Model Following Predictive Control for Network Congestion System

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Abstract: The goal of congestion control is to avoid congestion in network elements. This study derives an MFCS (Model Following Control System)-based control methodology for networks congestion system. The features of this design method are: Bounded property of the inner states for the control system is given and the utility of this control de because there is no necessary to make transformation of this system; It is confirmed on basis of a sign is guaranteed; Both the physical structure of the system and the physical system variables properties can be preserved because there is no necessary to make transformation of this system. It is confirmed on basis of a numerical example of the network congestion system that the output signal of the control system asymptotically follows the reference model signal in the case of the existence of disturbances.

Keywords: Model following control system, network congestion system, nonlinear system, time delay

INTRODUCTION

The design of some communication systems requires the implementation of time delays within the system. These time delays can be accomplished with a variety of optics technologies, which could be readily fabricated and integrated into the communication system without significant impacts on the system design.

Time-delay is very important for the modeling of networks, occurring both in the control of networks and in the control over networks (Chiasson and Loiseau, 2007). In the context of communication networks, the term "congestion control" is generally used to refer to the action of regulating various flows within a network. In recent years, intense research efforts have been devoted to the application of the Smith predictor for queue length control of ATM (asynchronous transfer mode) networks (Mascolo, 2000). The Transmission Control Protocol (TCP) was designed in the late 1980s by Van Jacobson, which is a critical part of the Internet machinery.

TCP/AQM (Transmission Control Protocol/ Active Queue Management) networks, which consist of TCP networks and AQM mechanisms (congestion controllers), are considered and dynamical models of the average TCP window size and the queue size in the bottleneck router are derived. Since dynamical models are nonlinear, the models are linearized at an equilibrium point and PI models depend on information delay, which is called time-delay and it is difficult to de sign controllers which are assured to have better stability and performance for models with time delay (Gu *et al.*, 2007; Hollot *et al.*, 2001a, b, 2002; Misra *et al.*, 2000; Azuma *et al.*, 2005a, b, c).

The purpose of this study is to design a congestion controller based on the Model Following Control System-MFCS (Wang *et al.*, 2009; Wu *et al.*, 2008; 2011; Wang and Okubo, 2008; Akiyama, 1998) control theory. The features of this design method are: Bounded property of the inner states for the control system is given and the utility of this control de because there is no necessary to make transformation of this system; It is confirmed on basis of a sign is guaranteed; Both the physical structure of the system and the physical system variables properties can be preserved because there is no necessary to make transformation of this system. It is confirmed on basis of a numerical example of the network congestion system that the output signal of the control system asymptotically follows the reference model signal in the case of the existence of disturbances.

THE EXPRESSION OF PROBLEMS

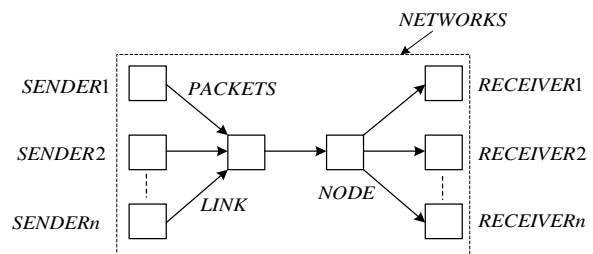A computer network considered in this study is shown in Fig. 1. This computer network consists of

Fig. 1: Computer networks with TCP

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Senders from No. 1 to n, one bottleneck router where congestion occurs and one receiver. Using TCP (Transmission Control Protocol), Data, which is divided into some pieces called packets, is sent from senders to the receiver through the bottleneck router.

Hollot *et al.* (2001b) and Misra *et al.* (2000) proposed nonlinear mathematic models which describe behaviors of the average window size in senders and the queue size in the bottleneck router in the network are given as follows:

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)}{2} \frac{W(t-R(t))}{R(t-R(t))} \overline{p}(t-R(t)) \qquad (1)$$

$$\dot{q}(t) = \frac{W(t)}{R(t)} N(t) - D(t) \qquad (2)$$

$$R(t) = \frac{\overline{q}(t)}{D(t)} + T_p \qquad (3)$$

where, $W(t)$ is the congestion window size in packets at time t, $q(t)$ is the queue length at the congested router in packets, $R(t)$ is the Round Trip Time (RTT) which represents the delay in TCP dynamics in seconds, $D(t)$ is the link capacity in packets per second, T_p is the propagation delay in seconds, $N(t)$ is the number of active TCP connections, $\overline{p}(t)$ is the packet mark/drop probability. Taking (W, q) as the state and p as the input and assuming $N(t) = N$, $R(t) = R_0$, $D(t) = D$, q (t) = q and W = 1, we define the set of operating points (W_0, q_0, p_0) by $\dot{W} = 0$ and $\dot{q} = 0$. Linearize the above mathematical model near the equilibrium point and use the Laplace transformation. $\delta W = W - W_0$, $\delta q = q - q_0$ and $\delta p = p - p_0$ are the state variables and input perturbations around the operating point. The control system block diagram (Fig. 2) and the control system is obtained as follows Hollot *et al.* (2002).

where,
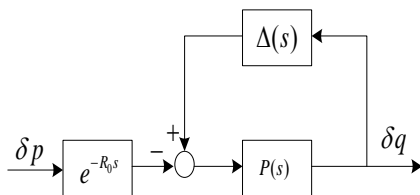
$$\Delta(s) = \frac{2N^2 s}{R_0^2 D^3}(1 - e^{-R_0 s})$$

Fig. 2: Block diagram of the linearized TCP dynamics

Then we have:

$$\delta \dot{W}(t) = -\frac{2N}{R_0^2 D}\delta W(t) - \frac{R_0 D^2}{2N^2}\delta p(t - R_0) \qquad (4)$$

$$\delta \dot{q}(t) = \frac{N}{R_0}\delta W(t) - \frac{1}{R_0}\delta q(t) \qquad (5)$$

$$R_0 = \frac{q_0}{D} + T_p \qquad (6)$$

The state vector, the output vector and the input vector are introduced as follows:

$$x(t) = \begin{bmatrix} \delta W(t) \\ \delta q(t) \end{bmatrix}$$

$$y(t) = \delta q(t)$$

$$u(t) = \delta p(t)$$

The TCP/AQM networks (4) and (5) can be described as the following linear system with information time-delay (Azuma *et al.*, 2005a, b, c):

$$\dot{x}(t) = Ax(t) + Bu(t - R_0) \qquad (7)$$

$$y(t) = Cx(t) \qquad (8)$$

where,

$$A = \begin{bmatrix} -\dfrac{2N}{R_0^2 D} & 0 \\ \dfrac{N}{R_0} & -\dfrac{1}{R_0} \end{bmatrix}, \quad B = \begin{bmatrix} -\dfrac{R_0 D^2}{2N^2} \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$P(s)$ is defined by:

$$P(s) = \frac{\dfrac{D^2}{2N}}{(s + \dfrac{2N}{R_0^2 D})(s + \dfrac{1}{R_0})} \qquad (9)$$

$$\dot{x}(t) = Ax(t) + Bu(t - R_0) + d(t) \qquad (10)$$

$$y(t) = Cx(t) + d_0(t) \qquad (11)$$

The reference model is given (12) and (13):

$$\dot{x}_m(t) = A_m x_m(t) + B_m r_m(t) \qquad (12)$$

$$y_m(t) = C_m x_m(t) \qquad (13)$$

In this study, we propose a design of model following control system with disturbances. We can proof that all the internal states are bounded and output error:

$$e(t) = y(t) - y_m(t) \tag{14}$$

Converges to zero asymptotically.

STATE PREDICTIVE CONTROL OF THE NETWORK CONGESTION SYSTEM WITH TIME DELAY

In this study, state predictive control techniques (Wang *et al.*, 2009b; Jalili-Kharaajoo and Araabi, 2004) are applied to congestion control synthesis problems for TCP/AQM network. State predictive control uses directly the time delay for input time delay systems and the input time delay systems can be transformed a linear time invariant systems by using a variable transformation. Thus it is easy to design controllers with enough stability and performance based on the linear control theory.

In this section, congestion controllers are designed by using MFCS state predictive control to satisfy the control object defined in the end of the previous section:

Let $p = d/dt$ and $\sigma = e^{pR_0}$, $\sigma u(t) = u(t - R_0)$.

From this, system (10) and (11) are given in system (15) and (16):

$$\dot{x}(t) = Ax(t) + B(\sigma)u(t) + d(t) \tag{15}$$

$$y(t) = Cx(t) + d_0(t) \tag{16}$$

where, $B(\sigma) = B\sigma$. Then the transfer function of controlled system and model are as follows:

$$C[pI - A]^{-1}B(\sigma) = N(\sigma, p)/D(p)$$
$$C_m[pI - A_m]^{-1}B_m = N_m(p)/D_m(p)$$

Here, $D(p) = |pI - A|$ and $D_m(p) = |pI - A_m|$
Then the representations of input-output equation are described as follows:

$$D(p)y(t) = N(p)u(t) + \omega(t) \tag{17}$$

$$\omega(t) = Cadj\ [pI - A]d(t) + D(p)d_0(t) \tag{18}$$

$$D_m(p)y_m(t) = N_m(p)r_m(t) \tag{19}$$

Define N (p), N_m(p) as:

$$N(p) = Cadj\ [pI - A]B(\sigma) \tag{20}$$

$$N_m(p) = C_m adj\ [pI - A_m]B_m \tag{21}$$

$$N_r(\sigma) = \overline{N}_r(\sigma) + \hat{N}_r \tag{22}$$

where, $\partial_{ri}\tilde{N}(p) < \eta_i$, $\partial_{ri}\tilde{N}_m(p) < \eta_{mi}$ $(i = 0,1,\cdots,l)$ is assumed to be regular. The disturbances $d(t)$ and $d_0(t)$ are satisfy the following conditions:

$$D_d(p)d(t) = 0, \ D_d(p)d_0(t) = 0 \tag{23}$$

where, $D_d(p)$ is a scalar characteristic polynomial of disturbances, $\partial D_d(p) = n_d$ and $D_d(p)$ is monic polynomial, thus $D_d(p)w(t) = 0$. Choose a stable polynomial T(p) which satisfy the following condition:

Degree of $T(p)$ is $\delta \geq n_d + 2n - n_m - 1 - \eta_i$

The coefficient of maximum degree term of $T(p)$ is the same as D(p). R(p) and S(p) can be obtained:

$$T(p)D_m(p) = D_d(p)D(p)R(p) + S(p) \tag{24}$$

where, degree of each term as the following:

$$\partial T(p) = \delta, \ \partial D_m(p) = n_m, \ \partial D_d(p) = n_d, \ \partial D(p) = n$$
$$\partial R(p) = \delta + n_m - n_d - n, \ \partial S(p) \leq n_d + n - 1$$

So, from (14), (17), (19) and (24), we can get easily:

$$T(p)D_m(p)e(t) = D_d(p)D(p)R(p)y(t)$$
$$+ S(p)y(t) - T(p)N_m(p)r_m(t) \tag{25}$$

We can rewrite $e(t)$ as:

$$e(t) = \{1/[T(p)D_m(p)]\}\{[D_d(p)R(p)N(p)$$
$$- Q(p)N_r(\sigma)]u(t) + Q(p)N_r(\sigma)u(t)$$
$$+ S(p)y(t) - T(p)N_m(p)r_m(t)\} \tag{26}$$

and $Q(p) = diag(p^{\delta + n_m - n + \eta_i}) + \tilde{Q}(p)$

$$\partial_{r_i}\widetilde{Q}(p) < \delta + n_m - n + \eta_i$$

So, $u(t)$ can be obtained by letting the right hand side of (26) be equal to zero:

$$u(t) = -\hat{N}_r^{-1}(\sigma)\bar{N}_r u(t)$$

$$-\hat{N}_r^{-1}(\sigma)Q^{-1}(p)\{D_d(p)R(p)N(p)$$

$$-Q(p)N_r(\sigma)\}u(t)$$

$$-\hat{N}_r^{-1}(\sigma)Q^{-1}(p)S(p)y(t)$$

$$+\hat{N}_r^{-1}(\sigma)Q^{-1}(p)T(p)N_m(p)r_m(t) \quad (27)$$

where, $i = 1, 2, \cdots, l$ and $|\hat{N}_r| \neq 0$, $\eta_i \geq \eta_{fi}$, $n_m - \eta_{mi} \geq n - \eta_i$. Control input $u(t)$ can be described using internal states as followings:

$$u(t) = -E_0 u(t) - H_1\xi_1(t) + E_2(\sigma)y(t)$$
$$- H_2\xi_2(t) + u_m(t) \quad (28)$$

$$u_m(t) = E_3\hat{r}_m(t) + H_3\xi_3(t) \quad (29)$$

Here,

$$E_0(\sigma) = \hat{N}_r^{-1}(\sigma)\bar{N}_r \quad (30)$$

$$H_1[pI - F_1]^{-1}G_1 = \hat{N}_r^{-1}Q^{-1}(p)\{D_d(p) \quad (31)$$
$$\cdot R(p)N(p) - Q(p)N_r\}$$

$$E_2(\sigma) + H_2[pI - F_2]^{-1}G_2 \quad (32)$$
$$= \hat{N}_r^{-1}(\sigma)Q^{-1}(p)S(p)$$

$$E_3 + H_3[pI - F_3]^{-1}G_3 \quad (33)$$
$$= \hat{N}_r^{-1}Q^{-1}(p)T(p)N_m(p)$$

The followings must be satisfied:

$$\dot{\xi}_1(t) = F_1\xi_1(t) + G_1 u(t) \quad (34)$$

$$\dot{\xi}_2(t) = F_2\xi_2(t) + G_2 y(t) \quad (35)$$

$$\dot{\xi}_3(t) = F_3\xi_3(t) + G_3 r_m(t) \quad (36)$$

where, $|pI - F_i| = |Q(p)|, (i = 1, 2, 3)$. If the internal states of control system are bounded, the design of MFCS for nonlinear system can be realized.

BOUNDED PROPERTY OF CONTROL SYSTEM INTERNAL STATES

System inputs are reference input signal $r_m(t)$ and disturbances $d(t), d_0(t)$ which are all assumed to be bounded.

First, the overall system can represented by state space in below:

$$\frac{d}{dt}\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ \xi_1(t) \\ \xi_2(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A - B(\sigma)E_2 C & -B(\sigma)H_1 \\ -G_1 E_2(\sigma)C & F_1 - G_1 H_1 \\ -G_2 C & 0 \\ -E_2(\sigma)C & -H_1 \end{bmatrix}$$

$$\begin{bmatrix} -B(\sigma)H_2 & -B(\sigma)E_0(\sigma) \\ -G_1 H_2 & -G_1 E_0(\sigma) \\ F_2 & 0 \\ -H_2 & -I + E_0(\sigma) \end{bmatrix}\begin{bmatrix} x(t) \\ \xi_1(t) \\ \xi_2(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} B(\sigma) \\ G_1 \\ 0 \\ I \end{bmatrix}u_m(t)$$

$$+ \begin{bmatrix} d(t) - B(\sigma)E_2(\sigma)d_0(t) \\ -G_1 E_2(\sigma)d_0(t) \\ G_2 d_0(t) \\ -E_2(\sigma)d_0(t) \end{bmatrix} \quad (37)$$

where, $z^T(t) = \begin{bmatrix} x(t) & \xi_1(t) & \xi_2(t) & u(t) \end{bmatrix}^T$.

Necessary part about bounded property is considered, state space can be simplified as:

$$E\dot{z}_s(t) = A_s(\sigma)z_s(t) + d_s(t) \quad (38)$$

$$y(t) = Cx(t) + d_0(t) \quad (39)$$

$$|pE - A_s(\sigma)| = |\hat{N}_r|^{-1} T(p)^l \quad (40)$$
$$\cdot D_m(p)^l |Q(p)|^2 V_s(\sigma, p)$$

The $Z_S(t)$ is bounded. The above result is summarized as follows:

For the nonlinear system of (10) and (11), using the method in this study to design model following control system, $A \in R^{n \times n}$. $d(t)$ and $d_0(t)$ which are all assumed to be bounded, $A(\sigma)$ is a stable system matrix.

$$E\dot{z}_s(t) = A_s(\sigma)z_s(t) + d_s(t)$$

all internal states are bounded if:

- $A_s(\sigma) \in R^{n \times n}$ is bounded
- $d(t), d_0(t)$ is a stable system matrix

SIMULATION RESULTS

System inputs are reference input signal $r_m(t)$ which is assumed to be bounded.

From Section 4, the control law $u(t)$ of the system can be obtained as:

$$u(t) = -E_0 u(t) - H_1 \xi_1(t)$$
$$+ E_2(\sigma)y(t) - H_2 \xi_2(t) + u_m(t)$$

$$u_m(t) = E_3 r_m(t) + H_3 \xi_3(t)$$

The network topology is simple and is shown in Fig. 3. But this small scale network has been developed by using four computers and some experiments have been done. This network consists of two senders with four data flows (this means four sessions and $N = 8$), one bottleneck router and one receiver, which receives data from senders through the bottleneck router. The reference queue size is $\bar{q}_0 = 20$ [packets]. Then we have:
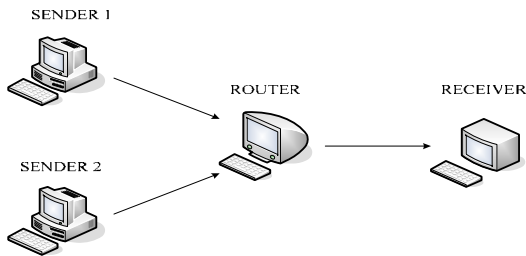

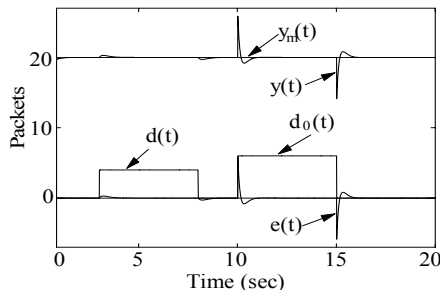Fig. 3: Test bed network topology

Fig. 4: Responses of the congestion system with white noises

$$\dot{x}(t) = \begin{bmatrix} -0.88 & 0 \\ 15 & 2 \end{bmatrix} x(t) + \begin{bmatrix} -18 \\ 0 \end{bmatrix} u(t - R_0)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

In this example, when the white noises $d(t)$ and $d_0(t)$ enter, the design of the congestion control system with time-delay is effective. The white noises are given as follows:

$$d(t) = 4, (2.5 \le t \le 8)$$

$$d_0(t) = 6, (10 \le t \le 15)$$

In the responses (Fig. 4) of the congestion system with white noises, the output errors converge to zero. The effectiveness of this method has thus been verified.

CONCLUSION

This study studied a control system for state predictive with time-delay with a model following method which was one of the effective means of solving time-delay problems in a control system. The method could efficaciously control time-delay under disturbances and had excellent practicability.

By using state predictive control, the linear input time-delay system which describes a TCP/AQM network is transformed into an equivalent linear time invariant system and it has been possible to design controllers based on linear control theories. For a congestion control problem, a round packet trip time is not stationary and depends on the actual traffic. From simulation results, it was verified that the congestion controller using the proposed approach is better.

ACKNOWLEDGMENT

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