

Performance Analysis of Bessel Beamformer in AWGN Channel Model Using Digital Modulation Technique

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Abstract: In this study, we simulate a communication system model using adaptive beamforming algorithm named as Bessel beamformer in Additive White Gaussian Noise (AWGN) environment and analyzed its performance employing 4 point Quadrature Amplitude Modulation (QAM) technique. This model describes the input and output relationship of physical systems. Bessel beamformer is used in smart antenna, a part of communication system to calculate weights adaptively to optimize Signal to Noise Ratio (SNR) of the desired signal in look direction. The QAM modulation technique allows us to send more bits per symbols to get higher throughput, to reduce interference and fading. QAM is used for systems like CDMA, WiMAX (IEEE802.16) and Wi-Fi (IEEE802.11) for beam formation, to measure Bit Error Rate (BER), noisy channel capacity, Mean Square Error (MSE), Power Efficiency for M-ary QAM signals, to study the effect of inter-symbol interference in terms of eyediagram for transmitted and received signal, elapsed time and array response gain towards desired user which leads not only for system capacity improvement but also enhancing spectrum efficiency.

Keywords: Affine projection adaptive filter, AWGN channel, bessel beam forming algorithm, quadrature amplitude modulation technique, smart antenna

INTRODUCTION

Highly bandwidth efficient digital modulation scheme is required for mobile communication to cater scarce resources (Lal, 1997) of the available radio spectrum. The QAM is one of the effective digital modulation techniques (Lal, 1997; Al-kebsi *et al.*, 2009) to achieve higher system capacity and spectrum efficiency for CDMA and for the next generation broadband wireless system through channel. It is therefore called a bandwidth-conservation scheme (Simon, 2005; Bruce Carlson *et al.*, 2002).

Communication channels introduce noise, fading, interference and other distortions into the signals that they transmit. Simulating a communication system involves modeling a channel based on mathematical descriptions. Different transmission media have different properties and are modeled differently. An AWGN channel adds white Gaussian noise to the signal that passes through it. The overall BER of QAM symbol is affected by the channel noise because the individual bits are affected independently (Modabbes, 2010; Laleh and Chintha, 2006). This research is the extension of our previous study done reported in Yasin

et al. (2010) and here the performance analysis of Bessel beamformer is carried out in AWGN environment using digital modulation technique to verify its efficiency.

METHODOLOGY

System model: Consider a communication system model as shown in Fig. 1.

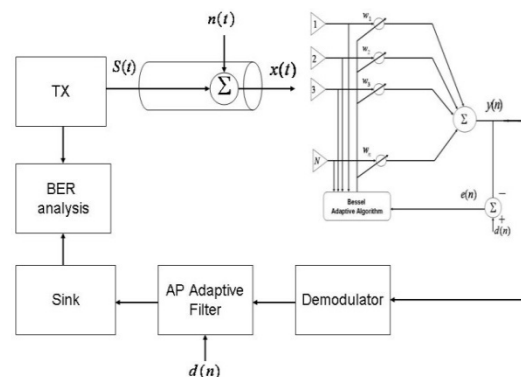


Fig. 1: Usage of bessel beamformer in AWGN model

When a digital signal (i.e., 1000001000 1000 1000 1001110000100011011010) is modulated using 4 point QAM technique in which the input signal takes one of four different values given by all possible combinations. This modulated signal is passed through AWGN channel which adds noise to it and becomes noisy. When this noisy signal is received at smart antenna's array then it is processed by Bessel beamformer. Bessel beamformer using Bessel function of the first kind (Yasin *et al.*, 2010b) can improve further smart/adaptive antenna technology for higher throughput. Bessel beamformer is an adaptive beamforming algorithm (Yasin *et al.*, 2010c), used in order to compute adaptive weights so that it minimizes the cost function and optimum results is obtained in form of beam towards desired direction and to place minimum gain towards interferers. Bessel beamformer has the ability to adapt the radiation pattern to the RF signal environment in real time. The output of beamformer is demodulated and computes the BER. The output of demodulator still has some noise parameters 10010111011101110010011000001000010 01010) i.e., higher frequency contents. To remove this noise and Inter-Symbol Interference (ISI), AP adaptive filter (Yasin *et al.*, 2010d) is used and gets a cleaned signal (i.e., 10000010001000100010011100001000110 11010) which is true copy of original signal as transmitted from transmitter.

Mathematical model:

Additive white Gaussian noise channel: Consider an AWGN channel at which data vector $S(k)$ is passed through it. An AWGN channel adds noise vector $n(k)$ to the data vector which is modeled as additive white Gaussian noise of zero mean (Haykin, 1996). Then we have:

$$E[s(k)n^H(k)] = 0 \tag{1}$$

where, H is the complex conjugate transpose-called Hermitian transpose and:

$$E[n(k)n^H(k)] = \sigma^2 I \tag{2}$$

From (1), it implies that noise vector $n(k)$ is uncorrelated with the data vector $S(k)$ and in (2), the elements of the noise vector are uncorrelated with each other and each element has a variance σ^2 and I is the identity matrix.

The signal received at the array of Bessel beamformer is given by:

$$x(k) = s(k) + n(k) \tag{3}$$

The mean square value of the transmission error is given by:

$$\varepsilon = E[|x(k) - s(k)|^2] = E|n(k)|^2 = E|n(k)n(k)^H| \tag{4}$$

From (2), we also have:

$$\varepsilon = \sum_{i=1}^M E|n_i(k)|^2 \tag{5}$$

It implies that each element $n_i(k)$ of the noise vector $n(k)$ has a variance σ^2 . Therefore,

$$\varepsilon = M \sigma^2 \tag{6}$$

where, M is the size of the noise vector:

Bessel beamformer: The algorithm is based on the Bessel function of the first kind of order (ν) (Yasin *et al.*, 2010e; John and Dimitris, 2009) and provides computationally efficient adaptive weights calculation. This is used for implementation of beamforming; therefore it is named as Bessel beamformer. It finds minimum Mean Square Error (MSE) and thus yields the set of optimum weights of the beamformer. The weight of Bessel beamformer is:

$$w(k+1) = w(k) - 2\mu e(k) J_\nu(N) x(k) \tag{7}$$

where, $J_\nu(N)$ represents the Bessel function of the first kind and defined by:

$$J_\nu(N) = \left(\frac{N}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{\left(\frac{-N^2}{4}\right)^k}{k! \Gamma(\nu + k + 1)} \tag{8}$$

where, ν denotes the order of the Bessel function of the first kind and must be a real number. The number of elements is presented by N and Γ is the gamma function. The signal array vector is written by:

$$x(k) = [x_1(k), x_2(k), \dots, x_M(k)]^T \tag{9}$$

This signal array vector can also be written as:

$$x(k) = s_d(k) a(\theta_d) + \sum_{i=1}^L s_i(k) a(\theta_i) + n(k) \tag{10}$$

where, S_d and s_i are the desired and interfering signals arriving at the array at an angle θ_d and θ_i , respectively.

L is the number of interfering signals and n is the noise at the array elements. $a(\theta_d)$ and $a(\theta_i)$ are the steering vectors for the desired and interfering signals, respectively. The steering vector is described as:

$$a(\theta) = [1, e^{-j\phi}, \dots, e^{-j(M-1)\phi}] \quad (11)$$

where, $\phi = \frac{2\pi d}{\lambda} \sin \theta$ is the phase shift observed at each sensor due to the angle of arrival of the wavefront and assume d is the uniform distance between array elements. $\lambda = \frac{c}{f}$, where, f is in Hertz. Therefore, the steering vector can be written as:

$$a(\theta) = [1, e^{-j\frac{2\pi}{\lambda}d \sin(\theta)}, \dots, e^{-j\frac{2\pi}{\lambda}d(M-1)\sin(\theta)}] \quad (12)$$

The error signal $e(k)$ is given by:

$$e(k) = d(k) - y(k) \quad (13)$$

where, $d(k)$ is the desired signal and $y(k)$ is the output of Bessel beamformer given by:

$$y(k) = x(k)^T w_k^\wedge(k) = x(k)w_k^\wedge(k)^T \quad (14)$$

where, $w_k^\wedge(k) = J_v(N)w_k(k)$ is the weight estimate vector using Bessel function of first kind $J_v(N)$ which conforms to the initial condition. Bessel function is a highly convergent series (Balanis, 2001) that helps the algorithm to coverage efficiently to compute the array factor (Balanis, 2001; Elisabet, 2009). Further, it is submitted that Bessel functions are Eigen functions which are all mutually orthogonal. Using the Orthogonality property to determine each of the coefficients that make the infinite series as a whole conform to the initial conditions. The infinite series is the solution of time-dependent problem involves a wave and forms a basis for series expansion, similar to Fourier series. Fourier series expresses a function in terms of frequency components. In applying Fourier series to signal processing, the individual terms should be what you would get if you applied a narrow bandpass filter to the signal. The Eigen functions may have little physical significance and are really just useful mathematical tools, because of the property of Orthogonality (Haykin, 1996; Abhayapala, 2008; Saxena *et al.*, 1956; Gopalan, 2001). Bessel beamformer employing Bessel functions have the ability to discriminate between the desired signal, noise

and other unwanted components using the principle of Orthogonality.

The weight of beamformer is selected adaptively so that sum of MSE is minimized. Thus we have:

$$\varepsilon(k) = \sum_{k=0}^M e^2(k) \quad (15)$$

where, ε is the MSE with time index k .

The parameter μ is the gain constant which is used to regulate the speed of adaptation and defined by:

$$0 < \mu < \frac{1}{\lambda_{\max}} \quad (16)$$

The gain constant also called step size, determines the amount of correction applied as the beamformer adapts from one iteration to the next.

The estimation of error $e(k)$ is computed by (13) and the weight matrix update approaches its true value, when the number of samples grows i.e., $k \rightarrow \infty$ and thus the estimated weights approaches the optimal weights ($w(k+1) \rightarrow w$) or w_{MSE} .

Affine projection adaptive filter: Affine Projection filter is an adaptive filter and used here due to its fast convergence. It is employed to extract a desired signal from a noise-corrupted signal by filtering out the noise (Yasin *et al.*, 2010f; Modabbes and Nasri, 2010).

The adaptive filter produces an output y that is a close replica of noise $n(k)$. The given system produces an output, $d-y$. The signal of interest d is equivalent to $S_d+n(k)$. Error signal is obtained when signal of interest is subtracted from the adaptive filter output. In other words, we get the cleaned signal error signal ε . Then system output is:

$$\varepsilon = s_d + n(k) - y \quad (17)$$

Squaring and taking expectation on both sides of (17) and assumption are made (Yasin *et al.*, 2010g). Then we get after calculation and adaptation, $y = n(k)$ and $\varepsilon = S_d$, means error signal is true copy of input desired signal. In this case, minimizing output power causes the output signal to be perfectly free of noise.

SIMULATION RESULTS

The QAM signal is applied for simulation purpose, to illustrate the effect of various parameters of digital communication system. It is used to study the

performance of radio links, including signal processing algorithm, such as channel estimation method, demodulation technique and beamforming algorithms which includes the effect of noise on transmitted bits, effect of Signal to noise ratio (Eb/No) on noise, to measure number of errors, BER estimation, performance analysis of Bessel Beamformer taking the effect of element spacing, number of elements and beam steering capabilities, performance analysis of AP filter and estimation of overall elapsed time.

The number of bits per symbol in QAM signal is given by:

$$K = \log_2(M) \tag{18}$$

where, M is the size of signal constellation but in our case it is 4.

Message signal in discrete form: The message input signal is discrete time signals (10000010001000 1000 1001110000100011011010) as shown in Fig. 2. This message signal is converted into symbol as shown in Fig. 3, to evaluate the performance of Bessel beamformer in AWGN model.

QAM modulated signal: QAM signal uses sine and cosine carriers to place information into four quadrants

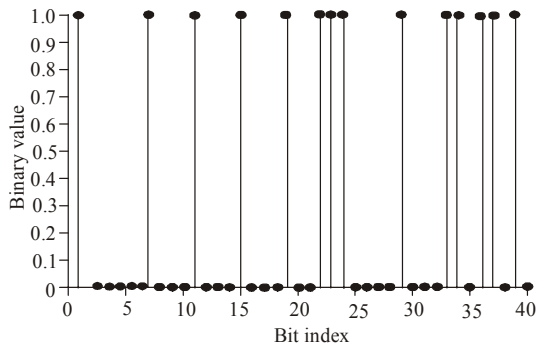


Fig. 2: Message signal in discrete form

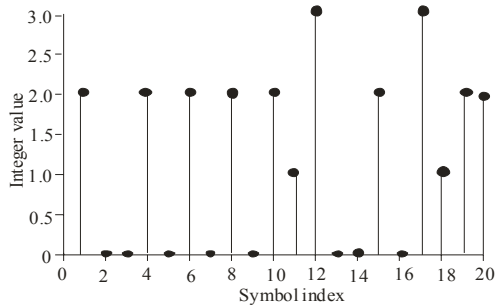


Fig. 3: Random bits of message signal is converted into symbols

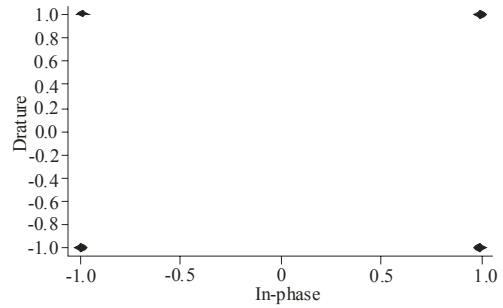


Fig. 4: 4-Quadrature Amplitude Modulated (QAM) signal

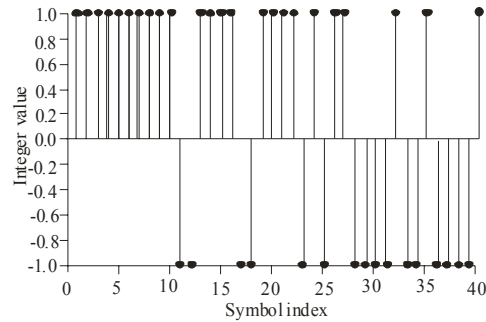


Fig. 5: 4-QAM signal in discrete symbol form

of an x-y plane. These four quadrants of an x-y plane are corner of a square lattice at angles $4\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$ as shown in Fig. 4. Four point QAM means that 4 points are arranged in a rectangular grid which is a power of two in digital communications the data is binary i.e., 0 and 1. The QAM signal in discrete symbol form is also shown in Fig. 5. In QAM modulation scheme, phase and amplitude are modulated to represent data where both I and Q are at the same frequency.

Effect of noise on QAM signal: When QAM signal is passed through AWGN channel it becomes noisy as shown in Fig. 6. It is to be noted that noise has very little affect on digital systems until the system fails completely means digital signals are less susceptible to noise.

Effect of number of elements on array factor: Uniform linear array is taken with element spacing $\lambda/2$ for simulation purpose with two hundred samples for $N = 16$. It is observed that the array directivity increases with the number of elements but at the same time number of side lobes also increases. The Angle of Arrival (AOA) for desired user is 10 degree. The array factor for $N = 16$ is shown in Fig. 7. The beam width is measured between the first two nulls of the array response function.

The data given in Table 1 is taken from Fig. 7.

Mean square error performance: The minimum MSE describes the performance of the given system as shown in Fig. 8. The weights values obtained at minimum MSE are the ones that minimize the power in the error signal indicating that system output has approached the desired output, is called optimum weights ($w(k+1) \rightarrow w$) or w_{MSE} .

Beamformer output: The Bessel beamformer output is shown in Fig. 9 as described by (14).

Demodulator: Digital demodulation receiver utilizes adaptive equalizer to negate the effects of signals arriving other than the desired signal. Demodulating a QAM signal is done by the system as shown in Fig. 10

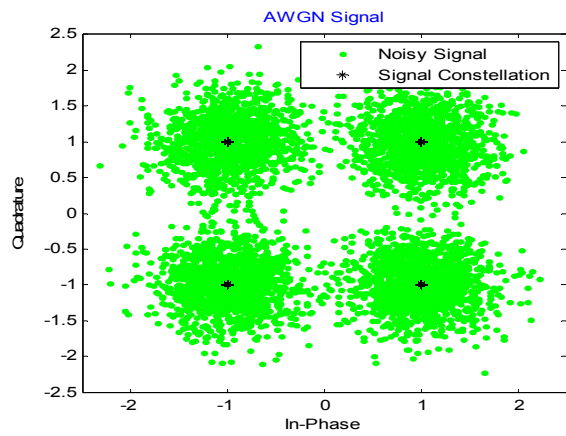


Fig. 6: Effect of noise on 4-QAM modulated signal in AWGN channel

Table 1: Effect of number of elements on beam width

Element Spacing	AOA d (degree)	Elements N	Step size μ	Beam width (degree)
$\lambda/2$	10	16	0.0001	25

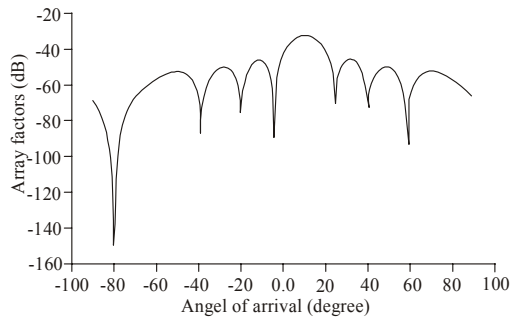


Fig. 7: Array factor plot for Bessel beamforming algorithm with AOA for desired user is 10 degree with constant space of $\lambda/2$ between elements for $N = 16$

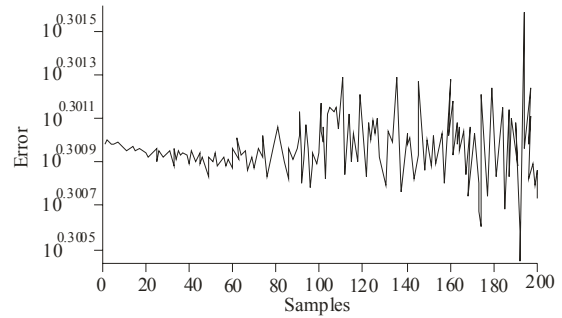


Fig. 8: Mean square error performance for Bessel beamforming algorithm for $N = 16$ with constant space of $\lambda/2$

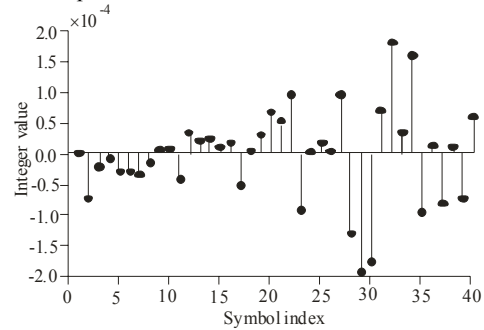


Fig. 9: Output signal of Bessel beamformer

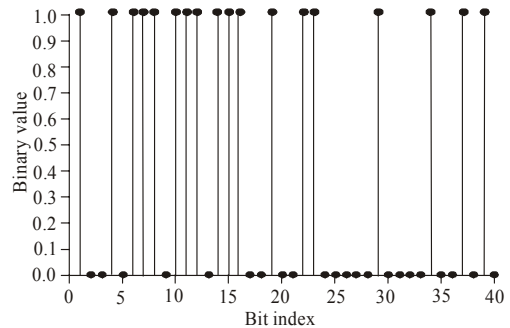


Fig. 10: Desired signal recovered by demodulator

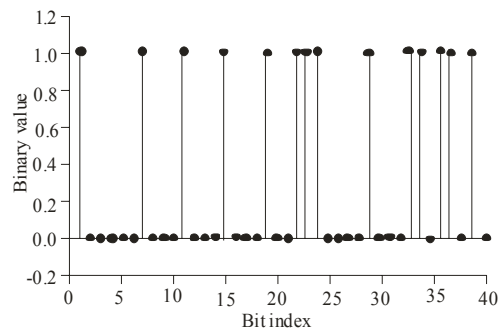


Fig. 11: Original signal recovered by affine projection adaptive filter

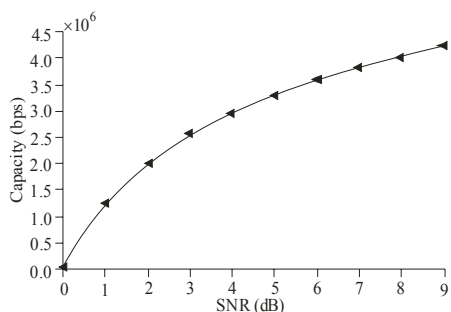


Fig. 12: AWGN channel capacity for 4-QAM signal

Table 2: Effect of SNR on channel capacity

SNR	0	1	2	3	4	5	6	7	8
Capacity	0	1.5	2	2.5	2.8	3.3	3.5	3.75	4

Table 3: Effect of SNR on bandwidth efficiency

SNR	0	1	2	3	4	5	6	7	8
η_B	0	1	1.59	2.0	2.32	2.59	2.81	3	3.17

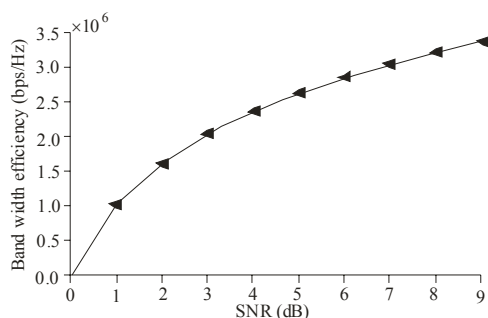


Fig. 13: Bandwidth efficiency of AWGN channel

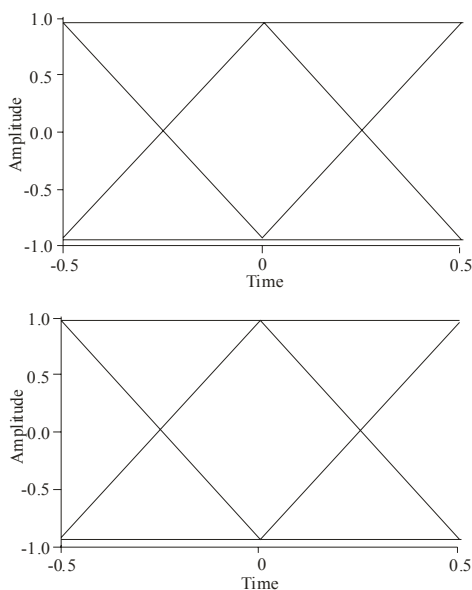


Fig. 14: Eye diagram of transmitted signal

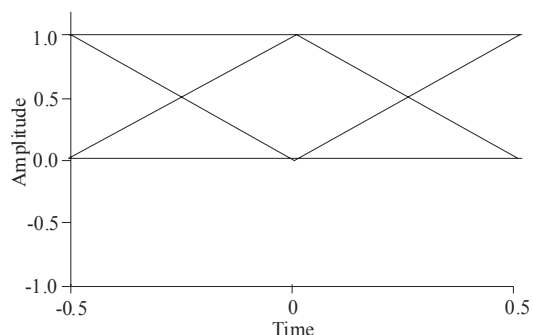


Fig. 15: Eye diagram of received signal via Bessel beamformer

but this desired signal still having some noise parameters (10010111011101110010011000001000010 01010) i.e., higher frequency contents.

Therefore it is passed through an AP adaptive filter which extracts desired signal i.e., streams of bits are filtered in order to remove higher frequency contents leaving only original data/signal (1000001000 1000 10001001110000100011011010) as shown in Fig. 11.

It is really true copy of transmitted message signal as shown in Fig. 2.

Shannon capacity of noisy channel: The channel capacity for AWGN channel is shown in Fig. 12 for analysis. It is indicated that by increasing SNR, the channel capacity increases. The results obtained from Fig. 12, are summarized in Table 2.

Bandwidth efficiency of noisy channel: The bandwidth efficiency of AWGN channel is shown in Fig. 13 for performance comparison. Table 3 compares various results obtained from Fig. 13.

Eye diagram of under study system: An eye diagram is a simple and suitable tool for studying the effects of Inter Symbol Interference (ISI) and other channel impairments in digital transmission/reception. It is used to look at the signal before transmission to ensure that the filter is behaving properly as shown in Fig. 14.

If the eye diagram is more open then the transmission of the data is less prone to errors due noise, ISI and timing drift.

Similarly, eye diagram of received signal via Bessel beamformer is shown in Fig. 15.

It is therefore confirmed that eye diagram for transmitted and received signal is same which proves the 100% efficiency of Bessel beamformer in the proposed digital system.

SNR improvement via adaptive array antenna: When number of elements (N_e) increases in an array then gain of the smart antenna increases as shown in

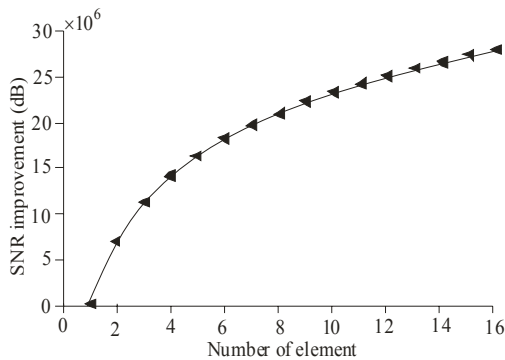


Fig. 16: Effect of number of elements on SNR

Table 4: Effect of number of elements on SNR

Ne	6	8	10	12	14	16
SNR	17.92	20.79	23.03	24.85	26.39	27.73

Fig. 16 and data is tabulated in Table 4. It is observed that the array directivity increases with the number of elements.

DISCUSSION AND COMMENTS

In this study, a digital communication system using smart antenna system having Bessel adaptive beamforming algorithm at receiving end is discussed in AWGN environment. A detailed system model is presented and analyzed, supported by mathematical and analytical model, which is further being utilized to develop simulation results for analysis. The role of Bessel algorithm and AP adaptive filter is highlighted for detection and extraction of desired signal from extremely noisy environment. Analysis of system model includes the effect of noise on transmitted bits, effect of Signal to noise ratio on noise, number of errors measurement, BER estimation, performance analysis of Bessel Beamformer taking the effect of element spacing, number of elements and beam steering capabilities, performance analysis of AP filter and overall elapsed time estimation performance.

The findings of simulation and mathematical analysis are:

- The Bessel adaptive beamforming algorithm has better capability to direct desired beam towards the desired user while suppressing interference in the proposed digital communication system. Due to these characteristics, the capacity and range of network equipped with smart antenna increases.
- The null depth and steering performance of the algorithm is good as shown in Fig. 7.

- The Communication System equipped with omni antennas keep the adjacent channels on standby during their transmission while System with smart antennas focus only on the desired users and allow the adjacent channels/users to communicate with each other without any interference.
- The Bessel adaptive beamforming algorithm is based on Space Division Multiple Access (SDMA) technique due to which all users in the network are able to exchange information at the same time using the same channel.
- The Bessel adaptive beamforming algorithm uses spatial filtering feature, due to this frequency reuse is efficient and effective in the proposed communication system employing smart antenna.
- The Bessel adaptive beamforming algorithm is more accurate and stable as it requires pilot signal for synchronization and convergence at the receiver which verifies the required output on the spot by judging against the desired signal.
- The simulation results also capitulate that the Bessel adaptive beamforming algorithm is much more stable and robust so that the degradation in its performance is minimum.
- The convergence property/capability of the algorithm is good.
- The algorithm has better capability to obtain minimum MSE by adjusting step size within bounded condition. It is also ascertained from simulation results as shown in Fig. 8.
- The Channel Capacity of AWGN as function of SNR is studied which verifies that Channel Capacity increases as SNR increases.
- The bandwidth efficiency of AWGN channel is also depends on SNR as given in Table 3. When SNR increases, bandwidth efficiency also increases.
- Power efficiency can be obtained for various QAM signals which confirm that power efficiency increases if number of bits increases but at the cost of more BER.
- Number of error and BER obtained are 64 and 0.32 during simulation respectively which is affordable for 4 point QAM.
- A QAM modulation technique for large number of constellation points is relatively more sensitive to phase noise because the constellation points are closer to each other, resulting a higher BER.
- An eye diagram is a suitable tool for studying the effects of ISI and other channel impairments in digital transmission/reception. It is confirmed from Fig. 14 and 15 that eye diagram for transmitted and received signal is same which proves the 100% efficiency of Bessel beamformer in the proposed digital system.

- The simulations are carried on Pentium (R) 4 CPU 3.00 @ 3.00 GHz, 512 MB of RAM hardware, using MATLAB version 7.8.0.347 (R2009a) software.
- The overall elapsed time for simulation is found as 5.892775 sec which is quite minimum. The software is also tested on Intel (R) Core (TM) 2 CPU E7400 @ 2.80 GHz, 1.98 GB of RAM hardware, using MATLAB version 7.8.0.347 (R2009a) software. The overall elapsed time is reduced to 1.32575 sec. If processor with higher speed is used for processing then estimation of elapse time can further be reduced.
- The performance of AP adaptive filter is good to extract desired signal from noisy environment and has fast convergence capability as compared to Least Mean Square (LMS) based adaptive filter.
- When number of elements in an array increases then gain of the smart antenna increases as tabulated in Table 4. It is therefore confirmed that directivity of the array increases with the number of elements.

CONCLUSION

A digital communication system model using Bessel beamformer and AP filter in AWGN environment is studied and analyzed its performance employing 4 point QAM technique. The performance of Bessel beamformer and AP filter is found optimum for beamforming, null steering and for noise cancellation, respectively. Therefore they can be employed as one of the better option to implement at base station of mobile communication systems in CDMA, WiMAX (IEEE802.16) and Wi-Fi (IEEE802.11) to reduce system overloading and to increase quality and capacity. However, if 16 point QAM or more is used then it allows us to send more bits per symbols to get higher throughput, to reduce interference and fading which leads towards increase in system capacity and spectrum efficiency but employing a higher order constellation QAM, the system is more susceptible to noise, resulting a higher BER. Therefore, it is recommended that 4 point QAM is suitable for usage to avoid noise and a balance between higher throughputs at the cost of BER must be met.

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