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Elastic Comparison Between Human and Bovine Femural Bone

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Abstract: In this study, the elastic stiffness and the degree of anisotropy will be compared for the femur human and bovine bones are presented. A scale for measuring the overall elastic stiffness of the bone at different locations is introduced and its correlation with the calculated bulk modulus is analyzed. Based on constructing orthonormal tensor basis elements using the form-invariant expressions, the elastic stiffness for orthotropic system materials is decomposed into two parts; isotropic (two terms) and anisotropic parts. The overall elastic stiffness is calculated and found to be directly proportional to bulk modulus. A scale quantitative comparison of the contribution of the anisotropy to the elastic stiffness and to measure the degree of anisotropy in an anisotropic material is proposed using the Norm Ratio Criteria (NRC). It is found that bovine femure plexiform has the largest overall elastic stiffness and bovine has the most isotropic (least anisotropic) symmetry.

Key words: Anisotropic materials, bulk modulus, human and bovine bones, overall stiffness

INTRODUCTION

It has been seen as a way of obtaining deeper insight into the intrinsic relation between structure and properties as well as of being important in understanding bone modeling. The remodeling of bone due to an implant is generally sophisticated. Bone is an inhomogeneous anisotropic, viscoelastic material, but experience has shown that it is reasonable to model bone as linearly elastic and anisotropic (Katz and Meunier, 1987). Although the symmetry of bone has been modeled as transversely isotropic symmetry, the most general degree of anisotropy assumed for bone is that of orthotropic material symmetry. An orthotropic material is characterized elastically by nine independent elastic constants. Hence, the contribution of anisotropy to the bone is an open question that was discussed in previous studies (Katz and Meunier, 1987; Yoon and Newnham, 1969; Berme *et al*., 1977; Ashman *et al*., 1984; Buskirk and Ashman, 1981; Katz and Thompson, 1977; Maharidge, 1984; Lang 1970) and is the goal of this study. Katz and Meunier (1987) worked on the microstructure of bone. He concluded that certain microstructural features suggest that the cortical haversian bone is transverse isotropic in its symmetry and that cortical lamellar bone is orthotropic bone is orthotropic. Buskirk and Ashman (1981) listed the anisotropic elastic constants for cortical bone using ultrasonic wave propagation from anatomical position; the Anterior, A, Medial M, Posterior P and Lateral L aspects at a number of different levels in the same bone. The specimens of different types of bone, or from different aspects of the same bone, or subject to different measurement technique, or measured as a function of temperature or humidity or aging to be compared both quantitatively and quantitatively is indispensible in biotechnology.

Physical properties are intrinsic characteristics of matter that are not affected by any change of the coordinate system. Therefore, tensors are necessary to define the intrinsic properties of the medium that relate an intensive quantity (i.e., an externally applied stimulus) to an extensive thermodynamically conjugated one (i.e. the response of the medium). Such intrinsic properties are the dielectric susceptibility and the elasticity tensors. Several studies were conducted to reveal the physical properties using decomposition methods for elastic stiffness tensors (Srinivasan, 1969, 1985; Spencer, 1983; Tu, 1968; Jerphagnon *et al*., 1978; Gaith and Akgoz, 2005; Sutcliffe, 1992). An interesting feature of the decompositions is that it simply and fully takes into account the symmetry properties when relating macroscopic effects to microscopic phenomena. One can directly show the influence of the crystal structure on physical properties, for instance, when discussing macroscopic properties in terms of the sum of the contributions from microscopic building units (chemical bond, coordination polyhedron, etc). A significant advantage of such decompositions is to give a direct display of the bearings of the crystal structure on the physical property. For the stress and strain, for instance, the decomposition allows one to separate changes in volume from changes of shape in linear isotropic elasticity; the bulk modulus relates to the hydrostatic part of stress and strain while the shear modulus relates the deviatoric part (Voigt, 1889).

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It is often useful, especially when comparing different materials or systems having different geometrical symmetries, to characterize the magnitude of a physical property. One may also have to make, in a given material, a quantitative comparison of the contribution of the anisotropy to a physical property (Tu 1968; Jerphagnon *et al.*, 1978; Gaith and Akgoz, 2005). The comparison of the magnitudes of the decomposed parts can give, at certain conditions, valuable information about the origin of the physical property under examination. These problems can be dealt with by defining the norm of a tensor. The norm is invariant and not affected by any change of the coordinate system. Invariance considerations are of primary importance when studying physical properties of matter, since these properties are intrinsic characteristics which are not affected by a change of the reference frame. Tu (1968) and Jerphagnon *et al*. (1978) proposed the norm criterion to quantify and then, quantitatively to compare the effect of elasticity using irreducible tensor theory. They compared the magnitude of elasticity of two materials only of the same symmetry using Cartesian and spherical framework. However, their method seemed to be valid only for elastic tensor. Gaith and Akgoz (2005) developed a decomposition procedure based on constructing orthonormal tensor basis elements using the form-invariant expressions (Srinivasan, 1969, 1985; Spencer, 1983). They introduced a method to measure the stiffness in fiber reinforced composite materials, using the norm criterion on the crystal scale. In their method, norm ratios proposed to measure the degree of anisotropy in an anisotropic material and compare it with other materials of different symmetries. They were able to segregate the anisotropic material property into two parts: isotropic and anisotropic parts. Of the new insights provided by invariance considerations, the most important is providing a complete comparison of the magnitude of a given property in different symmetries. From a device point of view, the new insights facilitate the comparison of materials; one is interested in maximizing the figure of merit by choosing the optimum configuration (crystal cut, wave propagation direction and, etc); and one wants to be able to state that a particular material is better than another for making a joint replacement (Katz and Meunier, 1987). It is most suitable for a complete quantitative comparison of the strength or the magnitude of any property in different materials belonging to the same crystal symmetry class, or different phases of the same material.

The goal of this study is to compare between two different types of bone; bovine and human femur. The effect of anisotropy on these tow bone will be investigated and the overall stiffness for each bone will be quantified.

DECOMPOSITION AND NORM CONCEPT

The Orthonormal Decomposition Method (ODM) (Tu, 1968) is established through constructing an orthonormal tensor basis using the form-invariant expressions (Srinivasan 1969, 1985; Spencer, 1983). The basis is generated for the corresponding symmetry medium of the tensor and the number of basis elements should be equal to the number of non-vanishing distinct stiffness coefficients that can completely describe the elastic stiffness in that medium. Accordingly, based on the Orthonormal Decomposition Method, the basis elements for isotropic material is obtained and consisted of two terms; shear and bulk moduli (Tu, 1968) and they are identical to those found in literature (Jerphagnon *et al*., 1978). The elastic stiffness matrix representation for the isotropic system can be decomposed in a contracted form as:

$$C_{ij} = \begin{bmatrix} 2C_{44}+C_{12} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & 2C_{44}+C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & 2C_{44}+C_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} = A_1 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + A_2 \begin{bmatrix} 4 & -2 & -2 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 & 0 \\ -2 & -2 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix} \quad (1)$$

where,

$$A_1 = \frac{1}{3}\left(C_{11} + 2C_{12}\right), \quad C_{11} = 2C_{44} + C_{12}$$
$$A_2 = \frac{1}{15}\left(C_{11} - C_{12} + 3C_{44}\right) \quad (2)$$

where, A_1 and A_2 are the Voigt average polycrystalline bulk *B* and shear *G* modulus, respectively. The decomposed parts of Eq. (1) designated as bulk and shear modulus are identical to those found in literature (Voigt, 1889; Hearmon, 1961; Pantea *et al*., 2009).

Bovine and human femurs are modeled with orthotropic symmetry. There are nine independent elastic stiffness coefficients that can describe the mechanical elastic stiffness for these materials. These elastic coefficients are function of elastic material parameters, namely, Young's modulus, shear modulus and Poisson's ratio. The values of the coefficients are listed in Table 1 for three different sections of the bovine femur bone and for femur and tibi human bones.

Thus, using the orthonormal decomposition procedure (Gaith and Akgoz, 2005), the elastic stiffness matrix representation for orthotropic system can be decomposed in a contracted form as shown in Eq. (3).

It can be shown that the sum of the four orthonormal parts on the right hand side of Eq. (3) and (4) is apparently the main matrix of orthotropic system (Pantea *et al*., 2009). Also, the first two terms on the right hand side are identical to the corresponding two terms obtained in Eq. (1) for the isotropic system (Hearmon, 1961). Hence, it can be stated that the orthotropic system is discriminated into the sum of two parts: isotropic part (first two terms) and anisotropic part (seven terms). The latter term resembles the contribution of the anisotropy on elastic stiffness in the orthotropic system. On the other hand, the first term on the right hand side of Eq. (1) and (3), designated as the bulk modulus, is identical to Voigt bulk modulus (Hearmon, 1961).

Since the norm is invariant for the material, it can be used for a Cartesian tensor as a parameter representing and comparing the overall stiffness of anisotropic materials of the same or different symmetry or the same material with different phases (Tu, 1968; Jerphagnon *et al*., 1978; Gaith and Akgoz; 2005).

The larger the norm value is, the more the elastic stiffness of the material is. The concept of the modulus of a vector, norm of a Cartesian tensor is defined as (Gaith and Akgoz, 2005):

$$N = \|C\| = \left\{ C_{ij} \bullet C_{ij} \right\}^{1/2} \tag{3}$$

$$\begin{aligned}
C_{ij} &= \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} = A_1 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&+ A_2 \begin{bmatrix} 4 & -2 & -2 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 & 0 \\ -2 & -2 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix} + A_3 \begin{bmatrix} -3 & -1 & -1 & 0 & 0 & 0 \\ -1 & -3 & -1 & 0 & 0 & 0 \\ -1 & -1 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} + \\
&A_4 \begin{bmatrix} -3 & -5 & 4 & 0 & 0 & 0 \\ -5 & -3 & 4 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} + A_5 \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} + \\
&A_6 \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} + A_7 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \\
&A_8 \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + A_9 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned} \tag{4}$$

where,

$$\begin{aligned}
A_1 &= \frac{1}{9}((C_{11} + C_{33} + C_{22}) + 2(C_{12} + C_{13} + C_{23})) \\
A_2 &= \frac{1}{45}((C_{11} + C_{33} + C_{22}) + 3(C_{44} + C_{55} + C_{66}) - (C_{12} + C_{13} + C_{23})) \\
A_3 &= \frac{1}{180}((15C_{33} - 3(C_{11} + C_{33} + C_{22}) - 4(C_{44} + C_{55} + C_{66}) \\
&-2(C_{12} + C_{13} + C_{23})) \\
A_4 &= \frac{1}{180}((3C_{33} + 18C_{13} + 18C_{23} - 3(C_{11} + C_{33} + C_{22}) + \\
&4(C_{44} + C_{55} + C_{66}) - 10(C_{12} + C_{13} + C_{23})) \\
A_5 &= \frac{1}{16}(8C_{44} + 8C_{55} - 2C_{13} - 2C_{23} + C_{33} - (C_{11} + C_{33} + C_{22}) + \\
&4(C_{44} + C_{55} + C_{66}) + 2(C_{12} + C_{13} + C_{23})) \\
A_6 &= \frac{1}{8}(-4C_{44} - 4C_{55} - 2C_{13} - 2C_{23} + C_{33} - (C_{11} + C_{33} + C_{22}) + \\
&4(C_{44} + C_{55} + C_{66}) + 2(C_{12} + C_{13} + C_{23})) \\
A_7 &= \frac{1}{2}(C_{11} - C_{22}) \\
A_8 &= \frac{1}{2}(C_{12} - C_{23}) \\
A_9 &= \frac{1}{2}(C_{44} - C_{55})
\end{aligned} \tag{5}$$

RESULTS AND DISCUSSION

There has been an increased interest in recent years in measuring the anisotropic properties of bone detail about p specimens. Measurements of the anisotropic

Table 1: Elastic coefficients *(GPa)* for bovine and human femur bones

	Bovine femur[a,b]			Human[c,d]	
	haversian	plexiform	phalanx	tibia	femur
C_{11}	21.1	22.4	19.7	11.2	20
C_{22}	21.0	25.0	19.7	14.4	21.7
C_{33}	29.0	35.0	32	22.5	30
C_{44}	6.3	8.2	5.4	4.91	6.56
C_{55}	6.3	7.1	5.4	6.92	11.5
C_{66}	5.4	6.1	3.8	2.41	4.74
C_{12}	11.7	14.0	12.1	7.95	10.9
C_{13}	12.7	15.8	12.6	6.1	11.5
C_{23}	11.1	13.6	12.6	3.56	5.85

a: Maharidge (1984); b: Lang (1970); c: Ashman *et al.* (1985); d: Buskirk and Ashman (1981)

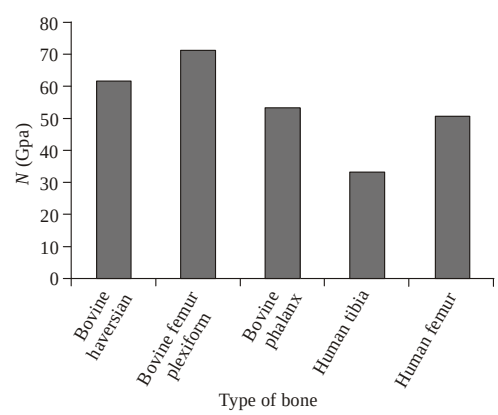

Fig. 1: The overall elastic stiffness *N* versus bone type

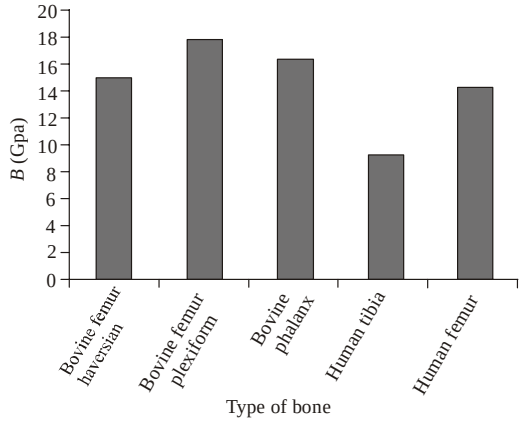

Fig. 2: The bulk modulus *B* versus bone type

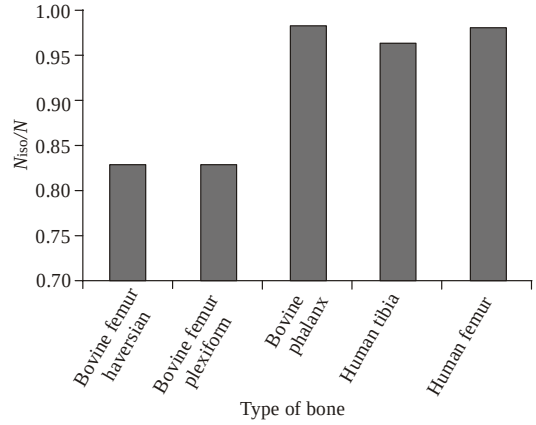

Fig. 3: Isotropic norm ratio *Niso*/*N* versus bone type

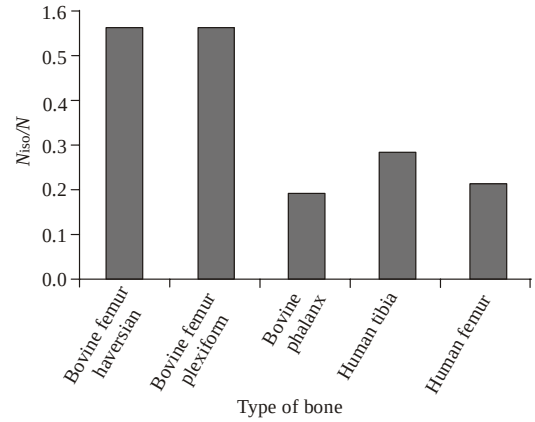

Fig. 4: Isotropic norm ratio *Naniso*/*N* versus bone type

properties provide much more ossible changes in bone remodeling. Decreases in the magnitude of the elastic constants have been correlated with aging microdamage accumulation, as well as bone diseases such as osteoporosis and osteogenesis imperfcta. Therefore, micromechanical models have been developed to investigate the structural origins of elastic inhomogeneity and anisotropy (Deuerling *et al*., 2009).

Based on the elastic stiffness coefficients listed in Table 1, overall elastic stiffness *N*, bulk modulus *B* are calculated for the five bone specimens in consideration. Figure 1 shows clearly the overall elastic stiffness for each bone.

Quantitatively, the overall elastic stiffness has the largest value (72 *GPa*) for bovine plexiform among the five bones while the human tibia bone has the smallest overall elastic stiffness. Figure 2 shows the bulk modulus *B* for each of the five bones. Clearly the behavior of the bulk modulus *B* is similar of the behavior of the overall elastic stiffness for all the bones except for bone phalanx which has the second largest bulk modulus after bovine femur plexiform. Hence, a conclusion can be states that the overall elastic stiffness and the bulk modulus are proportionally related. Therefore, the overall elastic stiffness and bulk modulus, the only elastic moduli possessed by all states of matter, reveal much about internal anisotropy and its affect on bonding strength. The bulk modulus also is the most often cited elastic constant to compare interatomic bonding strength among various materials (Pantea *et al*., 2009) and thereafter the overall elastic stiffness can be cited as well.

For the isotropic symmetry material, the elastic stiffness tensor is decomposed into two parts as shown in Eq. (1), meanwhile, the decomposition of the transversely isotropic symmetry material, from Eq. (3), is consisted of the same two isotropic decomposed parts and another three parts. It can be verified the validity of this trend for higher anisotropy, i.e., any anisotropic elastic stiffness will consist of the two isotropic parts and anisotropic parts. Their total parts number should be equal to the number of the non-vanishing distinct elastic coefficients for the corresponding anisotropic material. Anisotropic materials with orthotropic symmetry, for example, like fiber reinforced composites and bones should have two isotropic parts and seven independent parts. Consequently, The Norm Ratio Criteria (NRC) used in this study is similar to that proposed in Gaith and Akgoz (2005). For isotropic materials, the elastic stiffness tensor has two parts, Eq. (1), so the norm of the elastic stiffness tensor for isotropic materials is equal to the norm of these two parts, Eq. (5), i.e., $N = N_{iso}$. Hence, the ratio N_{iso}/N is equal to one for isotropic materials. For orthotropic symmetry materials, the elastic stiffness tensor has the same two parts that consisting the isotropic symmetry materials and other seven parts, will be designated as the other than isotropic or the anisotropic part. Hence, two ratios are defined as: N_{iso}/N for the isotropic parts and N_{aniso}/N for the anisotropic part (s). The norm ratios can also be used to assess the degree of anisotropy of a

material property as a whole. In this study the following criteria are proposed: when N_{iso} is dominating among norms of the decomposed parts, the closer the norm ratio N_{iso}/N is to one, the more isotropic the material is. When N_{iso} is not dominating, norm ratio of the other parts, N_{aniso}/N, can be used as a criterion. But in this case the situation is reversed; the closer the norm ratio N_{aniso}/N is to one, the more anisotropic the material is.

The norms and isotropic norm ratios for the five bones specimens are calculated and shown in Fig. 3. Clearly the bovine phalanx bone has highest isotropic norm ration (which means nearest to isotropic behavior) N_{iso}/N= 0.9817. This can be verified since the elastic coefficients, from Table 1, for phalanx are similar to the transversely isotropic symmetry in which it is closer to isotropy than other specimens. On the other hand bovine femur plexiform has the lowest isotropic norm ratio (most anisotropic) N_{iso}/N = 0. 8269. Hence, Fig. 4 confirms the findings of Fig. 3.

CONCLUSION

An interesting feature of the decompositions is that it simply and fully takes into account the symmetry properties when relating macroscopic effects to microscopic phenomena. Therefore, the decomposition of elastic stiffness for bovine and human bones with orthotropic symmetry materials into two parts; isotropic (two terms) and anisotropic parts is presented. A scale for measuring overall elastic stiffness is introduced and correlated to different bovine and human bones. The overall elastic stiffness and bulk modulus for these bones are calculated and found to have the largest value for bovine plexiform. Meanwhile human tibia has been found to be the smallest overall stiffness among these five bone specimens.

The Norm Ratio Criteria (NRC) is introduced to scale and measure the isotropy in the transversely isotropic symmetry cortical human bone. Hence, a scale quantitative comparison of the contribution of the anisotropy to the elastic stiffness and to measure the degree of anisotropy in an anisotropic material is proposed. Bovine plexiform is found to be the least isotropic (or nearest to anisotropic) among the five specimens and Bovine phalanx is the nearest to isotropy. These conclusions will be investigated on different types of bones and for orthotropic and transversely isotropic human, canine and bovine bones in the next study.

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