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Investigation of Buckling under Periodic and Uniform Loads in Rectangular Plates

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Abstract: In this research, local elastic buckling of the plates is studied with different boundary conditions under periodic and uniform compressive loadings by analytical direct method (Equivalency in Partial Differential Equation: EPDE) and FEM modeling in rectangular plates. Also, new formulations are presented for determination of critical buckling loads under uniform loading in rectangular plates. In this study, governing differential equation is used for thin plates under lateral and direct periodic and loadings. Next, the mentioned equation would be solved by assumed displacements by direct smart method. Therefore, the minimum critical buckling load is obtained for first mode of buckling by theoretical and finite element methods. These analytical results are validated by the FEM modeling. Finally, good agreements are found between the analytical and numerical predictions for the critical buckling loads.

Keywords: Analytical and numerical methods, buckling of plates, EPDE, periodic and uniform loadings, rectangular

INTRODUCTION

In practice, buckling is characterized by a sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. Plates are initially flat structural elements, having thickness much smaller than the other dimensions. Included among the more familiar examples of plates are street manhole covers, side panels and roofs of buildings, turbine disks, bulkheads and tank bottoms. In practice, members that carry transverse loads, such as end plates and closures of pressure vessels, pump diagrams, telephone and loudspeaker diagrams, thrust bearing plates, piston heads, diffusers, clutches, springs made of assemblages of plates, turbine disks and so on are usually circular in shape. Ships and offshore structures are some examples of complex thin walled structures that consist of plate elements, which are subjected to a diversity of load combinations. Thus, many of these significant applications fall within the scope of the formulas derived for circular plates. According to the criterion often applied to define a thin plate (for purpose of technical calculations) the ratio of the thickness to the smaller span length for rectangular plate should be less than 0.05. We assume that plate and shell materials are homogeneous and isotropic. The fundamental assumptions of the small-deflection theory of bending or so-called classical or customary theory for isotropic, homogeneous, elastic, thin plates are based on the geometry of deformations. These assumptions, known as the Kirchhoff hypotheses. Semi-analytical buckling analysis of heterogeneous variable thickness viscoelastic circular plates on elastic foundations was presented (Alipour and Shariyat, 2011). Elastic buckling of plates subjected to distributed tangential loads was investigated (Brown, 1991). Buckling analysis of Reissner-Mindlin plates subjected to in-plane edge loads using a shear-locking-free and meshfree method has been studied (Bui *et al*., 2011). Stability to plate buckling in new regulations was investigated (Horvatić and Živni, 2000). Prediction of mechanical behavior of PZT and SMA by finite element analysis was presented (Monfared and Khalili, 2011). New analytical formulations for contact stress and prediction of crack propagation path in rolling bodies and comparing with Finite Element Model (FEM) results statically have been introduced (Monfared, 2011). Reduction factor has been determined using geometrical parameters so as to reduce the classical buckling load to a more realistic value based on the post-buckling load, That is reduction factor for buckling load of spherical cap shells was determined (Khakina and Zhou, 2011). Buckling of standing vertical plates under body forces has been investigated (Wang *et al*., 2002). Buckling of a heavy standing plate with top load was studied (Wang, 2010). Elastic stability of a simply supported plate under linearly variable compressive stresses was studied (Wang and Sussman, 1967). Numerical problems of nonlinear stability analysis of elastic structures was presented (Waszcyszin, 1983). An analytical method was presented

for estimating elastic local buckling load of rectangular plates subjected to uniform and periodic loadings. Also, results of the assumed displacements for determination of critical buckling loads were compared. Through a comparison of calculated results by analytical smart method with FEM eigen value analysis, acceptable accuracy of the proposed formulation was demonstrated. Next, new formulations were obtained for determination of critical buckling loads subjected to periodic and uniform loadings in rectangular plates. In this research, the buckling of rectangular plates is discussed with two different boundary conditions under periodic and uniform loadings. The Finite Element Method (FEM) is used to predict buckling characteristics of complete rectangular plates. The theoretical results are compared with results of the finite element (ANSYS). In this investigation, for calculation of buckling loads, governing differential equations, are used for the thin plate under combined lateral and in-plane loads, with assumed deflection, which satisfies the boundary conditions. Buckling of the plates is investigated with different boundary conditions and periodic and axial loading for first mode of buckling by theoretical direct method and FEM modeling. Then, results of analytical and numerical method will be compared. By solving the governing equation, we can determine the minimum of the force by direct method (equivalent method). Therefore, this minimum value is the buckling load. That is, critical load is determined by Differential Equivalent Method (DEM or EPDE) in different boundary conditions (simply and clamped edges). Good agreements are found between the analytical and numerical methods. Eventually, local buckling of the rectangular plates will be studied with different boundary conditions under periodic and uniform loadings by analytical direct method and FEM modeling in rectangular plates. Moreover, novel formulations will be introduced for obtaining of critical buckling loads under uniform loading in rectangular plates. Buckling analysis is performed for prevention of buckling phenomenon and its dangers.

MATERIALS AND METHODS

This study was conducted in department of mechanical engineering of islamic azad university, zanjan, Iran; between August 2011 to April 2012. When load is constantly being applied on a member, such as plate, it will ultimately become large enough to cause the member to become unstable. When a plate is compressed in its midplane, it becomes unstable and begins to buckle at a certain critical value of the in-plane force. Buckling of plates is qualitatively similar to column buckling. Governing differential equation for a thin plate, under combined lateral and in-plane forces is presented by Eq. (1).

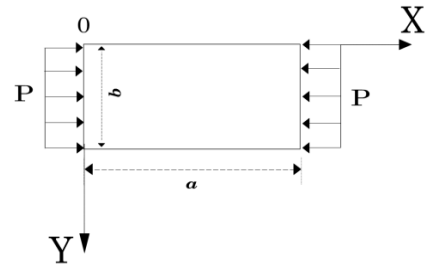

Fig. 1: Rectangular plate subjected to uniaxial compressive force

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left(p+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}\right) \quad (1)$$

where D is the flexural rigidity of the plate, that is:

$$D=\frac{Et^3}{12\left(1-v^2\right)} \quad (2)$$

where E, t, v are elastic module, thickness and poison ratio respectively. A rectangular plate subjected to uniaxial in-plane compressive force P is depicted in Fig. 1 generally.

In order to reduction of calculations in Eq. (1), it was assumed that the lateral load is zero (P = 0) Material was assumed steel for verifying the results of FEM and analytical new method. Governing differential equation is solved by direct method.

In this section, the critical loads are calculated in rectangular plate under periodic (harmonic) and uniform loads with different boundary conditions by smart direct method. Then the mentioned problem would be simulated by finite element method.

Presentation of displacements and boundary conditions: Simply supported edge is shown as “S”, clamped edge as “C”, four simply supported edges as “S-S-S-S” and four clamped edges as “C-C-C-C”. First the critical buckling load in S-S-S-S boundary conditions under periodic loading is determined, (Fig. 2).

In Fig. 2, N_x equal to $N\cos\left(\frac{\pi y}{b}-\frac{\pi}{2}\right)$ and the equivalent force is $\frac{2Nb}{\pi}$. Assumed deflections for simply supported edges are, Eq. (3)-(5):

$$W_{1s}=A_1\sinh\left[xy(x-a)(y-b)\right] \quad (3)$$

$$W_{2s}=B_1\sin\left(\frac{\pi(x-a)}{a}\right)\sin\left(\frac{\pi(y-b)}{b}\right) \quad (4)$$

$$W_{3s}=\sum_{i=1}^{10}\sum_{j=1}^{10}C_{ij}\sin\left(\frac{\pi i(x-a)}{a}\right)\sin\left(\frac{\pi j(y-b)}{b}\right) \quad (5)$$

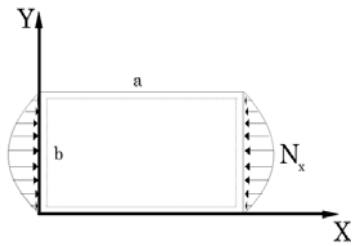

Fig. 2: Rectangular plate with S-S-S-S boundary condition subjected to N_x

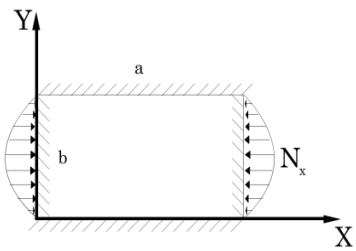

Fig. 3: Rectangular plate with C-C-C-C boundary condition subjected to N_x

where the above deflections satisfy the boundary conditions and then the Eq. (1) must be satisfied by Eq. (3)-(5). In above formulations A_1, B_1, C_{ij} are constants. The boundary conditions for simply supported edges are:

$$\begin{cases} w=0,\ x=0,\ a \\ w=0,\ y=0,\ b \end{cases} \tag{6}$$

$$\begin{cases} \dfrac{\partial^2 w}{\partial x^2}=0,\ x=0,\ a \\ \dfrac{\partial^2 w}{\partial y^2}=0,\ y=0,\ b \end{cases} \tag{7}$$

The assumptions are the following relations:

$$\begin{cases} P=0 \\ N_x = N\cos\left(\dfrac{\pi y}{b}-\dfrac{\pi}{2}\right) \\ N_y=0 \\ N_{xy}=0 \\ r=\dfrac{a}{b} \end{cases} \tag{8}$$

Now the minimum of N is determined by derivation of N with respect to r generally. That is:

$$\frac{dN}{dr}=0 \tag{9}$$

Now the critical buckling load in C-C-C-C boundary condition under periodic loading is determined, (Fig. 3).

The below displacements are deflection for clamped boundary conditions, Eq. (10)-(12):

$$W_{1c}=A_2\cosh\left[xy(x-a)(y-b)\right]-1 \tag{10}$$

$$W_{2c}=B_2x(x-a)\sin\left(\frac{\pi(x-a)}{a}\right)\left(1-\cos\frac{2\pi y}{b}\right) \tag{11}$$

$$W_{3c}=\sum_{i=1}^{10}\sum_{j=1}^{10}C_{ij}^{'}\left(1-\cos\frac{2\pi xi}{a}\right)\left(1-\cos\frac{2\pi yi}{b}\right) \tag{12}$$

In above formulations A_2, B_2, C'_{ij} are constants. The boundary conditions for clamped boundary conditions are:

$$\begin{cases} w=0,\ x=0,\ a \\ w=0,\ y=0,\ b \end{cases} \tag{13}$$

$$\begin{cases} \dfrac{\partial w}{\partial x}=0,\ x=0,\ a \\ \dfrac{\partial w}{\partial y}=0,\ y=0,\ b \end{cases} \tag{14}$$

Eq. (3), (4), (5), (10), (11) and (12) substitute in governing differential Eq. (1) and therefore, critical buckling loads in rectangular plate with C-C-C-C, S-S-S-S boundary conditions subjected to uniform and periodic loadings were determined. Therefore, relation between two critical loads is:

$$N_{cr}\big|_{rect(s-s-s-s)} \cong \frac{1}{2}N_{cr}\big|_{rect(c-c-c-c)} \tag{15}$$

Values of critical buckling loads were determined by using of W_{3s}, W_{3c} and governing differential equation by smart direct method. That is, these results have been determined by direct solution of the governing differential equation by assumed displacements W_{3s}, W_{3c}.

Presentation of new formulation: Critical buckling load in rectangular plate with S-S-S-S and C-C-C-C edges boundary condition subjected to uniform load for $r=\frac{a}{b}\geq 1$ would be the following form:

$$N_{cr} = f(a, b, D) \tag{16}$$

$$N_{cr} \propto \left(\frac{k_1ab+k_2b^2}{k_3}\right)\times D\times k_4 \tag{17}$$

where, k_i′s are constant coefficients for the proportion of equations. With attention to analytical and numerical

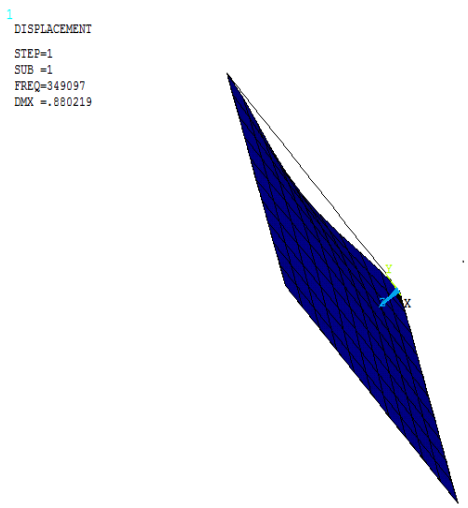

Fig. 4: First buckling mode for S-S-S-S conditions in rectangular plate under mentioned periodic loading (y = 0.3b)

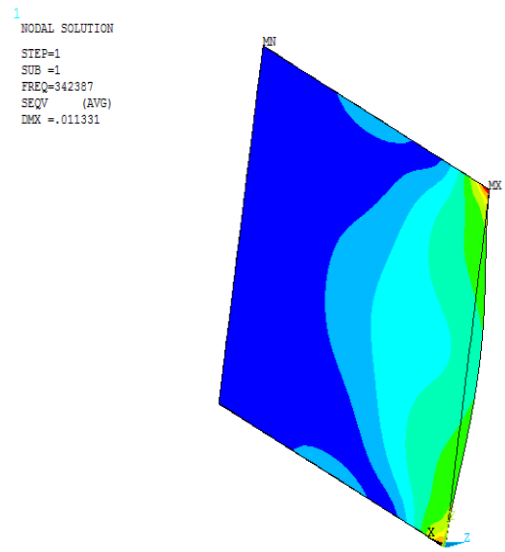

Fig. 5: First buckling mode for S-S-S-S conditions in rectangular plate under mentioned periodic loading (y = 0.4b)

results with FEM's considerations, the values of k_1, k_2, k_3 and k_4 for SSSS edge conditions are equal to 9, 5, 2, 9.86, respectively. The values of k_1, k_2, k_3 and k_4 for CCCC edge conditions are equal to 9, 5, 1, 9.86, respectively. Therefore, the new formulations for critical buckling loads under uniform loading are:

$$N_{cr}\big|_{rect(s-s-s-s)} = \frac{\left(\frac{9a-5b}{2b}\right)D\pi^2}{b^2} \tag{18}$$

$$N_{cr}\big|_{rect(c-c-c-c)} = \frac{\left(\frac{9a-5b}{b}\right)D\pi^2}{b^2} \tag{19}$$

Therefore, critical buckling load in rectangular plate with S-S-S-S and C-C-C-C edge boundary conditions subjected to periodic (harmonic) load for $r=\frac{a}{b}\geq 1$ would be the following form:

$$N_{cr}\big|_{rect(s-s-s-s)} = \frac{\left(\frac{9a-5b}{2b}\right)D\pi^2}{b^2\cos\left(\frac{\pi y}{b}-\frac{\pi}{2}\right)} \tag{20}$$

$$N_{cr}\big|_{rect(c-c-c-c)} = \frac{\left(\frac{9a-5b}{b}\right)D\pi^2}{b^2\cos\left(\frac{\pi y}{b}-\frac{\pi}{2}\right)} \tag{21}$$

And critical stress would be the below equation:

$$\sigma_{cr} = \frac{N_{cr}}{bt} \tag{22}$$

Now, results of the finite element method for rectangular plate with two boundary conditions subjected to periodic loading is investigated, (Fig. 4 and 5).

According to the Fig. 4 and 5 and mentioned formulations, it is seen that the results of the new formulations and FEM are similar to the analytical direct method.

RESULTS AND DISCUSSION

To examine the validity of the present analytical method, the steel is chosen as a test case. For comparison purpose, the finite element numerical calculations of local buckling behavior of these rectangular plates are also carried out using the finite element commercial code of ANSYS. The model geometry is chosen as shown in Fig. 1-5. The axisymmetric approach with linear quadratic element of shell 3D 4node is used for FEM analysis. This element has good buckling modeling capability and ratio of a to b was equal to 2. Also 162 elements and 190 nodes are applied in this finite element approach.

The critical buckling loads were determined by direct method with assumption of the deflections in different boundary conditions. Ratio of buckling load in C-C-C-C to buckling load in S-S-S-S in rectangular plate subjected to periodic loading is equal to 2, approximately. Also, new formulations were presented for determination of critical buckling loads under uniform loading in rectangular plates. New displacements were presented for simply supported and clamped plates. Next, the mentioned equation would be solved by assumed

Table 1: Critical loads $\left(\frac{N}{m},\frac{N}{m^2}\right)$ in S-S-S-S edge conditions under periodic load in rectangular plate with $\frac{a}{b}=2$

y/b	P_{cr}, analytical	P_{cr}, FEM	σ_{cr}, analytical	σ_{cr}, FEM
0.1	964.66	962.5	482330	482318
0.2	507.12	500.9	253560	253551
0.3	368.41	349.0	184205	184200
0.4	313.34	342.0	156670	156661
0.5	295.50	294.1	147750	147743

Table 2: Critical loads $\left(\frac{N}{m},\frac{N}{m^2}\right)$ in C-C-C-C edge conditions under periodic load in rectangular plate with $\frac{a}{b}=2$

y/b	P_{cr}, analytical	P_{cr}, FEM	σ_{cr}, analytical	σ_{cr}, FEM
0.1	1930.12	1925.20	964661.60	964671.9
0.2	1010.59	998.30	507110.71	507132.8
0.3	736.20	721.50	368451.00	368461.4
0.4	621.10	620.89	313449.00	313471.0
0.5	595.63	600.80	295590.00	295613.4

Table 3: Comparison of critical loads $\left(\frac{N}{m},\frac{N}{m^2}\right)$ in S-S-S-S edge conditions under uniform load N in rectangular plate with $\frac{a}{b}=2$ by using of assumed displacements

W	P_{cr}, analytical	P_{cr}, FEM	σ_{cr}, analytical	σ_{cr}, FEM
W_{1s}	295.50	294.1	147750	147062
W_{2s}	294.10	294.1	147050	147062
W_{3s}	296.95	294.1	148475	147062
New formulation	293.92	294.1	146960	147062

Table 4: Comparison of critical loads $\left(\frac{N}{m},\frac{N}{m^2}\right)$ in C-C-C-C edge conditions under uniform load N in rectangular plate with $\frac{a}{b}=2$ by using of assumed displacements

W	P_{cr}, analytical	P_{cr}, FEM	σ_{cr}, analytical	σ_{cr}, FEM
W_{1c}	591.00	599.4	295500	299812
W_{2c}	588.20	599.4	294100	299812
W_{3c}	593.90	599.4	296950	299812
New formulation	587.84	599.4	293920	299812

displacements by direct smart method which determination of critical loads were performed by these displacements and governing equation correctly. At last, good agreements are found between the analytical and numerical predictions for the critical buckling loads. Therefore we can use the finite element method instead of analytical new direct approach and vice versa. The mentioned smart new direct method can be used instead of complicated methods and approaches. These values of critical buckling loads and stresses are optimum values of N. (Table 1, 2, 3 and 4).

CONCLUSION

In this research, buckling of the plates was investigated with different boundary conditions subjected to periodic and uniform loadings by differential equivalent direct method and FEM modeling. The governing differential equation was solved with deflections by analytical direct method that boundary conditions were satisfied by deflections. In addition, applied loads were periodic and uniform, for rectangular plate. Next, results of new formulations were similar to the FEM and analytical methods. Values of analytical critical buckling loads were coincided to the FEM and new formulations. Also, results of buckling loads by assumed deflections and governing equation are similar to the FEM and analytical methods. Finally, the ratio of buckling load in C-C-C-C to buckling load in S-S-S-S in rectangular plate subjected to periodic and uniform loadings was equal to 2, approximately. At last, good agreements were found between the analytical and numerical predictions for the critical buckling loads. Therefore, we could declare that results of analytical new direct and new formulations methods coincide to the finite element method logically. Many of intricate differential equations can be solved by analytical new direct method with smart vision and consideration and satisfied relations. Probably, most of researches will move and proceed to the analytical smart direct method for solution of complex differential equations by satisfied relations in future.

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