$$

From the initial conditions, when t = 0, $n(0) = n_0$ and $C(0) = C_0$, Eq. (11) and (13) reduces to:

$$n_0 = A_1 + A_2 - \frac{q\Lambda}{\rho_k} \tag{14}$$

$$C_0 = B_1 + B_2 - \frac{\beta q}{\lambda\rho_k} \tag{15}$$

Differentiating Eq. (13) we get:

$$\frac{dC(t)}{dt} = B_1\left(\frac{-\rho_m + k}{2\Lambda}\right)\exp\left(\frac{-\rho_m + k}{2\Lambda} t\right) + B_2\left(\frac{-\rho_m - k}{2\Lambda}\right)\exp\left(\frac{-\rho_m - k}{2\Lambda} t\right) \tag{16}$$

where, $k = \sqrt{\rho_m^2 + 4\lambda\rho_k\Lambda}$

Substituting Eq. (11), (13), (16) into (2), gives:

$$B_1\left(\frac{-\rho_m+k}{2\Lambda}\right)\exp\left(\frac{-\rho_m+k}{2\Lambda}t\right)+B_2\left(\frac{-\rho_m-k}{2\Lambda}\right)\exp\left(\frac{-\rho_m-k}{2\Lambda}t\right)= \frac{\beta}{\Lambda}\left[A_1\exp\left(\frac{-\rho_m+k}{2\Lambda}t\right)+A_2\exp\left(\frac{-\rho_m-k}{2\Lambda}t\right)-\frac{q\Lambda}{\rho_k}\right]- \lambda\left[B_1\exp\left(\frac{-\rho_m+k}{2\Lambda}t\right)+B_2\exp\left(\frac{-\rho_m-k}{2\Lambda}t\right)-\frac{\beta q}{\lambda\rho_k}\right] \tag{17}$$

The constant terms on the right hand side of Eq. (17) cancels out and by grouping like terms on both sides of the equation we get:

$$B_1\left(\frac{-\rho_m+k}{2\Lambda}\right)\exp\left(\frac{-\rho_m+k}{2\Lambda}t\right)=\frac{A_1\beta}{\Lambda}\exp\left(\frac{-\rho_m+k}{2\Lambda}t\right)-B_1\lambda\exp\left(\frac{-\rho_m+k}{2\Lambda}t\right) \tag{18}$$

and

$$B_2\left(\frac{-\rho_m-k}{2\Lambda}\right)\exp\left(\frac{-\rho_m-k}{2\Lambda}t\right)=\frac{A_2\beta}{\Lambda}\exp\left(\frac{-\rho_m-k}{2\Lambda}t\right)-B_2\lambda\exp\left(\frac{-\rho_m-k}{2\Lambda}t\right) \tag{19}$$

Further simplification of Eq. (18) reduces to:

$$A_1=B_1\left(\frac{-\rho_m+k2\Lambda\lambda}{2\beta}\right) \tag{20}$$

Similarly, simplifying Eq. (19) we get:

$$A_2=B_2\left(\frac{-\rho_m-k+2\Lambda\lambda}{2\beta}\right) \tag{21}$$

Adding Eq. (20) and (21) gives:

$$A_1+A_2=B_1\left(\frac{-\rho_m+k+2\Lambda\lambda}{2\beta}\right)+B_2\left(\frac{-\rho_m-k+2\Lambda\lambda}{2\beta}\right) \tag{22}$$

But from Eq. (14), $A_1+A_2=n_{00}+\frac{q\Lambda}{\rho_k}$. Thus,

$$n_0+\frac{q\Lambda}{\rho_k}=B_1\left(\frac{-\rho_m+k+2\Lambda\lambda}{2\beta}\right)+B_2\left(\frac{-\rho_m-k+2\Lambda\lambda}{2\beta}\right) \tag{23}$$

And after simplifying Eq. (23) we get:

$$2\beta n_0+\frac{2\beta q\Lambda}{\rho_k}=B_1(-\rho_m+k+2\Lambda\lambda)+B_2(-\rho_m-k+2\Lambda\lambda) \tag{24}$$

But from Eq. (15):

$$B_2=C_0+\frac{\beta q}{\lambda\rho_k}-B_1 \tag{25}$$

Substituting Eq. (25) into (24) we get:

$$2\beta n_0+\frac{2\beta q\Lambda}{\rho_k}=B_1(-\rho_m+k+2\Lambda\lambda)-\rho_m C_0-\frac{\rho_m\beta q}{\lambda\rho_k}+\rho_m B_1 -kC_0-\frac{k\beta q}{\lambda\rho_k}+kB_1+2\Lambda\lambda C_0+\frac{2\Lambda\beta q}{\rho_k}-2\Lambda\lambda B_1 \tag{26}$$

Further simplification of Eq. (26) leads to:

$$B_1=\frac{(\lambda\rho_k C_0+\beta q)}{2\rho_k\lambda}+\frac{\rho_m(\rho_k\lambda C_0+\beta q)+2\lambda\rho_k(\beta n_0-\Lambda\lambda C_0)}{2k\lambda\rho_k} \tag{27}$$

Substitute Eq. (27) into (25) to get:

$$B_2=\frac{\rho_k C_0\lambda+\beta q}{2\lambda\rho_k}-\left[\frac{\rho_m(\rho_k\lambda C_0+\beta q)+2\lambda\rho_k(\beta n_0-\Lambda\lambda C_0)}{2k\lambda\rho_k}\right] \tag{28}$$

Substituting Eq. (27), (28) into (20) and (21) respectively gives:

$$A_1=\frac{-\rho_m+k+2\Lambda\lambda}{2\beta}\times\left[\frac{\rho_k C_0\lambda+\beta q}{2\lambda\rho_k}+ \frac{\rho_m(\rho_k\lambda C_0+\beta q)+2\lambda\rho_k(\beta n_0-\Lambda\lambda C_0)}{2k\lambda\rho_k}\right] \tag{29}$$

and

$$A_2=\frac{-\rho_m-k+2\Lambda\lambda}{2\beta}\times\left[\frac{\rho_k C_0\lambda+\beta q}{2\lambda\rho_k}- \frac{\rho_m(\rho_k\lambda C_0+\beta q)+2\lambda\rho_k(\beta n_0-\Lambda\lambda C_0)}{2k\lambda\rho_k}\right] \tag{30}$$

But $k=\sqrt{\rho_m^2+4\lambda\rho_k\Lambda}$. Therefore Eq. (29) and (30) becomes after substituting for k:

$$A_1=\frac{-\rho_m+\sqrt{\rho_m^2+4\lambda\rho_k\Lambda}+2\Lambda\lambda}{2\beta}\times\left[\frac{\rho_k C_0\lambda+\beta q}{2\lambda\rho_k}+ \frac{\rho_m(\rho_m\lambda C_0+\beta q)+2\lambda\rho_k(\beta n_0-\Lambda\lambda C_0)}{2k\lambda\rho_k}\right] \tag{31}$$

and

$$A_2=\frac{-\rho_m-\sqrt{\rho_m^2+4\lambda\rho_k}+2\Lambda\lambda}{2\beta}\times\left[\frac{\rho_k C_0+\beta q}{2\lambda\rho_k}- \frac{\rho_m(\rho_k\lambda C_0+\beta q)+2\lambda\rho_k(\beta n_0-\Lambda\lambda C_0)}{2k\lambda\rho_k}\right] \tag{32}$$

Equation (31) and (32) are substituted into Eq. (11) to obtain the final solution to the time dependent neutron density.

Using the prompt jump approximation and a constant neutron source, at t = 0 when $n(t) = n_0$:

$$C(t)=C_0=\frac{\beta n_0}{\Lambda\lambda} \tag{33}$$

Substituting Eq. (33) into (1) we get:

$$\frac{dn(t)}{dt}=\frac{\rho(t)-\beta}{\Lambda}n(t)+\frac{\beta n_0}{\Lambda}+q \tag{34}$$

Then using the prompt jump approximation:

$$n(t)=\frac{\beta n_0+q\Lambda}{\beta-\rho(t)}=\frac{\beta n_0+q\Lambda}{\beta+|\rho_0|-rt} \tag{35}$$

Table 1: Comparison of the results obtained using Eq. (37) and the Euler method at $t_0 = 5$, $\rho_0 = -0.04$ and r = 0.0005

t (s)	Euler method h = 0.01 s	Euler method h = 0.001 s	This study	Percentage difference (%)
0.1	$3.239197x10^6$	$3.174848x10^6$	$3.161223x10^6$	0.4292
1.0	$3.333852x10^6$	$3.333853x10^6$	$3.191490x10^6$	4.2702
5.0	$3.336069x10^6$	$3.336068x10^6$	$3.333334x10^6$	0.0819
50.0	$3.360500x10^6$	$3.360504x10^6$	$3.360097x10^6$	0.0121
60.0	$3.365807x10^6$	$3.365811x10^6$	$3.365407x10^6$	0.0120
80.0	$3.376289x10^6$	$3.376293x10^6$	$3.375894x10^6$	0.0118

Table 2: Comparison of the results obtained using Eq. (37) and the Euler method at $t_0 = 5$, $\rho_0 = -0.04$ and r = 0.0001

t (s)	Euler method h = 0.01 s	Euler method h = 0.001 s	This study	Percentage difference (%)
0.1	$3.117131x10^6$	$3.059251x10^6$	$3.158560x10^6$	-3.2461
1.0	$3.191966x10^6$	$3.191967x10^6$	$3.164557x10^6$	0.8587
5.0	$3.193998x10^6$	$3.193998x10^6$	$3.191490x10^6$	0.0785
50.0	$3.216390x10^6$	$3.216394x10^6$	$3.217322x10^6$	-0.0288
60.0	$3.221253x10^6$	$3.221257x10^6$	$3.222178x10^6$	-0.0286
80.0	$3.230857x10^6$	$3.230861x10^6$	$3.231722x10^6$	-0.0266

and

$$n(t_0) = \frac{\beta n_0 + q\Lambda}{\beta - \rho(t_0)} = \frac{\beta n_0 + q\Lambda}{\beta + |\rho_0| - rt_0} \quad (36)$$

Thus the time dependent neutron density is expressed as:

$$\begin{cases} n(t) = A_1 \exp\left(\frac{-\rho_m + \sqrt{\rho_m^2 + 4\lambda\rho_k\Lambda}}{2\Lambda} t\right) + \\ A_2 \exp\left(\frac{-\rho_m - \sqrt{\rho_m^2 + 4\lambda\rho_k\Lambda}}{2\Lambda} t\right), \\ for \quad t \geq 0 \; and \\ n(t) = \frac{\beta n_0 + q\Lambda}{\beta + |\rho_0| - rt} \quad for \;\; 0 \leq t \leq t_0 \end{cases} \quad (37)$$

where, A_1 and A_2 are as expressed in Eq. (31) and (32).

RESULTS

In order to verify the analytical method proposed in this study for the prediction of the neutron density distribution, the Euler method was used for the numerical solution of the point kinetics equations, Eq. (1) and (2). The nuclear parameters used in the validation of formulation proposed in this study against the formulation proposed by Zhang *et al*. (2008) Eq. (4) and the numerical method are: $\lambda = 0.001s^{-1}$, $\Lambda = 0.0015s$, $\beta = 0.0075$, q = $1x10^8$ neutrons / cm^3 s and the sub-criticality $\rho_0 = -0.04$. To obtain accurate solution, the time step for the Euler method was chosen to be h = 0.001s and h = 0.01s.

The neutron density responds of inserting discontinuous linear reactivity using Eq. (37) and the Euler method with time step h = 0.01s and h = 0.001s when durations are the same and speeds are different are compared in Table 1 and 2. It is observed that for higher speed (Table 1) r = 0.0005 the Euler method predicted higher values than the method presented in this study. However with r = 0.0001 (Table 2), the Euler method predicted lower values than the method presented in this study except for t = 1.0 and 5.0 s, respectively.

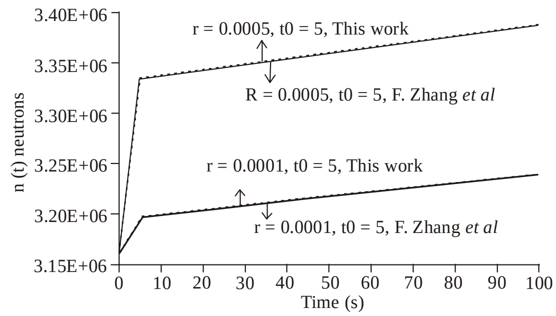

Fig. 1: Comparison between calculation methods for neutron density response when speeds are different and durations are the same

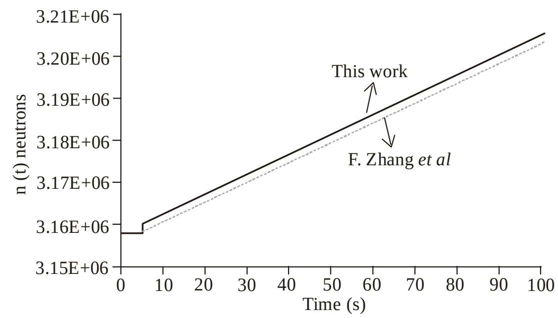

Fig. 2: Comparison between calculation methods for neutron density response at $t_0 = 5$ and r = 0.0000001

Again it is observed from Table 1 and 2 that the percentage difference between the Euler method at h = 0.001 and the formulation presented in this study generally decreases with increasing time. In other words, as the time increases, the difference between the analytical method and the numerical method (Euler method) reduces.

It is also observed from Table 1 and 2 that for higher speed the maximum percentage difference between the Euler method and the formulation presented in this study is 4.2702% and for lower speed, the maximum percentage difference between the Euler method and the formulation presented in this study is 0.8587%.

Figure 1 show the response of neutron density distribution when speeds are different and durations are the same. In Fig. 1, the results obtained using the formulation proposed in this study is compared with the formulation of Zhang *et al*. (2008) and one can see the similarity between the two methods.

In Fig. 2 the neutron density response of the formulations in this study against the formulation of Zhang *et al*. (2008) for a relatively low speed or a relatively big time interval for control rods withdrawal is shown. Again as can be seen in Fig. 2, the similarity between the two methods can be observed.

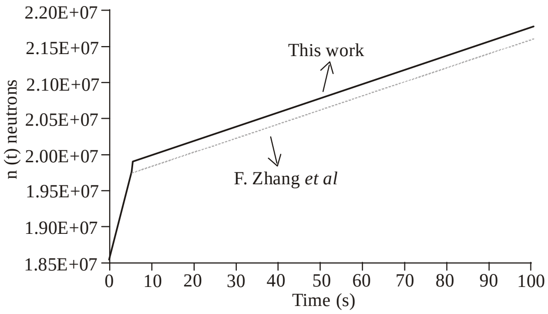

Fig. 3: Comparison between calculation methods for neutron density response at $t_0 = 5$, $\rho_0 = -0.0006$ and r = 0.0001

Figure 3 shows the neutron density response of the formulations in this study against the formulation of Zhang *et al*. (2008) for a reactivity insertion expressed by $\rho(t) = -0.0006+0.0001t$ such as might occur when the reactor is close to criticality. It is observed from Fig. 3 that the two results agree well with each other with the formulation in this study predicting slightly higher values.

CONCLUSION

An analytical solution has been presented in this study to seek predicting the neutron density response when reactivity is linearly introduced discontinuously or when reactivity is linearly introduced in a short period of time. The formulation consists of the solution of the point kinetics equations for one-group of delayed neutrons. Calculations were performed using the analytical method and the results compared with a numerical solution using Euler's method as well as formulation proposed by Zhang *et al*. (2008). The results obtained have shown to be compatible with the numerical method as well as those obtained by Zhang *et al*. (2008).

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