Research Journal of Applied Sciences, Engineering and Technology 4(22): 4636-4645, 2012
ISSN: 2040-7467

Submitted: March 19, 2012 Accepted: April 10, 2012 Published: November 15, 2012

# A New Method in the Location Problem using Fuzzy Evidential Reasoning

Sh. Rahgan and A. Mirzazadeh
Department of Industrial Engineering, University of Kharazmi, Tehran, Iran

**Abstract:** One of the most important factors leading to the success of a company is its location. Branches have a strategic importance on an organization's performance and its competitiveness. The purpose of this study is to present a decision-making model for selecting the most appropriate location for a bank branch. This research is the first study in the bank branch location researches considering various types of uncertainties. This model involves both quantitative and qualitative attributes as well as various types of uncertainty. So, we propose a methodology by integrating the fuzzy AHP and Evidential Reasoning approach. The fuzzy extent analysis is used to determine the weights of attributes and sub-attributes and the interval ER algorithm is used to rank the alternatives. This methodology can be used to help the decision makers, because it is capable of handling incomplete and imprecise judgments. We have demonstrated the applicability of the methodology through a case study.

**Keywords:** Bank branches, decision with uncertainties, evidence theory, fuzzy AHP, location, Multiple Attribute Decision Analysis (MADA)

## INTRODUCTION

Selection of location is a strategic decision in the company which is very important for decision maker. A poor choice of location might results in excessive transportation costs, lost of qualified labor, competitive advantage or some similar condition that would be detrimental to operations (Stevenson, 1993). Selecting the proper location of bank branch plays an important role in the bank's profitability in at least three ways. First, a new bank branch, located more conveniently than existing offices, may be sufficient reason for another branch's clients to switch banks. Second, the proper location of branch offices may prevent overlapped banking services within the trading area of potential competitors, thereby increasing future market share. Third, a good decision may ease financial pressure on the banking operation, because the proper location of a new bank branch may reduce capital investment costs, such as leasing costs or property taxes (Kramer, 1971). Branches have a strategic importance on a bank's performance and competitiveness. So, to maintain the continued growth of banking organizations, bank managers must determine and select the best location of new bank branch (Min, 1989).

The general procedure of making location decision usually consists of the following steps: Decide on the attributes that will be used to evaluate location alternatives; select the important attribute; develop location alternatives and select of the alternatives evaluated (Stevenson, 1993).

Selection of location is a multi-attribute decision because it requires to take into consideration both qualitative and quantitative factors. Therefore, Multi-Attribute Decision Analysis (MADA) is an appropriate technique to evaluate the alternatives. The literature including bank branch location has also shown that the selection process is multi attribute decision (Min, 1989; Miliotis *et al*., 2002; Cinar, 2010). MADA techniques have been used to solve location problems in the literature. The Analytical Hierarchy Process (AHP) described by Saaty (1980) is a popular method used to finding solution of the location problems. Tzeng *et al*. (2002), Aras *et al*. (2004), Wu *et al*. (2007) and Fernandez and Ruiz (2009) proposed the use of the AHP to deal with location selection. AHP can be combined with the fuzzy set theory of Zadeh (1965) to achieve more flexibility in judgment and decision making. Fuzzy AHP (FAHP) retains many of the conventional AHP advantages, in particular, the relative ease with which it handles multiple attribute and a combination of qualitative and quantitative data. Like AHP, it also provides a hierarchical structure, facilitates decomposition and pair wise comparison, reduces inconsistency and generates priority vectors. Finally, FAHP reflects human thinking in which approximate information and uncertainty is used to generate decisions (Kahraman *et al*., 2004). FAHP has been used to location problems, such as locating convenience stores and other facilities (Kuo *et al*., 1999, 2002; Kahraman *et al*., 2003; Partovi, 2006), site selection (Chi and Kuo, 2001; Witlox, 2003; Vahidnia *et al*., 2009;

**Corresponding Author:** S. Rahgan, Department of Industrial Engineering, University of Kharazmi, Tehran, Iran, Tel.: (009821) 4445 1990

Chou *et al*., 2008), local park planning (Zucca *et al*., 2008) and selecting the best environment-watershed plan (Chen *et al*., 2011). The approaches such as ELECTRE, TOPSIS and PROMETHEE have been used to rank alternatives sites (Norese, 2006; Gligoric *et al*., 2010; Marinoni, 2005). But, many complex multiple attribute decision analysis problems involve both quantitative and qualitative information with various types of uncertainties such as local and global ignorance (incomplete or no information) and fuzziness (vague information). Fuzzy logic based approaches have been extensively used to consider vagueness and ambiguity, but, it cannot deal with such uncertainties as incomplete, imprecise and missing information (Chin *et al*., 2009). These complex problems can be consistently modeled using the Evidential Reasoning (ER) approach (Yang and Singh, 1994; Yang, 2001; Yang and Xu, 2002 a, b; Wang *et al*., 2006; Xu *et al*., 2006; Yang *et al*., 2006; Wang and Elhag, 2007; Guo *et al*., 2007; Yang *et al*., 2010). In conventional methods, a MADA problem is modeled using a decision matrix, with each criterion assessed at each alternative decision by a single value. In the ER approach, a MADA problem is described using a belief decision matrix, with each criterion assessed at each alternative by a belief structure-attribute values (or assessment grades) and their associated degrees of belief. This provides a novel way to model MADA problems and allows different types of uncertainties to be modeled in the same framework (Chin *et al*., 2008).

The main objective of this study is to present a new decision-making model for selecting the most appropriate location for a bank branch with using ER approach. We created a hierarchical model with five main attributes: Cost, Demographic, Banking, Geographical condition and Accessibility. Also, fourteen sub-attributes have been considered to select the bank branch location. This model involves both quantitative and qualitative attributes as well as various types of uncertainties. So, we propose to use Fuzzy AHP to elicit the weights of attributes and then because of facing with these uncertainties to evaluate of alternatives, we use ER approach for ranking the alternatives to select the most appropriate location for a new bank branch. In the rest of study first, an explanation of our theoretical method is presented and then, a case study, with data collection procedure, calculation and results are described. Finally, the conclusions are stated.

## THEORETICAL BACKGROUND

In this section we explain the theoretical methods which are Fuzzy AHP and Interval Evidential Reasoning.

**Fuzzy analytic hierarchy process:** The AHP (Saaty, 1980) is used to solve multiple attribute decision problems. It is widely used to tackle multi-attribute decision making problems in real situations. In spite of its popularity, the AHP is often criticized for its inability to incorporate the inherent uncertainty and imprecision associated with mapping the decision maker's perceptions to exact numbers (Deng, 1999). In the traditional formulation of the AHP, human judgments are represented as crisp values. However, in many practical cases the human preference model is uncertain and decision makers might be reluctant or unable to assign crisp values to the comparison judgments (Chan and Kumar, 2007). Since fuzziness is a common characteristic of decision making problems, the FAHP method was developed to address this problem (Mikhailov and Tsvetinov, 2004). It allows decision makers to express approximate or flexible preferences using fuzzy numbers where adding fuzziness to the input, implies adding fuzziness to the judgment (Wang *et al*., 2008).

**Fuzzy set theory:** The fuzzy set theory introduced by Zadeh (1965) is suitable for handling problems involving the absence of sharply defined criteria (Chou *et al*., 2008). It is essentially a generalization of set theory where the classes lack sharp boundaries (Van Laarhoven and Pedrycz, 1983). In a universal set of discourse X, a fuzzy subset A of X is defined by a membership function $f_A(x)$ which maps each element x in A to a real number in the interval [0, 1]. The function value $f_A(x)$ represents the grade of membership of in FAHP uses a range of values to express the decision maker's uncertainty (Lee *et al*., 2008). The decision maker is free to select a range of values that reflects his confidence. Alternatively, he can specify his attitude in general terms as optimistic, pessimistic or moderate, representing high, low and middle ranges of values respectively. A fuzzy number A in R (real line) is a triangular fuzzy number if its membership function $f_A : R \rightarrow [0, 1]$ is:

$$f_A(x) = \begin{cases} (x-1)/(m-1), 1 \le x \le m \\ (u-x)/(u-m), m \le x \le u \\ O \ \ otherwise \end{cases} \tag{1}$$

The triangular fuzzy number A can be denoted by (1, m,u). The parameter gives the maximal grade of $f_A(x)$ and it is the most possible value of the evaluated data. and are the lower and upper bounds of the available area for the evaluated data. They are used to reflect the fuzziness of the evaluation data.

There are many fuzzy AHP methods proposed by various authors such as Van Laarhoven and Pedrycz (1983), Buckley (1985), Chang (1996) and Leung and Cao (2000). In this study, we prefer Chang's extent analysis method (Chang, 1996) since the steps of this approach is relatively easier than the other fuzzy AHP approaches and similar to the conventional AHP.

**Fuzzy extent analysis:** When the expert judgments are expressed as triangular fuzzy numbers, the triangular fuzzy comparison matrix is:

$$A = (a_{ij})_{n\times n} = \begin{bmatrix} (1,1,1) & (1_{12}, m_{12}, u_{12}) & \cdots & (1_{1n}, m_{1n}, u_{1n}) \\ (1_{21}, m_{21}, u_{21}) & (1,1,1) & \cdots & (1_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (1_{n1}, m_{n1}, u_{n1}) & (1_{n2}, m_{n2}, u_{n2}) & \cdots & (1,1,1) \end{bmatrix}$$

where, $a_{ij} = (I_{ij},m_{ij},u_{ij})$and $a^{-1}{}_{ij} = (1/u_{ji},1/m_{ji},1/I_{ji})$

For i, j = 1, ..., n and $i \neq j$ (2)

The steps of Chang's extent analysis can be given as the following:

**Step 1:** Sum each row of the fuzzy comparison matrix A. Then normalize the row sums by the fuzzy arithmetic operation.

$$S_i = \sum_{j=1}^{n} a_{ij} \otimes \left[ \sum_{K=1}^{n} \sum_{j=1}^{n} a_{kj} \right]^{-1}$$

$$= \left( \frac{\sum_{j=1}^{n} 1_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} m_{kj}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} 1_{kj}} \right)$$

i = 1, …, n. (3)

where, $\otimes$ denotes the extended multiplication of two fuzzy numbers.

**Step 2:** Compute the degree of possibility for $Si \geq S_j$ by the following equation:

$$V(S_i \geq S_j) = \underset{y \geq x}{SUP} \left[ \min\left( S_j(x), S_i(y) \right) \right] \qquad (4)$$

This formula can be equivalently expressed as:

$$V(S_i \geq S_j) = \left\{ \begin{array}{cc} 1 & m_i \geq m_j \\ \dfrac{u_i - 1_i}{(u_i - m_i) + (m_j - 1_j)} & 1_j \leq u_i, i, j = 1, \ldots, n; j \neq i \\ 0 & otherwise \end{array} \right\}$$

where,

$S_i = (I_i, m_i, u_i)$ and $S_j = (I_j, m_j, u_j)$ (5)

Table 1: Triangular fuzzy conversion scale

| Linguistic scale reciprocal | Triangular fuzzy scale | Triangular fuzzy scale |
|---|---|---|
| Just equal | (1, 1, 1) | (1, 1, 1) |
| Equally important | (1/2, 1, 3/2) | (2/3, 1, 2) |
| Weakly important | (1, 3/2, 2) | (1/2, 2/3, 1) |
| Strongly more important | (3/2, 5/2, 3) | (1/3, 2/5, 2/3) |
| Very strongly more Important | (2, 5/2, 3) | (1/3, 2/5, 1/2) |
| Absolutely more important | (5/2, 3, 7/2) | (2/7, 1/3, 2/5) |

**Step 3:** Assume that:

$$d'(A_i) = \min V(S_i \geq S_k) \; for \; k = 1, 2, \ldots, n; k \neq i \qquad (6)$$

Then the weight vector is given by:

$$W' = (d'(A_1), d'(A_2), \ldots d'(A_n))^T \qquad (7)$$

where, $A_i$(i=1, 2 ,..., n) are n elements.

**Step 4:** Via normalization, the normalized weight vectors are

$$W' = (d(A_1), d(A_2), \ldots, d(A_n))^T \qquad (8)$$

where, W is a non fuzzy number.
In order to perform a pair wise comparison among fuzzy parameters, linguistic variables have been defined for several levels of preference. The triangular fuzzy conversion scale given in Table 1 is used to determine the weights of attributes.

**The evidential reasoning approach:** The ER approach is developed to handle Multiple Attribute Decision Analysis (MADA) problems having both quantitative and qualitative information with uncertainties and subjectivity (Wang and Elhag, 2008; Wang and Elhag, 2007; Yang, 2001; Yang and Xu, 2002).

To produce rational and consistent assessment outcomes and reveal the combined effects of any uncertainty in the performances of an alternative, the ER approach employs the ER algorithm to aggregate belief degrees, which has been developed on the belief decision matrix, decision theory and the evidence combination rule of the Dempster-Shafer theory (Beynon *et al*., 2001; Shafer, 1976). The Evidential Reasoning (ER) algorithm is developed to aggregate multiple attributes based on a belief decision matrix and the evidence combination rule of the Dempster-Shafer (D-S) theory (Dempster, 1967; Shafer, 1976). Different from traditional MADA approaches that describe a MADA problem using a decision matrix, the ER approach uses the belief decision matrix, in which each attribute of an alternative is described by a distribution assessment using a belief structure. The advantages of doing so is that using a distribution assessment can both model precise data and capture various types of uncertainties such as ignorance and vagueness in subjective judgments (Wang *et al*., 2006). Where there are uncertainties, the ER algorithm can generate a lower bound and an upper bound of the performance of an alternative to indicate the sensitivity of the outcomes to uncertainties (Chin *et al*., 2008). In this study we use the Interval Evidential Reasoning (IER) presented by Xu *et al*. (2006). The main difference between the ER and IER algorithm is the aggregation process of two attribute assessments. Suppose $A_m$ and $A_n$ are two basic attributes, in parallel to the ER algorithm, $A_m$ and $A_n$ assess using the grades. In the IER algorithm the performance of $A_m$ or $A_n$ can be assessed to an individual grade or a grade interval. Then decision makers are not restricted to provide assessments to only individual assessment grade and ignorance is assigned to the whole space of grades.

**The analytical interval evidential reasoning algorithm:** Suppose a MADA problem has a simple two level hierarchy of attributes with a father attribute at the top level and basic attributes at the bottom level denoted by:

$$A=\{A_1 A_2 \ldots A_m \ldots A_n \ldots A_M\}, \quad (9)$$

And the weights of the attributes are given by:

$$\omega = \{\omega_1\ \omega_2 \ldots \omega_m \ldots \omega_M\}, \quad (10)$$

where, $\omega_i$ is the relative weight of thebasic attribute $A_i$ with $0 \le \omega_i \le 1$and $\sum_{i=1}^{M} \omega_i = 1$ A number of alternatives need to be assessed.

N distinctive evaluation grades are defined that collectively provide a complete set of standards to assess an attribute, as represented by:

$$\{H_1 H_2 \ldots H_i \ldots H_N\} \quad (11)$$

Then, the complete set of all individual grades and grade intervals, denoted by H, to assess each attribute can be represented by:

$$H = \left\{ \begin{array}{cccc} H_{11}\ H_{12} & \ldots H_{1(N-1)} & H_{1N} \\ H_{22}\ \ \ldots & H_{2(N-1)} & H_{2N} \\ \ddots & \vdots & \vdots \\ H_{(N-1)(N-1)} & H_{(N-1)N} & \\ & H_{NN} & \end{array} \right\} \quad (12)$$

where, $H_{ii}$ is the ith evaluation grade. Without loss of generality, it is assumed that grade $H_{(i+1)(i+1)}$ is preferred to grade $H_{ii}$.

Suppose there are P evaluators and each of them is assigned a weight (t = 1,2 , ..., P) with:

$$\sum_{t=1}^{P} \theta_{t=1} \quad (13)$$

$$\text{Let} \left\{ \left( H_{ij}, \beta_{ij,m}^{(t)} \right), i, \ldots, N;\ j = i, \ldots, N \right\} \quad (14)$$

be the belief structure provided by evaluator on the assessment of an alternative on attribute $A_m$. The collective assessment of the P evaluator for this alternative is also a belief structure, which is denoted as:

$$S(A_m) = \left\{ \left( H_{ij}, \beta_{ij,m} \right); i = 1, \ldots, N;\ j = i, \ldots, N; i \le j \right\} \quad (15)$$

and the attribute $A_n$ is given by:

$$S(A_n) = \left\{ \left( H_{ij}, \beta_{ij,n} \right); i = 1, \ldots, N;\ j = i, \ldots, N; i \le j \right\} \quad (16)$$

where,

$$\beta_{ij,m} = \sum_{t=1}^{P} \theta_t \beta_{ij,m}^{(t)}, i = 1, \ldots, N;\ j = i, \ldots, N \quad (17)$$

$$\beta_{ij,m} = \sum_{t=1}^{P} \theta_t \beta_{ij,n}^{(t)}, i = 1, \ldots, N;\ j = i, \ldots, N \quad (18)$$

And $\beta_{ij,m}, \beta_{ij,m}$ is the belief degree associated with the grade interval $H_{ij}$ and by definition the total belief degrees should be 1:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij,m} = 1 \quad (19)$$

and,

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij,n} = 1 \quad (20)$$

Suppose $\omega_m$ and $\omega_n$ are the weights for $A_m$ and $A_n$. Then the basic probability masses assigned to the grade interval $\{H_{ij}\}$ by $S(A_m)$ are given by:

$$m_{ij} = \omega_m\ \beta_{ij,m}\ (i = 1, \ldots, N;\ j = i, \ldots, N) \quad (21)$$

$$m_H = 1 - \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_m \beta_{ij,m} = 1 - \omega_m \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij,m} = 1 - \omega_m \quad (22)$$

$m_H$ is the remaining probability mass that is to be assigned depending on the relative importance of other attributes. $m_H$ should eventually be assigned to individual grades and grade intervals in the set H defined by Eq. (12).

Similarly, the basic probability masses assigned to the grade interval $\{H_{ij}\}$ by $S(A_n)$ are given by:

$$n_{ij}\ \omega_n\beta_{ij},_{\,n}(i = 1, \ldots, N;\ j = i, \ldots, N), \tag{23}$$

$$n_H = 1 - \sum_{i=1}^{N}\sum_{j=1}^{N}\omega_n\beta_{ij,m} = 1 - \omega_n \tag{24}$$

By aggregating the two assessments, the combined probability mass for each grade interval $\{H_{ij}\}$ with $i \leq j$, denoted by $C_{ij}$ which is given below:

$$C_{ij} = \frac{1}{1-K}\left[\begin{array}{l} -m_{ij}n_{ij} + \sum_{K=1}^{i}\sum_{1=j}^{N}\left(m_{K1}n_{ij} + m_{ij}n_{K1}\right) + \sum_{K-1}^{i-1} \\ \sum_{1=j+1}^{N}\left(m_{Kj}n_{ij} + m_{i1}n_{Kj}\right) + m_H n_{ij} + m_{ij}n_H \end{array}\right] \tag{25}$$

And the probability mass at large in defined by is given by:

$$C_H = \frac{m_H n_H}{1-K}, \tag{26}$$

and,

$$K = \sum_{i=1}^{N}\sum_{j=i}^{N}\sum_{k=1}^{i=1}\sum_{1=k}^{i=1}\left(m_{k1}n_{ij} + m_{ij}n_{k1}\right) \tag{27}$$

By applying the above aggregation process recursively until all the M basic attribute assessments are aggre gated and assuming that the final resultant probability masses are shown as in Eq. (25)-(27), the overall assessment of A can be expressed as:

$$S(A) = \{(H_{ij,}\ \beta_{ij})(i = 1, \ldots, N,\ j = i, \ldots i, \ldots N)\}$$

with

$$\beta_{ij} = \frac{c_{ij}}{1-c_H}\left(i = 1, \ldots, N\right) \tag{28}$$

To rank alternatives, expected utilities or values can be calculated. Suppose $u(H_{ii})$ is the value of the grade $H_{ii}$ with $u(H_{i+1,i+1}) > u(H_{ii})$ as it is assumed that the grade $H_{i+1,i+1}$ is preferred to $H_{ii}$. Because of interval uncertainty, again the maximum, minimum and average expected values are calculated. As the belief degree $\beta_{ij}$ could be assigned to the best grade in the interval $H_{ij}$, which is $H_{jj}$, if the uncertainty turned out to be favorable to the assessed alternative, then the maximum value could be calculated as:

$$u_{\max}(A) = \sum_{i=1}^{N}\sum_{j=1}^{N}\beta_{ij}u(H_{ij}) \tag{29}$$

Similarly, in the worst case, if the uncertainty turned out to be against the assessed alternative, the belief degree $\beta_{ij}$ assigned to $H_{ii}$, the worst grade in the interval $H_{ij}$ and then the minimum value would be given by:

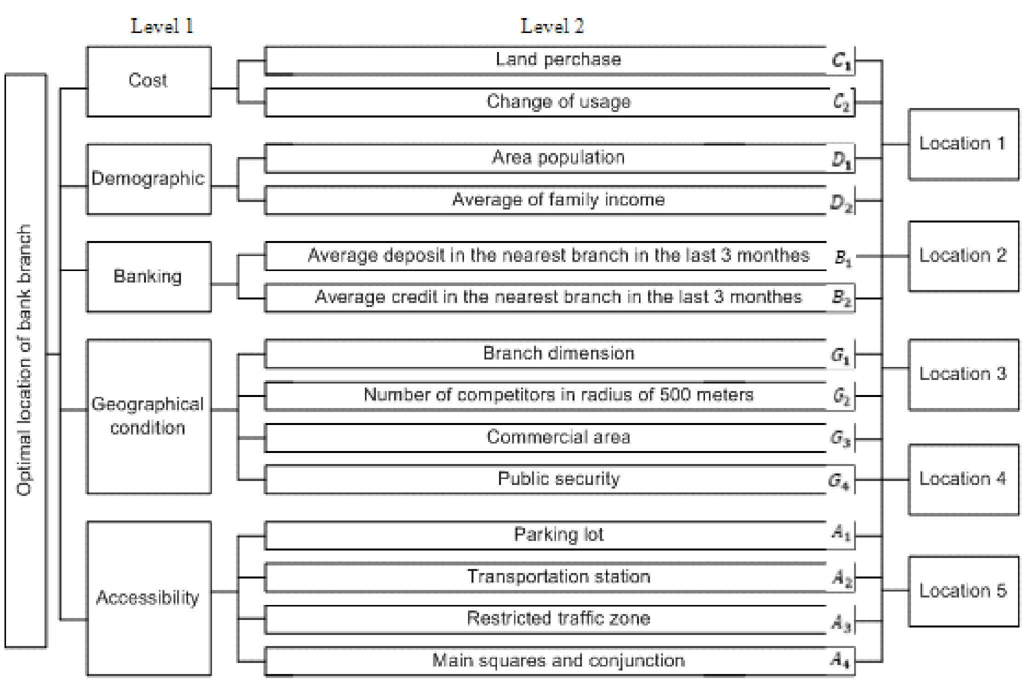


Fig. 1: The hierarchical structure of bank branch location selection

$$u_{\min}(A) = \sum_{i=1}^{N}\sum_{j=1}^{N} \beta_{ij} u(H_{ij}) \quad (30)$$

The average of the two is given by

$$u_{avg}(A) = \frac{u_{\max}(A) + u_{\min}(A)}{2} \quad (31)$$

The average value can be utilized to determine the priority of each alternative. The degree of preference is calculated using the equation below developed by Wang *et al*. (2005):

$$P(a > b) = \frac{\max(0, a_2 - b_1) + \max(0, a_1 - b_2)}{(a_2 - a_1) + (b_2 - b_1)} \quad (32)$$

where, $a = [a_1,a_2]$ and $b = [b_1,b_2]$ are two positive interval numbers.

## CASE STUDY

This section presents a case study to demonstrate the application of the proposed methodology to selection of bank branch location. The case study is Sina bank which is a private bank in Tehran. The five alternatives in various areas of Tehran are available as candidate locations to this case study ($L_1$, $L_2$, $L_3$, $L_4$ $L_5$).

**The attributes of bank branch location:** Each organization should consider meaningful attributes, suitable to its mission and strategy for location in order to make an efficient and effective strategic decision. The considered location factors vary from business to business but it is emphasized that the objection of the decision is to maximize the benefits of firm location.

The banks identify meaningful attribute for their location considering their mission and strategies. Also, we consider the studies in the literature review and the discussion with the bank managers, for attributes selection. There are five main attributes in level 1 including Cost, Demographic, Banking, Geographical condition and Accessibility which decomposed fourteen sub-attributes in level 2. The hierarchical structure is shown in Fig. 1.

**Fuzzy weights of attributes for each level:** Four bank's manager indicated their pair wise comparisons to obtain the weights of main and sub-attributes using the Linguistic scale which is presented in Table 1. The overall results could be obtained by taking the geometric mean of individual evaluations. This geometric mean of main attributes is shown in Table 2.

The values of fuzzy synthetic extent with respect to the main attributes are calculated as below:

$S_1 = (3.249,4.591,6.257)\otimes(0.027,0.037,0.05) = (0.088,0.17,0313)$
$S_2 = (2.778,3.241,4.486)\otimes(0.027,0.037,0.05) = (0.075,0.12,0.224)$
$S_3 = (5.59,7.77,10.136)\otimes(0.027,0.037,0.05) = (0.151,0.287,0.507)$
$S_4 = (4.051,5.625,7.45)\otimes(0.027,0.037,0.05) = (0.109,0.208,0.372)$
$S_5 = (4.451,6.066,8.09)\otimes(0.027,0.037,0.05) = (0.12,0.224,0.404)$

The degrees of possibility are:

$V(S_1 \geq S_2) = 1$, $V(S_1 \geq S_3) = 0.581$, $V(S_1 \geq S_4) = 0.84$, $V(S_1 \geq S_5) = 0.781$
$V(S_2 \geq S_1) = 0.731$, $V(S_2 \geq S_3) = 0.304$, $V(S_2 \geq S_4) = 0$, $V(S_2 \geq S_5) = 0.5$
$V(S_3 \geq S_1) = 1$, $V(S_3 \geq S_2) = 1$, $V(S_3 \geq S_4) = 1$, $V(S_3 \geq S_5) = 1$
$V(S_4 \geq S_1) = 1$, $V(S_4 \geq S_2) = 1$, $V(S_4 \geq S_3) = 0.738$, $V(S_4 \geq S_5) = 0.94$
$V(S_5 \geq S_1) = 1$, $V(S_5 \geq S_2) = 1$, $V(S_5 \geq S_3) = 0.801$, $V(S_5 \geq S_4) = 1$

For each pair wise comparison, the minimum of the degrees of possibility is found as below:

$\text{Min}(S_1 \geq S_i) = 0.581$ Min, $V(S_2 \geq S_i) = 0.304$, Min $V(S_3 \geq S_i) = 1$
Min $V(S_4 \geq S_i) = 0.738$, Min $V(S_5 \geq S_i) = 0.801$
These values yield the following weights vectors:

$$W' = (0.581, 0.304, 1, 0.738, 0.801)^T$$

Via normalization, the importance weights of the main attributes are calculated as follow:

w = (0.17, 0.089, 0.292, 0.215, 0.234).

Table 2: Fuzzy pair wise comparison of main attributes

| | Cost | Demographic | Banking | Geographical situation | Accessibility |
|---|---|---|---|---|---|
| Cost | (1, 1, 1) | (0.841, 1.355, 1.861) | (0.408, 0.516, 0.816) | (0.5, 0.904, 1.355) | (0.5, 0.816, 1.225) |
| Demographic | (0.537, 0.738, 1.198) | (1, 1, 1) | (0.333, 0.4, 0.577) | (0.5, 0.587, 0.904) | (0.408, 0.516, 0.816) |
| Banking | (1.225, 1.938, 2.451) | (1.733, 2.5, 3.003) | (1, 1, 1) | (0.816, 1.225, 2) | (0.816, 1.107, 1.682) |
| Geographical situation | (0.738, 1.106, 2) | (1.106, 1.703, 2) | (0.5, 0.816, 1.225) | (1, 1, 1) | (0.707, 1, 1.225) |
| Accessibility | (0.816, 1.225, 2) | (1.225, 1.938, 2.451) | (0.594, 0.903, 1.225) | (0.816, 1, 1.414) | (1, 1, 1) |

Table 3: The weights of attributes in each level

| Level 1 | Weight | Level 2 | Weight |
|---|---|---|---|
| Cost | 0.17 | $C_1$ | 0.758 |
| | | $C_2$ | 0.242 |
| Demographic | 0.089 | $D_1$ | 0.26 |
| | | $D_2$ | 0.74 |
| Banking | 0.292 | $B_1$ | 0.568 |
| | | $B_2$ | 0.432 |
| Geographical | 0.215 | $G_1$ | 0.027 |
| | | $G_2$ | 0.373 |
| | | $G_3$ | 0.466 |
| | | $G_4$ | 0.134 |
| Accessibility | 0.234 | $A_1$ | 0.147 |
| | | $A_2$ | 0.08 |
| | | $A_3$ | 0.349 |
| | | $A_4$ | 0.424 |

The same method was used to obtain the weights of the sub-attributes. The weights are presented in Table 3.

Table 4: Evaluation grades

| Evaluation grades | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| Definition | Weak situation | Poor situation | Average situation | Good situation | Best situation |

**Assessment of the alternatives:** Five evaluators ($E_1$,$E_2$,$E_3$,$E_4$,$E_5$) assessed the candidate locations with respect to each attribute. First, we defined the evaluation grades for assessment of candidate locations with respect to each qualitative sub-attribute. The evaluation grades used by the evaluators are represented in Table 4.

Then, the complete set of all individual grades and grade intervals is:

Table 5: Assessment information provided by evaluators

| Attributes | | Alternatives $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ |
|---|---|---|---|---|---|---|
| **Quantitative** | | | | | | |
| $C_1$ | | 665 | 972 | 572 | 624 | 399 |
| $C_2$ | | 80 | 125 | 90 | 140 | 160 |
| $D_1$ | | 105000 | 69000 | 125000 | 87000 | 91000 |
| $D_2$ | | 1.2 | 1.6 | 0.6 | 0.7 | 0.9 |
| $B_1$ | | 0.085 | 0.099 | 0.045 | 0.06 | 0.079 |
| $B_2$ | | 60.48 | 53.25 | 48.25 | 41.57 | 70.28 |
| $G_1$ | | 190 | 180 | 260 | 240 | 210 |
| $G_2$ | | 3 | 2 | 4 | 4 | 2 |
| **Qualitative** | | | | | | |
| $G_3$ | $E_1$(25%) | 5-7 | 7-9:60% | 3:80% | 7:60%, 7-9:40% | 5 |
| | $E_2$(20%) | 5 | 7:30%,9:45% | 1-3:75%, 1:5% | 5:30%,7:70% | 5:65%,3:20% |
| | $E_3$(20%) | 5:80%,7:20% | 9 | 1-3:70% | 7:40%,9:20% | 3-5 |
| | $E_4$(20%) | 5:60%,7:30% | 7-9 | 1 | 7:90% | 5:70% |
| | $E_5$(15%) | 5 | 9 | 3:40%, 1:20% | 7 | 3-5 |
| $G_4$ | | | | | | |
| | $E_1$(25%) | 7:50%, 5:50% | 5 | 7 | 5 | 7-9 |
| | $E_2$(20%) | 5 | 3-5:60%, 3:30% | unknown | 5 | 7-9 |
| | $E_3$(20%) | 5-7:50% | 3-5 | 5-7:70%, 7:20% | 3-5 | 9 |
| | $E_4$(20%) | 5-7 | unknown | unknown | 3-5 | 9 |
| | $E_5$(15%) | 5 | 3-5 | 7 | 5 | 9 |
| $A_1$ | | | | | | |
| | $E_1$(25%) | 5 | 7 | 5-7 | 1 | 7:80% |
| | $E_2$(20%) | 5:60%, 3:40% | 7-9 | 5:90% | 1 | 7 |
| | $E_3$(20%) | 5:50%,3:50% | 9:80% | 5 | 1 | unknown |
| | $E_4$(20%) | 3-5 | 9:75% | 5-7 | 1 | 7-9:55%,7:25% |
| | $E_5$(15%) | 3-5:70%,1:30% | 9 | 5-7 | 1 | 7 |
| $A_2$ | | | | | | |
| | $E_1$(25%) | 5 | 1 | 9 | 7 | 9 |
| | $E_2$(20%) | 3 | 1-3 | 9 | 5-7 | 7-9 |
| | $E_3$(20%) | unknown | 3:40%,1:50% | 9 | 7:65%,5:35% | 9 |
| | $E_4$(20%) | 3-5 | 3 | 9 | unknown | 7 |
| | $E_5$(15%) | 3 | 3 | 9 | 5-7:60% | 7-9:80% |
| $A_3$ | | | | | | |
| | $E_1$(25%) | 5 | 9 | 9 | 1 | 5 |
| | $E_2$(20%) | 5-7 | 9 | 9 | 1 | 3-5:80% |
| | $E_3$(20%) | 5:45% | 9 | 9 | 1 | 5:75%, 3:15% |
| | $E_4$(20%) | 5-7:60%, 5:15% | 9 | 9 | 1 | 3-5 |
| | $E_5$(15%) | 5-7 | 9 | 9 | 1 | 5 |
| $A_4$ | | | | | | |
| | $E_1$(25%) | 7-9 | 5:65% | 9 | 9 | 3 |
| | $E_2$(20%) | 7-9:85% | 7 | 9 | 7-9 | 3-5 |
| | $E_3$(20%) | 7-9:70%% | 7:10%, 5:50% | 9 | 7-9 | 3:75% |
| | $E_4$(20%) | 7-9:50%, 7:50% | 3-5 | 9 | 9 | 5:20%, 3:70% |
| | $E_5$(15%) | 7-9 | 5:75% | 9 | 9 | 3-5:65% |

$$H = \begin{bmatrix} 1 & 1-3 & 1-5 & 1-7 & 1-9 \\ & 3 & 3-5 & 3-7 & 3-9 \\ & & 5 & 5-7 & 5-9 \\ & & & 7 & 7-9 \\ & & & & 9 \end{bmatrix}$$

The assessment information provided by evaluators on the each attribute is shown in Table 5. There are various types of uncertainties in the assessment data such as probability, unknown and grade intervals. The five evaluators are assumed to be of different importance. Bank manager assigned the weight of each evaluator. Their weights are given in the parentheses after the evaluator's name.

Table 6 shows the aggregated assessment for the qualitative attributes which are given by Eq. (17) and (18). The belief degrees assigned to the interval grade 1-9 in Table 6 represent the ignorance information.

By implementing the Interval ER algorithm Eq. (21)-(31) we obtained the ranking order of alternatives. Note that to facilitate the evaluation process, a software system is developed on the basis of a software shell called Intelligent Decision System (IDS). The calculated results generated by using the IDS are presented In Table 7.

The ranking order of the five alternatives can be generated as:

77.45% 86.14% 100% 100%
$L_2 \succ L_5 \succ L_1 \succ L_3 \succ L_4$

where, the degrees of preference are computed by Eq. (32). The ranking shows that $L_2$ and $L_5$ are definitely ranked ahead of the other locations.

## CONCLUSION

The proper location of bank branch plays an important role in the bank's profitability and it is critical to the success of the banking business. This study presents a method to select a location for a bank branch. The method employs the Fuzzy AHP and interval ER algorithm for this selection and it can improve decision making quality. The fuzzy extent analysis is used to determine the weights of attributes and sub-attributes and the interval ER algorithm is used to rank the alternatives. Selection of bank branch location involves both quantitative and qualitative attributes as well as various types of uncertainties. Incomplete data and Incomplete data and human inability of making accurate

Table 6: Aggregated assessment

| | Alternatives | | | | |
|---|---|---|---|---|---|
| Attributes | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ |
| $G_3$ | 5:63%<br>5-7:25%<br>7:10%<br>1-9:2% | 7:6%<br>7-9:35%<br>9:44%<br>1-9:15% | 1:24%<br>1-3:29%<br>3:26%<br>1-9:21% | 5:6%<br>7:70%<br>7-9:10%<br>9:4%<br>1-9:10% | 3:4%<br>3-5:35%<br>5:52%<br>1-9:9% |
| $G_4$ | 5:47.5%<br>5-7:30%<br>7:12.5%<br>1-9:10% | 3:6%<br>3-5:47%<br>5:25%<br>1-9:22% | 5-7:14%<br>7:44%<br>1-9:42% | 3-5:40%<br>5:60% | 7-9:45%<br>9:55% |
| $A_1$ | 1:4.5%<br>3:18%<br>3-5:30.5%<br>5:47% | 7:25%<br>7-9:20%<br>9:46%<br>1-9:9% | 5:38%<br>5-7:60%<br>1-9:2% | 1 | 7:60%<br>7-9:11%<br>1-9:29% |
| $A_2$ | 3:35%<br>3-5:20%<br>5:25%<br>1-9:20% | 1:35%<br>1-3:20%<br>3:43%<br>1-9:2% | 9 | 5:7%<br>5-7:29%<br>7:38%<br>1-9:26% | 7:20%<br>7-9:32%<br>9:45%<br>1-9:3% |
| $A_3$ | 5:37%<br>5-7:47%<br>1-9:16% | 9 | 9 | 1 | 3:3%<br>3-5:36%<br>5:55%<br>1-9:6% |
| $A_4$ | 7:10%<br>7-9:81%<br>1-9:9% | 3-5:20%<br>5:37.5%<br>7:22%<br>1-9:20.5% | 9 | 7-9:40%<br>9:60% | 3:54%<br>3-5:29.75%<br>5:4%<br>1-9:12.25% |

Table 7: Ranking generated by the IER algorithm

| | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ |
|---|---|---|---|---|---|
| Maximum | 0.6560 | 0.7554 | 0.4070 | 0.3558 | 0.7094 |
| Minimum | 0.5950 | 0.6751 | 0.3603 | 0.3283 | 0.6376 |
| Average | 0.6255 | 0.7153 | 0.3837 | 0.3421 | 0.6735 |
| Average rank | 3 | 1 | 4 | 5 | 2 |

judgments are common in practice, which need to be addressed for better decision making. The proposed method is able to deal with these uncertainties. We have demonstrated the applicability of the methodology through a case study. We created a hierarchical model with five main attributes which are Cost, Demographic, Banking, Geographical condition and Accessibility. From the result of weights of attributes, the Banking and Accessibility are the first and second important attributes in the five main attributes. Therefore, the managers focus on this attributes to select the location for a new branch. The case study establishes that the methodology can be effectively used to selection of bank branch location. The study shows that the method can provide a framework to assist decision makers in analyzing location factors and making a dispassionate and objective selection. The findings are focused on a specific organization but the methodology provides a flexible way to solve the location problems. Therefore, for future, the methodology can be used easily for location problems in the different organizations.

## ACKNOWLEDGMENT

The authors would like to thank Research Department of Sina Bank for their kind cooperation for presenting necessary information used in this study.

## REFERENCES

Aras, H., S. Erdogmus and E. Koc, 2004. Multi-criteria selection for a wind observation station location using analytic hierarchy process. Renew. Energ., 29(8): 1383-1392.

Beynon, M., D. Cosker and D. Marshall, 2001. An expert system for multi-criteria decision making using Dempster Shafer theory. Exp. Syst. Appl., 20(4): 357-367.

Buckley, J.J., 1985. Fuzzy hierarchical analysis. Fuzzy Sets Syst., 17(3): 233-247.

Chan, F.T.S. and N. Kumar, 2007. Global supplier development considering risk factors using fuzzy extended AHP-based approach. Omega, 35(4): 417-431.

Chang, D.Y., 1996. Applications of the extent analysis method on fuzzy AHP. Euro. J. Oper. Res., 95(3): 649-655.

Chen, V.Y.C., H.P. Lien, C.H. Liu, J.J.H. Liou, G.H. Tzeng and L.S. Yang, 2011. Fuzzy MCDM approach for selecting the best environment-Watershed plan. Appl. Soft Comput., 11(1): 265-275.

Chi, S.C. and R.J. Kuo, 2001. Examination of the influence of fuzzy analytic hierarchy process in the development of an intelligent location selection support system of convenience store. IFSA World Congress and 20th NAFIPS International Conference, pp: 1312-1316.

Chin, K.S., D.L. Xu, J.B. Yang and J.P.K. Lam, 2008. Group-based ER-AHP system for product project screening. Exp. Syst. Appl., 35(4): 1909-1929.

Chin, K.S., Y.M. Wang, J.B. Yang and K.K. Poon, 2009. An evidential reasoning based approach for quality function deployment under uncertainty. Exp. Syst. Appl., 36(3): 5684-5694.

Chou, T.Y., C.L. Hsu and M.C. Chen, 2008. A fuzzy multi-criteria decision model for international tourist hotels location selection. Int. J. Hospit. Manage., 27(2): 293-301.

Cinar, N., 2010. A decision support model for bank branch location selection. Int. J. Hum. Soc. Sci., 5(13): 846-851.

Dempster, A.P., 1967. Upper and lower probabilities induced by a multi-valued mapping. Ann. Math. Statist., 38: 325-339.

Deng, H., 1999. Multi criteria analysis with fuzzy pair wise comparisons. Int. J. Approx. Reason, 21(3): 215-231.

Fernandez, I. and M.C. Ruiz, 2009.Descriptive model and evaluation system to locate sustainable industrial areas. J. Clean. Prod., 17(1): 87-100.

Gligoric, Z., C. Beljic and V. Simeunovic, 2010. Shaft location selection at deep multiple orebody deposit by using fuzzy TOPSIS method and network optimization. Exp. Syst. Appl., 37(2): 1408-1418.

Guo, M., J.B. Yang, K.S. Chin and H. Wang, 2007. Evidential reasoning based preference programming for multiple attribute decision analysis under uncertainty. Euro. J. Oper. Res., 182(3): 1294-1312.

Kahraman, C., D. Ruan and I. Dogan, 2003. Fuzzy group decision-making for facility location selection. Inform. Sci., 157: 135-153.

Kahraman, C., U. Cebeci and D. Ruan, 2004. Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey. Int. J. Prod. Econ., 87(2): 171-184.

Kramer, R.L., 1971. Forecasting branch bank growth patterns. J. Bank Res., 1(4): 17-27.

Kuo, R.J., S.C. Chi and S.S. Kao, 1999. A decision support system for locating convenience store through fuzzy AHP. Comput. Ind. Eng., 37(1-2): 323-326.

Kuo, R.J., S.C. Chi and S.S. Kao, 2002. A decision support system for selecting convenience store location through integration of fuzzy AHP and artificial neural network. Comput. Ind., 47(2): 199-214.

Lee, A.H.I., W.C. Chen and C.J. Chang, 2008. A fuzzy AHP and BSC approach for evaluating performance of IT department in the manufacturing industry in Taiwan. Exp. Syst. Appl., 34(1): 96-107.

Leung, L. and D. Cao, 2000. On consistency and ranking alternatives in fuzzy AHP. Euro. J. Oper. Res., 124(1): 102-113.

Marinoni, O., 2005. A stochastic spatial decision support system based on PROMETHE. Int. J. Geograp. Inform. Sci., 19: 51-68.

Mikhailov, L. and P. Tsvetinov, 2004. Evaluation of services using a fuzzy analytic hierarchy process. Appl. Soft Comput., 5(1): 23-33.

Miliotis, P.M., Dimopoulou and I. Giannikos, 2002. A hierarchical location model for locating bank branches in a competitive environment. Int. T. Oper. Res., 9-5: 549-565.

Min, H., 1989. A model-based decision support system for locating bank. Inform. Manage., 17(4): 207-215.

Norese, M.F., 2006. ELECTRE III as a support for participatory decision-making on the localization of waste-treatment plant. Land Use Pol., 23(1): 76-85.

Partovi, F.Y., 2006. An analytic model for locating facilities strategically. Omega, 34(1): 41-55.

Saaty, T.L., 1980. The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation. McGraw-Hill, New York.

Shafer, G.A., 1976. Mathematical Theory of Evidence. Princeton University Press, New Jersey.

Stevenson, W.J., 1993. Production/ Operations Management. 4th Edn., Richard D. Irwin., Homewood.

Tzeng, G.H., M.H. Teng, J.J. Chen and S. Opricovic, 2002. Multi criteria selection for a restaurant location in Taipei. Int. J. Hospit. Manage., 21(2): 171-187.

Vahidnia, M.H., A.A. Alesheikh and A. Alimohammadi, 2009. Hospital site selection using fuzzy AHP and its derivatives. J. Env. Manage., 90(10): 3048-3056.

Van Laarhoven, P.J.M. and W. Pedrycz, 1983. A fuzzy extension of Saaty's priority theory. Fuzzy Sets Syst., 11(1-3): 199-227.

Wang, Y.M. and T.M.S. Elhag, 2007. A comparison of neural network, Evidential reasoning and multiple regression analysis in modeling bridge risks. Exp. Syst. Appl., 32(2): 336-348.

Wang, Y.M., J.B. Yang and D.L. Xu, 2005. Interval weight generation approaches based on consistency test and interval comparison matrices. Appl. Math. Comput., 167(1): 252-273.

Wang, Y.M., J.B. Yang and D.L. Xu, 2006. The evidential reasoning approach for multiple attribute decision analysis using interval belief degrees. Euro. J. Oper. Res., 175(1): 35-66.

Wang, Y.M. and T.M.S. Elhag, 2008. Evidential reasoning approach for bridge condition assessment. Exp. Syst. Appl., 34(1): 689-699.

Wang, Y.M., Y. Luo and Z. Hua, 2008. On the extent analysis method for fuzzy AHP and its applications. Euro. J. Oper. Res., 186(2): 735-747.

Witlox, F., 2003. MATISSE: A relational expert system for industrial site selection. Exp. Syst. Appl., 24(1): 133-144.

Wu, C.R., C.T. Lin and H.C. Chen, 2007. Optimal selection of location for Taiwanese hospitals to ensure a competitive advantage by using the analytic hierarchy process and sensitivity analysis. Build. Env., 42(3): 1431-1444.

Xu, D.L., J.B. Yang and Y.M. Wang, 2006. The ER approach for multi-attribute decision analysis under interval uncertainties. Euro. J. Oper. Res., 174(3): 1914-1943.

Yang, J.B. and M.G. Singh, 1994. An evidential reasoning approach for multiple attribute decision making with uncertainty. IEEE T. Syst. Man Cyb., 24(1): 1-18.

Yang, J.B., 2001. Rule and utility based evidential reasoning approach for multi-attribute decision analysis under uncertainties. Euro. J. Oper. Res., 131(1): 31-61.

Yang, J.B. and D.L. Xu, 2002a. On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty. IEEE T. Syst. Man Cy. A., 32(3): 289-304.

Yang, J.B. and D.L. Xu, 2002b. Nonlinear information aggregation via evidential reasoning in multi attribute decision analysis under uncertainty. IEEE T. Syst. Man Cy. A., 32(3): 376-393.

Yang, J.B., Y.M. Wang, D.L. Xu and K.S. Chin, 2006. The evidential reasoning approach for MCDA under both probabilistic and fuzzy uncertainties. Euro. J. Oper. Res., 171(1): 309-343.

Yang, J.B., D. Xu, X. Xie and A.K. Maddulapalli, 2010. Evidence theory and multiple criteria decision making: The evidential reasoning approach. Proceedings of the Conference on the Theory of Belief Functions, France, April 1-2.

Zadeh, L., 1965. Fuzzy sets. Inform. Contr., 8(3): 338-353.

Zucca, A., A.M. Sharifi and A.G. Fabbri, 2008. Application of spatial multi-criteria analysis to site selection for a local park: A case study in the Bergamo Province, Italy. J. Env. Manage., 88(4): 752-769.