

## Transfer Function Modelling as a Tool for Solving Manufacturing System Dysfunction

<sup>1</sup>A.C. Igboanugo and <sup>2</sup>C.C. Nwobi-Okoye

<sup>1</sup>Department of Production Engineering, University of Benin, Benin City, Edo State, Nigeria

<sup>2</sup>Emeagwali Centre for Research, Anambra State University, Uli, Anambra State, Nigeria

**Abstract:** Wastes, poor quality products and downtimes are serious problems usually encountered during production which are often caused by ineffective maintenance and improper determination of process capability. The sensitivity and versatile applicability of transfer function modelling as an off-line tool for dealing with these types of problems is discussed. This study seeks to investigate the relationship among process variability, maintenance, process noise and controlled output in relation to the transfer function characteristics. A conceptual model of single-input-single-output production system, considering maintenance as one of the inputs, was developed and the transfer function synthesised. A random survey of twenty firms in Nigeria was conducted and the outcomes suggest that maintenance and related production variables contribute significantly to degraded output and process variability. Our result showed that there is a strong relationship between the ratio of controller output  $\theta(s)$  and process noise  $\theta_c(s)$ , on the one hand and the transfer function characteristics namely  $k$  and  $\tau_R$ , which can be manipulated to appreciably reduce product degradation and control process variability. Moreover, we showed that transfer function characteristics can be used to determine the need for process facilities maintenance. The study has ably demonstrated that transfer function is very effective in fault diagnosis.

**Keywords:** Maintenance, manufacturing dysfunction, process noise, production process, transfer function

### INTRODUCTION

Wastes in production output, low quality outputs and production machines' downtimes appear to be the major problems which manufacturers of product encounter in the course of production of goods. Whereas production system comprises the input, the process (black box) and the output, the causes of low quality output and hence waste can be traced to the nature of output, the organization and maintenance of the black box. Since wastes and low quality products invariably affect manufacturers' economic performance, it is essential that the causes be researched with a view to eliminating them.

The traditional approach of adopting specific maintenance policy coupled with adjustments to controllers are no longer considered cost effective in today's modern technology besides the fact that it entails holding more workforce to maintain such traditional system. The transfer function modelling approach advocated is predictive by nature in the sense that it foresees the imminence of process capability deterioration as well as the magnitude or degree of such dysfunction and based on such information adequate corrective action could then be taken.

Transfer function modelling is a veritable control theory tool for error minimization in different areas of endeavour in manufacturing. The goal is to minimize waste and there exists a vast pool of literature to support this capability of the transfer function (TF). For example Igboanugo and Nwobi-Okoye (2011a and 2011b) applied TF modelling to production process capability measurements, Horejs *et al.* (2010) applied the TF in machining operation for compensation of machine tool thermal errors. The authors demonstrated the applicability and robustness of the TF model experimentally on a real machine tool using indirect software compensation. Some research studies on thermal errors reduction using TF algorithm include Horejs (2009), Bryan (1990), Brecher and Hirsch (2004) and Barta (2007).

Ramesh (2009) provides a good review of error compensation on machine tools. Soysal and Semlyen (1993) reported on the estimation of transfer function parameter using least square method in conjunction with Singular Value Decomposition (SVD). The author found out that the condition number increases with observation frequency. They also noted that shifting row and column scaling were helpful in improving the condition of the problem.

Table 1: Frequency table for Q<sub>1</sub>

S/N	Rating(r)	Frequency(f)	(rf)
1	4	0	0
2	3	0	0
3	2	0	0
4	1	3	3

Table 2: Frequency table for Q<sub>4</sub>

S/N	Rating(r)	Frequency(f)	(rf)
1	4	3	3
2	3	0	0
3	2	0	0
4	1	0	0

Feser *et al.* (2009) reported that TF method seems to represent a quite sensitive method to detect faults in test set-ups and transformers. They concluded by stating that their results demonstrate the sensitivity and applicability of the transfer function as an off-line tool for diagnostics checking.

The foregoing review of literature has lucidly confirmed the versatility and applicability of transfer functions in detecting faults generally. The sample survey so far points to the fact that transfer function is relevant in maintenance and production system's management. The balance of literature on production system appears to offer little on transfer function applications. Evidently the current research is an attempt to create a breach in this knowledge frontier.

The aim of this study therefore is to demonstrate the plausibility of applying the transfer function in dealing with the problem of maintenance and process variability in production systems in Nigeria. A survey of twenty firms experiencing manufacturing dysfunction was conducted with a view to establishing the fact that the problems are not just farcical but real.

### THEORETICAL FRAMEWORK

Consider a data matrix R constructed with a four-point Rensis Likert's attitudinal scale comprising: often (4), occasionally (3), rarely (2) and never (1). If  $\alpha_{ij} \in I^+$  is the element of  $i^{th}$  row and  $j^{th}$  column, where  $\alpha_{ij}$  is the response of the  $i^{th}$  respondent to  $j^{th}$  question crafted with the scale item and entries in R are recorded in descending order of  $\alpha_{ij}$ , it follows that:

$$\alpha_{ij} \in R \text{ a}_{(i+1)j} \leq \alpha_{ij} \forall_j$$

Establish 4 quartiles of R, namely: Q<sub>1</sub>, Q<sub>2</sub>, Q<sub>3</sub> and Q<sub>4</sub> where:

$$Q_i \subset R, I = 1, 2, \dots, 4$$

Discard Q<sub>2</sub> and Q<sub>3</sub> and use Q<sub>1</sub> and Q<sub>4</sub> to calculate discrimination power (DP).

**DP calculations:** Consider Table 1 and 2, let the weighted average WA be given by:

$$WA = \frac{\sum_{i=1}^4 r_i f_i}{\sum_{i=1}^4 r_i} \tag{1}$$

where,  $r_i$  is the 4-points Rensis Likert's attitudinal rating and  $f_i$  is the frequency of occurrence of  $r_i$ .

Let WA<sub>1</sub> be weighted average in quartile 1 (Q<sub>1</sub>) and WA<sub>2</sub> be weighted average in quartile 4 (Q<sub>4</sub>). It follows that the DP is given by:

$$DP = WA_1 - WA_2 \tag{2}$$

The theorems that underpin the foregoing discriminative power concept are briefly presented here.

**Theorem 1:** Let R represent reordered data matrix in merit order of respondent's scores. If all entries  $\alpha_{ij} \in R$  are entirely uniform, the resulting DP = 0, leading to conclusiveness or consensuality or complete agreement.

**Proof:** From Eq. (1) we have:

$$WA = \frac{\sum_{i=1}^4 r_i f_i}{\sum_{i=1}^4 r_i}$$

Let WA<sub>1</sub> be weighted average in quartile 1 (Q<sub>1</sub>) and WA<sub>2</sub> be weighted average in quartile 4 (Q<sub>4</sub>):

$$\text{Then } WA_1 = \frac{\sum_{i=1}^4 r_{1i} f_{1i}}{\sum_{i=1}^4 f_{1i}} ; \text{ also } WA_2 = \frac{\sum_{i=1}^4 r_{2i} f_{2i}}{\sum_{i=1}^4 f_{2i}}$$

Since,  $r_{1i} = r_{2i}$  on account of uniform entries of R, then WA<sub>1</sub> - WA<sub>2</sub> = 0 (infimum).

**Theorem 2:** Let all elements of Q<sub>1</sub>  $\subset$  R be 4 and those of Q<sub>4</sub>  $\subset$  R be 1. Then the resulting DP is a Supremum leading to controvertibility.

**Proof:** Let  $\alpha_{ij} \in R$  be uniform and consider  $j^{th}$  question. Also let WA<sub>1</sub> be weighted average in quartile 1 (Q<sub>1</sub>) and WA<sub>2</sub> be weighted average in quartile 4 (Q<sub>4</sub>):

$$\text{Then } WA_1 = \frac{\sum_{i=1}^4 r_{1i} f_{1i}}{\sum_{i=1}^4 f_{1i}} ; \text{ also } WA_2 = \frac{\sum_{i=1}^4 r_{2i} f_{2i}}{\sum_{i=1}^4 f_{2i}}$$

Since,  $r_{1i} = 4r_{2i}$  on account of uniform entries of R, then WA<sub>1</sub> = WA<sub>2</sub>. Therefore, DP = 3WA<sub>2</sub>.

Table 3: Frequency table for Q<sub>1</sub> of sample 1

S/N	Rating(r)	Frequency(f)	(rf)
1	4	0	0
2	3	3	9
3	2	0	0
4	1	0	0

Table 4: Frequency table for Q<sub>4</sub> of sample 1

S/N	Rating(r)	Frequency(f)	(rf)
1	4	0	0
2	3	0	0
3	2	0	0
4	1	3	3

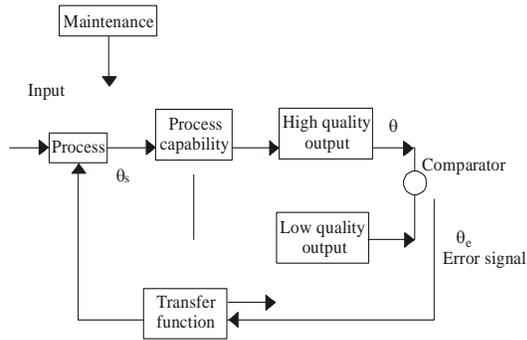


Fig. 1: Conceptual model of the processing system

**Theorem 3:** Let all elements of Q<sub>1</sub> ∈ R be r<sub>1</sub> and those of Q<sub>4</sub> ∈ R be r<sub>2</sub> where, r<sub>1</sub>-r<sub>2</sub> = 1. Then the resulting DP is a middling signifying inconclusiveness or is up in the air.

**Proof:** Let WA<sub>1</sub> be weighted average in quarter 1 (Q<sub>1</sub>) and WA<sub>2</sub> be weighted average in quarter 4 (Q<sub>4</sub>):

$$\text{Then } WA_1 = \frac{\sum_{i=1}^4 r_{1i} f_{1i}}{\sum_{i=1}^4 f_{1i}} \text{ also } WA_2 = \frac{\sum_{i=1}^4 r_{2i} f_{2i}}{\sum_{i=1}^4 f_{2i}}$$

$$\text{Since, } f_{1i} = f_{2i} \text{ and } r_{1i} - r_{2i} = 1 \text{ then, } WA_2 - WA_1 = \frac{\sum_{i=1}^4 f_{1i} (r_{1i} - r_{2i})}{\sum_{i=1}^4 f_{1i}} = 1$$

Thus the DP depends on the difference between ratings of the upper and lower quartiles. The maximum difference of 3 is obtained for controvertibility. In the second case, the minimum difference of 0 is found for consensuality. In the third case, the middle of the road difference of 1 is got for up in the air or unresolved or open case or inconclusiveness.

**Sample calculation:**

Consider Table 3 and 4.

And applying Eq. (1) and (2), we have:

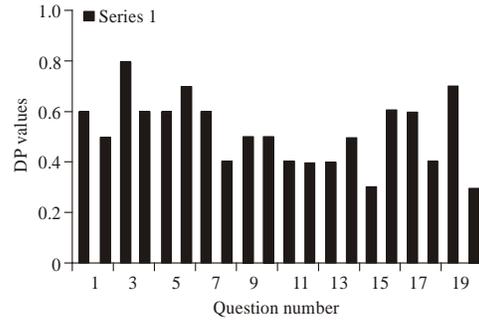


Fig. 2: DP values for twenty production system variables

WA<sub>1</sub> = 0.9  
 WA<sub>2</sub> = 0.3  
 DP = 0.6

The DP values for the remaining 19 samples were similarly determined and shown in Fig. 2.

**METHODOLOGY**

The method adopted consists of developing a conceptual model of production system that comprises input, maintenance, process, output and comparator. It is assumed that the system incorporates manual proportional control as well as floating actions both of which are common features of a modern process. Time series modelling of input and output system was considered using the backshift operator B and lag variable b. Possible model structures for pre- and post-maintenance status were considered. Also a time domain differential-integro-process equation was developed incorporating proportional control and floating action parameters (k and k<sub>R</sub>) respectively. This equation was then transformed using Laplace transforms in order to convert the unstructured differential-integro equation, with its associated messy computation, to ordinary algebra. In this regard, the analysis gave a clear picture of key control variables that optimise desired output and the associated process noise.

Furthermore, a multidimensional survey of factors that influence quality of output was conducted and Rensis Likert's 4-point attitudinal scale was used in dimensioning the respondents' responses, which were captured in a data matrix that was subsequently analysed with opinion discrimination analytical tool to obtain the discriminative powers (DP). The DP values were plotted as bar chart and interpretations rendered. The DP-values are segmented as follows:

- **Lower bandwidth:** 0.2 < DP ≤ 0.5  
 In this regime, the respondents are in total agreement on the issue considered. In other words there is consensuality.

- **Middle bandwidth:**  $0.5 < DP \leq 0.6$   
In this group, the respondents are fairly in agreement on the subject matter.
- **Upper bandwidth:**  $DP > 0.6$   
In this category, the respondents are weakly in agreement with what was put forth.

**Methodology of relating transfer functions to maintenance:** The production system transfer function is hereby represented by the general model that describes dynamic systems in differential equation form:

$$(1 + \bar{E}_1 D + \dots + \bar{E}_r D^r) Y(t) = g(1 + H_1 D + \dots + H_s D^s) X(t-b) \quad (3)$$

Since the production process is better represented by discrete dynamic equation, Eq. (3) is replaced by an equivalent difference equation to obtain (Box *et al.*, 1994):

$$(1 + \xi_1 \nabla + \dots + \xi_r \nabla^r) Y(t) = g(1 + \eta_1 \nabla + \dots + \eta_s \nabla^s) X(t-b) \quad (4)$$

Using the backward shift operator on Eq. (4) and (5) is obtained:

$$(1 - \delta_1 B^{-1} - \dots - \delta_r B^{-r}) Y_t = (\omega_0 - \omega_1 B - \dots - \omega_s B^s) X_{t-b} \quad (5)$$

$$\delta(B) Y_t = \omega(B) X_{t-b} \quad (6)$$

$$Y_t = \delta^{-1}(B) \omega(B) X_{t-b} \quad (7)$$

$$\Omega(B) = \omega(B) B^b \quad (8)$$

$$\delta(B) Y_t = \Omega(B) X_t \quad (9)$$

$$Y_t = \delta^{-1}(B) \Omega(B) X_t \quad (10)$$

Let  $v(B)$  represent the system transfer function. It follows that:

$$v(B) \delta^{-1}(B) \Omega(B) \quad (11)$$

$$Y_t = v(B) X_t \quad (12)$$

From Eq. (12),

$$Y_t = v(B) X_t$$

If  $Y$  lags  $X$ , then Eq. (12) could be written as:

$$Y_t = v(B) X_{t-b} \quad (13)$$

Let the input-output relationship before and after maintenance be respectively given by:

$$Y_{bt} = v_b(B) X_{t-b1} \quad (14)$$

$$Y_{at} = v_a(B) X_{t-b2} \quad (15)$$

$$\text{If } v_b(B) \neq v_a \quad (16)$$

$$\text{or } b1 \neq b2,$$

then the transfer function,  $v(B)$ , could be used as the predictor tool, with the variables serving as maintenance status indicators. Moreover, progressive growth in the lag variable  $b$  is a strong indication that there is some level of system dysfunction.

Figure 1 shows the production system incorporating the proportional and flow action controls.

The time domain process equation is given by:

$$\theta_s = \theta(s) = k\theta_e + k_R \int \theta_e dt \quad (17)$$

This differential-Integro equation embodies both manual control ( $k$ ) and automatic reset ( $k_R$ ). The Laplace transform leads to the equation:

$$\theta(s) = (k + \frac{k_R}{s}) \theta_e(s) \quad (18)$$

$$\theta(s) = \theta_e(s) \frac{k_R}{s} (\frac{k}{k_R} s + 1)$$

$$\theta(s) = \theta_e(s) \frac{k}{s(k/k_R)} (\frac{k}{k_R} s + 1)$$

$$\frac{\theta(s)}{\theta_e(s)} = \frac{k_R}{s} (\tau_R s + 1) \quad (19)$$

$$\tau_R = \frac{k}{k_R}$$

$$\frac{\theta(s)}{\theta_e(s)} = \frac{k}{\tau_R s} (\tau_R s + 1) \quad (20)$$

## RESULTS

Equation (14) and (15) compare pre- and post-maintenance transfer functions and where the transfer function characteristics differ significantly, a deterioration of output quality needing corrective action is therefore called for.

On the other hand, Eq. (20) shows the proportional relationship between target output  $\theta$  and the process noise  $\theta_e$  in the  $s$ -domain and the associated transfer function

Table 5: Lower bandwidth DP values of variables that affect production system efficiency

Variable number	Description	DP value
2	Poor housekeeping	0.5
8	Unqualified personnel	0.4
9	Inadequate maintenance tools	0.5
10	Inexperience	0.5
11	Lack of predictive tools	0.4
12	Aged equipment	0.4
13	Poorly designed equipment	0.4
15	Poor communication	0.3
18	Maintenance procedure violations	0.4
20	Poor supervision and work assignment	0.3

Table 6: Middle bandwidth DP values of variables that affect production system efficiency

Variable number	Description	DP value
1	Poor work place design	0.6
4	Poor motivation	0.6
5	Physical disability	0.6
7	Poor on- the- job training	0.6
16	Poor interpretation of manuals	0.6
17	Unavailability of manuals	0.6

Table 7: Upper bandwidth DP values of variables that affect production system efficiency

Variable number	Description	DP value
3	Fatigue	0.8
6	Mental unfitness and behavioural factor	0.7
19	Behavioural violations	0.7

characteristics. It is evident from (20) that undegraded output  $\theta$  as well as process noise is dependent upon the proportional gain ( $k$ ) and the ratio ( $k/k_R$ ) respectively. The relation developed can enable production output quality target to be constantly kept on course. Figure 2 shows the DP pattern for the 20 production system variables studied.

The DP values in Table 5 are low indicating that there is high level of agreement among respondents that these variables are material to degraded process capability and hence significant process error  $\theta_e$  and degraded output. See Theorem 1.

In Table 6, the respondents are fairly in agreement with certain attributions made concerning these variables. Any decisions reached about this cluster are therefore considered fairly conclusive. The reason for this is evident in Theorem 3.

As shown in Table 7 the DP values are high, the variables clustered therein are relatively controversial, as the respondents are weakly in agreement with the attributions made concerning such variables. The reason for this is shown in Theorem 2.

### DISCUSSION

At the onset it was claimed that a transfer function model capable of assessing process capability and maintenance effectiveness would be developed. Justification of this claim is pursued in what follows:

Equation (14), (15) and (20) represent the transfer functions. Whereas (14) and (15) deals with comparing

pre- and post-maintenance relativities, (20) compares desired output  $\theta$  and the error signal (process noise)  $\theta_e$ . Thus, by quantifying these process transfer functions, the variance can help to track product quality variabilities, thus tracking production process dysfunction as well as serving as a production manager's guide to action.

The DP-values obtained have enabled us to appreciate the main factors to blame for manufacturing dysfunction. In particular their relative influence is indicative of the magnitude of the DP-values. This is also another manager's guide to action.

Overall, the transfer function models developed in conjunction with the DP-values established have been helpful in dealing with manufacturing dysfunction common in small and medium scale enterprises especially in developing countries where manufacturing functions are stymied by economic depression. Thus the use of the transfer function tools as an aid in the manufacturing sector would likely engender economic growth and development in the industrial sector.

### NOMENCLATURE, SYMBOLS AND NOTATIONS

- $\theta_s$  = Controller output
- $\theta_e$  = Control error/ process noise
- $k$  = Proportional gain
- $k_R$  = Automatic reset gain
- $\tau_R$  = Automatic reset time constant
- (s) = Laplace transfer domain (s-domain)
- T = Time domain
- v(B) = Transfer function
- B = Backshift operator
- Y = Process output
- X = Process input
- b = Lag variable
- $\Omega$  = Difference equation variable for input
- $\omega$  = Difference equation variable for input
- $\delta$  = Difference equation variable for output
- DP = Discriminative power
- Infimum = Greatest lower bound
- Supremum = Least upper bound

### CONCLUSION

Several factors work conjointly to affect manufacturing dysfunction. The transfer function modelling approach proposed is effective in dealing with the problem of process variability and degraded output.

Manufacturing industries are in concordance that maintenance variables contribute to degradation of outputs. This is evidenced by the low DP-values obtained. This empirical evidence is supportive to the objectives of the study. Also TF models have been shown to possess

inherent properties for predicting imminence of process capability deterioration as exemplified by Eq. (20). Finally, the theory underpinning the discriminative power index is also developed and demonstrated.

#### REFERENCES

- Barta, P., 2007. Thermal transfer function based control method of a machine cooling system. Proceedings of the Topical Meeting: Thermal Effects in Precision Systems, Maastricht, EUSPEN, pp: 16-18.
- Box, G.E.P., G.M. Jenkins and G.C. Reinsel, 1994. Time Series Analysis Forecasting and Control. McGraw-Hill Inc., USA.
- Brecher, C. and P. Hirsch, 2004. Compensation of thermal-elastic machine tool deformation based on central internal data. *Ann. CIRP*, 53(1).
- Bryan, P., 1990. Intermediate status of thermal error research. *Ann. CIRP*, 39(2): 645-656.
- Feser, O., J. Christian, T. Liebfried, M. Loppacher, A. Kachler, C. Neumann and U. Sundermann, 2009. Transfer Function Method for Detection of Winding Displacements on Power Transformers after Transport, Short Circuit or 30 Years Service. Technical Report, Haefely High Voltage Test. Retrieved from: <http://www.haefely.com/>, (Accessed on: May 23, 2011).
- Horejs, O., 2009. Thermal transfer function based control method of a machine cooling system. Proceedings of the Topical Meeting: Thermal Effects in Precision Systems, 2-5 June 2009, San Sebastian, EUSPEN, pp: 16-18.
- Horejs, O., M. Mares, P. Kohut, P. Barta and J. Hornych, 2010. Compensation of machine tool thermal errors based on transfer functions. *Modern Manufact. Sci. J.*, pp: 25-28.
- Igboanugo, A.C. and C.C. Nwobi-Okoye, 2011a. Production process capability measurement and quality control using transfer functions. *J. Nigerian Assoc. Math. Phys.*, 19: 453-464.
- Igboanugo, A.C. and C.C. Nwobi-Okoye, 2011b. Optimisation of transfer function models using genetic algorithms. *J. Nigerian Assoc. Math. Phys.*, 19: 439-452.
- Ramesh, R., 2009. Error compensation in machine tools-a review. *Int. J. Machine Tool Manufact.*, 40: 1257-1284.
- Soysal, A.O. and A. Semlyen, 1993. Practical transfer function estimation and its application to wide range representation of transformers. *IEEE T. Power Delivery*, 8(3): 1627-1637.